

Credit volatility : Options and beyond

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1 Agenda

- CDS options
- Index options
 - payoff specificity
 - back to Black
 - market
- Exotic vol products & the need for new models

2 What is a CDS option ?

- warning : no market on single-name CDS options !
 - except a few risky names (autos, etc.)
- right to enter at exercise date T into a CDS at a given spread k (European)
- payer option = right to pay k against protection
 - a.k.a. call on protection, or put on risk
- knock-out in case of default before exercise

3 Underlying CDS

- $0 = T_0 < T_1 < \dots < T_n$ (quarterly payment dates)
- Assume exercise at $T = T_1 = 3m$ in what follows
- RBP = *Risky* Basis Point value, a.k.a. *Risky* DVO1 or *Risky* Duration
- RBP_t is the present value at t of 1bp paid between T and T_n :

$$RBP_t = \sum_{i=2}^n (T_i - T_{i-1}) RZC_{t,T_i}$$

where $RZC_{t,u}$ is the value at t of a *risky* zero-coupon with maturity u

- if S_T is the market spread at T , the MtM for a CDS with spread k is:

$$MtM_T = (S_T - k) RBP_T$$

4 Option payoff: credit Vs. rates

- option payoff at T :

$$MtM_T^+ = (S_T - k)^+ RBP_T$$

where $x^+ = \max(0, x)$

- reminds of Interest-Rate swaptions
- natural numeraire RBP_t knocks out upon default \Rightarrow not $> 0!$
- but change of probability still works as the option also knocks out upon default

5 Reminder on numeraires

- a *numeraire* is simply a price process
- a positive numeraire $\mathcal{N} > 0$ can be used to define a *pricing* probability $\mathbf{Q}^{\mathcal{N}}$
 - e.g. if $\mathcal{N} =$ bank account then $\mathbf{Q}^{\mathcal{N}} = \mathbf{Q} =$ risk-neutral probability
 - then an asset price X expressed in units of numeraire \mathcal{N} is a $\mathbf{Q}^{\mathcal{N}}$ –martingale
- if $\bar{\mathcal{N}} \geq 0$ is a numeraire, can still define new proba $\mathbf{Q}^{\bar{\mathcal{N}}}$ by:

$$\frac{d\mathbf{Q}^{\bar{\mathcal{N}}}}{d\mathbf{Q}^{\mathcal{N}}} = \frac{\bar{\mathcal{N}}_T}{\bar{\mathcal{N}}_0} \cdot \frac{\mathcal{N}_0}{\mathcal{N}_T}$$

- can prove that $X/\bar{\mathcal{N}}$ is a $\mathbf{Q}^{\bar{\mathcal{N}}}$ –martingale, IF $X = 0$ when $\bar{\mathcal{N}} = 0$!

6 Welcome to the survival world

- use $\bar{\mathcal{N}} := RBP \geq 0$ as numeraire
- define *survival* probability $\bar{\mathbb{Q}} := \mathbb{Q}^{RBP}$ (Schönbucher)
- option payoff knocks-out upon default
 \Rightarrow option price can be computed as expectation under $\bar{\mathbb{Q}}$
- option price Π_0 at 0 now involves equity-like option calculations:

$$\Pi_0 := RBP_0 \cdot \mathbf{E}^{\bar{\mathbb{Q}}} \left[(S_T - k)^+ \right]$$

- need distribution for S_T under $\bar{\mathbb{Q}}$

7 (Forward) spread = martingale

- S_T seen as the T -value of the spread $S_{t,T}$ of a *forward* CDS starting at T
- MtM of this forward CDS is given by:

$$MtM_t = (S_{t,T} - k) RBP_t$$

- MtM_t is a price process and knocks out upon default
 - ⇒ it is a $\bar{\mathbf{Q}}$ -martingale when expressed in units of numeraire
 - ⇒ spread $S_{t,T}$ is a $\bar{\mathbf{Q}}$ -martingale
 - ⇒ can assume *driftless* lognormal dynamics:

$$\frac{dS_{t,T}}{S_{t,T}} = \sigma dW_t$$

8 Back to Black

- we are left with a call on a lognormal underlying !
- therefore we get a Black formula:

$$\Pi_0 = RBP_0 \cdot [S_{0,T}N(d_+) - kN(d_-)]$$

with the usual:

$$d_{\pm} = \frac{\ln\left(\frac{S_{0,T}}{k}\right) \pm \frac{1}{2}\sigma^2T}{\sigma\sqrt{T}}$$

9 Reminder on the credit index

- $0 = T_0 < T_1 < \dots < T_n$: quarterly index dates ($T_0 = \text{roll}$)
- $T = \text{index trade date}$ (for an option T will be the exercise date)
- $S_T = \text{quoted "market" spread}$
- $C = \text{contractual spread}$: *all* index trades pay contractual spread C instead of "market" spread \Rightarrow need to pay/receive some upfront amount
- Index Upfront defined from the the pseudo "market" spread:

$$(S_T - C) \cdot \underbrace{FRBP_T(S_T)}_{\text{flat RBP} = ???} \cdot \underbrace{N_T}_{\text{index notional}}$$

10 Flat RBP

- Single-name CDS naturally involves RBP
- Index contract introduces *Flat* RBP
- $FRBP_t(S)$ is the present value at t of 1bp paid between T and T_n

$$FRBP_t(S) = \sum_{i=2}^n (T_i - T_{i-1}) RZC_{t,T_i}(S)$$

where the risky ZC is computed assuming a *flat* spread curve at S

- examples : usual vs. flat, with curves as of Feb 08
 - iTraxx Main 5y : 3.64 Vs. 3.63
 - iTraxx XO 5y : 3.15 Vs. 3.11

11 Index trade example

- assume contractual spread at $C = 500$ bps for iTraxx XO 5y
- today ($T = 20$ Feb 08), trader quotes index at 450/455 bps
- client wants to sell protection on \$10m
 - he should receive 450 bps
 - in practice, he will receive 500bps and pay upfront for the 50bps difference an amount equal to:

$$(50 \cdot 10^{-4}) \times 3.11 \times 10^7 = \$155,500$$

12 Index option payoff

- European option with exercise date T
 - e.g. $T = T_1 = 3\text{m IMM}$, $T = T_2 = 6\text{m IMM}$, or longer ?
- on exercise of a payer option with strike k :
 1. sell risk on physical index contract at C
 2. receive Option Upfront $(C - k) \cdot FRBP_T(k) \cdot N_T$
 3. receive losses L_T (unlike single-name options)
- MtM of physical index contract = index upfront = $(S_T - C) \cdot FRBP_T(S_T) \cdot N_T$

13 Which index spread in the payoff ?

- final payoff involves 2 Flat RBP terms:

$$[(S_T - C) \cdot FRBP_T(S_T) \cdot N_T - (k - C) \cdot FRBP_T(k) \cdot N_T + L_T]^+$$

- can introduce CDS-like spread S^* , satisfying :

$$(S_T - C) \cdot FRBP_T(S_T) \cdot N_T = (S_T^* - C) \cdot RBP_T^*$$

where RBP_T^* depends on the curve of CDS-like spreads up to T_n

- can bootstrap curve of CDS-like spreads from quoted "market" spreads
- can rewrite payoff with CDS-like quantities

14 Which numeraire ?

- Can use the *forward* CDS-like RBP
- No natural definition for the *forward* flat RBP
 - can introduce a definition making it a martingale (when discounted)
 - ... but can only approximate this definition

15 Armageddon?

- payoff should be 0 when numeraire is 0, i.e. all names default
- Could split payoff as sum of
 - term that becomes 0 upon Armageddon event
 - maximum index loss
- Otherwise neglect this event:

$$L_T \simeq L_T \mathbf{1}_{N_T=0}$$

16 How to incorporate the loss term ?

- successive approximations yield a Black-like formula
- semi-analytical
 - build loss distribution *conditional* on spread
 - use *adjusted* spread that pays running the realised losses (Pedersen)

17 On the road to Black

- To get analytical formula, keep only $FRBP_T(S_T)$ in payoff
- Approximation ($S_{t,T}$ forward spread at t of index entered at T)

$$\underbrace{\frac{FRBP_T(k)}{FRBP_T(S_T)}}_{\text{Flat RBPs}} \simeq \underbrace{\frac{FRBP_0(k)}{FRBP_0(S_{0,T})}}_{\text{forward Flat RBPs}} =: \alpha$$

- seen from 0, no bias on the payoff upon exercise
- for the 5Y Crossover S8: $\alpha = 110.6\%$ for $k = 325\text{bps}$ and $T = \text{jun-08}$

18 Revisited payoff

- We can approximately rewrite the payoff as:

$$\left[(S_T - K) \cdot \underbrace{FRBP_T(S_T) \cdot N_T}_{A_T} + L_T \right]^+$$

with an adjusted strike $K = \alpha k - (\alpha - 1)C$

19 Towards a Black formula ?

- *without* the loss term, back to payoff of single-name option:

$$(S_T - K)^+ A_T$$

- spread $S_{t,T}$ is martingale under proba associated to numeraire A_T
- *assume* lognormal diffusion for spread
- get Black formula as for single-name option

20 Adjusted spread / adjusted forward

- *standard* forward given by usual no-arbitrage argument:
 - (fee leg spot $0 \rightarrow T$) + (fee leg fwd $T \rightarrow T_n$) = (fee leg long $0 \rightarrow T_n$)
- *adjusted* forward = money of the option = ATM spread
 - let $\tilde{S}_{t,T}$ be the adjusted forward
 - let $\tilde{S}_T := \tilde{S}_{T,T}$ be the adjusted spread at T
 - we want:

$$(S_T - \tilde{S}_T) A_T + L_T = 0$$

- concept only makes sense in option context, due to odd loss term in payoff

21 Back to Black again

- clearly the adjusted spread depends on:
 - standard spread
 - model for loss term

$$\tilde{S}_T = S_T + \frac{1}{A_T} L_T$$

- rewrite payoff:

$$\left(\tilde{S}_T - K\right)^+ A_T$$

- now get Black formula by *assuming* that *adjusted* spread is lognormal
- how do we model the adjustment, i.e. the loss term ?

22 Loss term : Naive approach

- approximate the *random* loss by the *expected* loss

$$L_T \simeq \mathbf{E}[L_T] = (\text{spot spread with maturity } T) \times (\text{RBP between 0 and } T)$$

- Reminder: T is the exercise date, typically $T = T_1 = 3m$ or $T = T_2 = 6m$
- Example : for a 6m option on XO 5y,
 - $\mathbf{E}[L_T] \simeq 4.8\% \times 0.5 = 2.4\%$
 - $A_T \simeq 3.15$
 - spread adjustment $\simeq 0.76\%$

23 Loss term : Conditional loss approach

- loss correlated with spread \implies expect high loss when payer option is exercised
- use expected loss *conditional* on spread realised at exercise:
$$L_T \simeq (\text{forward spd starting at } T \text{ with maturity } T + T) \times (\text{RBP between } 0 \text{ and } T)$$
- issue : we model only the *forward* spread starting at T with maturity date T_n
- get the short forward spread from the (modelled) long forward spread by assuming either of spread curve translation or homothecy
- expected spread adjustment (pure translation) similar to naive (current curve almost flat)

24 Homogeneous Pool with Stochastic Intensity

1. assume:

- (a) homogeneous pool with common spread/intensity dynamics (e.g. mean-reverting)
- (b) default independence conditionally on spread

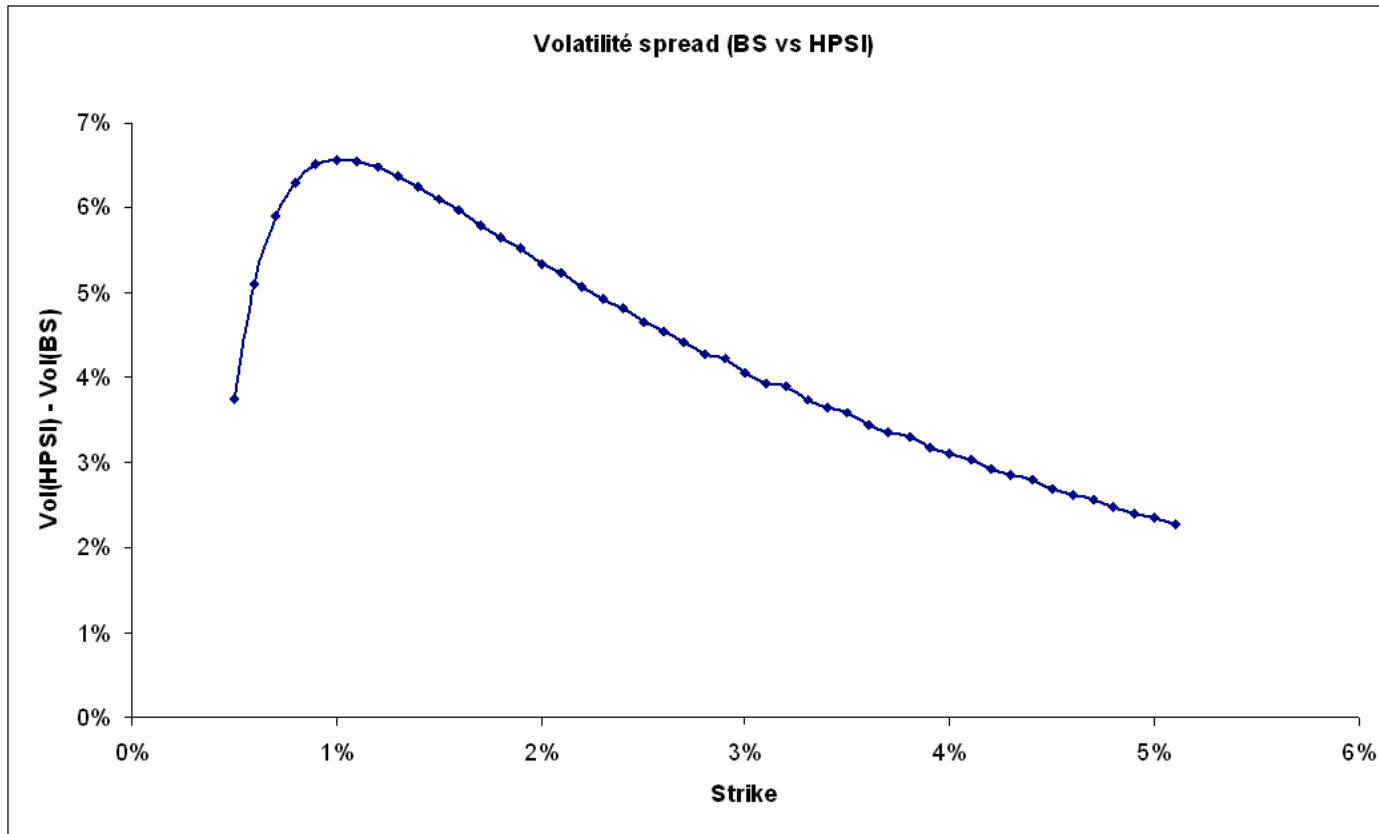
2. conditionally on spread:

- (a) compute loss distribution
- (b) compute option premium

3. integrate over spread distribution

→ quite appropriate, but cannot become market standard...

25 HPSI : smile impact



26 Market

- liquidity keeps improving since late 2006
- option features:
 - underlying: iTraxx Main less liquid than iTraxx XO
 - payers, receivers, straddles
 - maturities : 3m, 6m at roll date (if longer than 6m, series will be off-the-run on exercise date)
 - strikes : market practice (ad hoc)

27 Bloomberg runs

Europe S8	Ref=127.5					
Strike	MAR-08	100	105	110	115	120
Volatility		91%/97%	92%/99%	93%/100%	94%/101%	95%/102%
Payer		149/152	131/134	115/118	99/104	86/90
Receiver		4/7	7/10	11/15	16/20	24/27
Straddle		154/159	139/145	126/133	116/124	110/117
Forward=135.52						

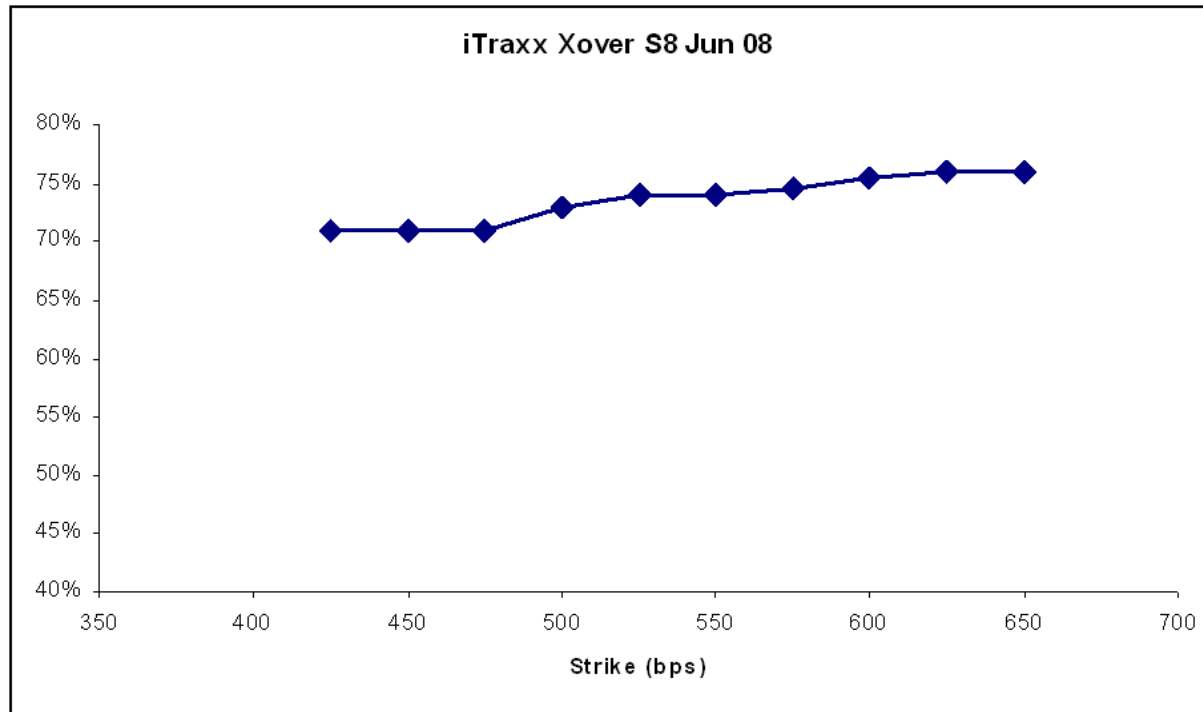
Strike	MAR-08	125	130	135	140	145
Volatility		109%/116%	111%/118%	112%/119%	114%/122%	-
Payer		75/78	65/68	56/59	48/52	-
Receiver		33/36	43/46	53/57	66/70	-
Straddle		108/114	107/114	109/116	114/121	-
Forward=135.52						

Xover S8	Ref=592					
Strike	JUN-08	425	450	475	500	525
Volatility		70%/72%	70%/72%	70%/72%	72%/74%	73%/75%
Payer		829/833	760/765	696/702	643/650	593/600
Receiver		39/43	54/59	73/78	101/107	129/136
Straddle		868/876	815/824	769/780	744/756	722/736
Forward=674.61						

Strike	JUN-08	550	575	600	625	650
Volatility		73%/75%	73%/76%	74%/77%	75%/77%	75%/77%
Payer		544/551	498/511	461/474	428/437	394/403
Receiver		158/165	189/201	227/239	267/275	304/313
Straddle		702/717	687/712	688/713	695/713	698/717
Forward=674.61						

28 Market smile

Relatively flat as of 3 march 2008:



29 Exotic products

- non standard index options:
 - long-dated maturities: require extrapolation of the volatility term structure
 - no loss term in payoff
 - no FRBP term in payoff → "pure" option on the index spread
- spread corridor (range accrual on spread), etc.

30 models reproducing smile and/or term structure

- **CEV**

$$\frac{dS_{t,T}}{S_t^\beta} = \sigma dW_t$$

- **Variance Gamma**
- **HPSI** with more appropriate spread/intensity dynamics:
 - CEV
 - stochastic volatility
- **SABR**