Monte Carlo Computation of Small Loss Probabilities

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joint work with **Stéphane Crépey**

Evry, June 26, 2008

Credit Portfolios Small Probabilities

- $\{L(t)\}_{t \geq 0}$ loss process for a credit portfolio
- Typically

$$L(t) = V(\mathbf{X}_t)$$

where

  - $\mathbf{X}_t = (X_t^1, \cdots, X_t^d)$ is a Markov process
  - $V : \mathbf{x} \hookrightarrow V(\mathbf{x})$ deterministic

Typical Computations

- $\mathbb{P}\{L(T) = x\}$ for x large
- $\mathbb{P}\{L(T_2) = x_2 | L(T_1) = x_1\}$ for $T_1 < T_2$ and $x_1 < x_2$

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Three Related Papaers

- **P. Del Moral and J. Garnier** Genealogical particle analysis of rare events. *Annals of Applied Probability,* 15(4):2496–2534, November 2005.
- **R. Frey, and J. Backhaus** Pricing and Hedging of Portfolio Credit Derivatives with Interacting Default Intensities. *Working Paper*, 2007.
- **D. Vestal, R. Carmona and J.-P.Fouque**. Interacting Particle Systems for the Computation of CDO Tranche Spreads with Rare Defaults. *Submitted*.

IPS Algorithm Implementation

As in **Del Moral - Garnier**

Choice of the potential function for **mutation** stage

$$\begin{aligned} G_i^\alpha(y_i) &= G_i^\alpha(x_0, x_1, \cdots, x_i) \\ &= = G^\alpha(x_{i-1}, x_i) \\ &= e^{\alpha(V(x_i) - V(x_{i-1}))} \end{aligned}$$

for a free parameter α

- **Importance Sampling**
  - Use Girsanov to *twist* the distribution in path space

$$I = \mathbb{E}^{\mathbb{Q}}\left\{ f(X_T)\exp[\int_0^T \nabla h(X_s)dW_s + \frac{1}{2}\int_0^T |\nabla h(X_s)|^2 ds]\right\}$$

  - Generate Monte Carlo samples from twisted distribution $\mathbb{Q}$ and compute

$$I \approx \frac{1}{M}\sum_{m=1}^{M} f(X_T(\omega_m))\exp[\int_0^T \nabla h(X_s)ds + \frac{1}{2}\int_0^T |\nabla h(X_s)|^2 ds](\omega_m)$$

- **Interacting Particles System**
  - Use Feynman-Kac *twisted distributions*

$$I = \mathbb{E}^{\mathbb{Q}}\{f(X_T)e^{\int_0^T V(X_s)ds}\} \qquad \text{with} \qquad \frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{e^{-\int_0^T V(X_s)ds}}{\mathbb{E}^{\mathbb{P}}\{e^{-\int_0^T V(X_s)ds}\}}$$

  - Generate **ALL** Monte Carlo samples from **ORIGINAL** distribution $\mathbb{P}$

IS versus IPS to Compute $I = \mathbb{E}^{\mathbb{P}}\{f(X_T)\}$

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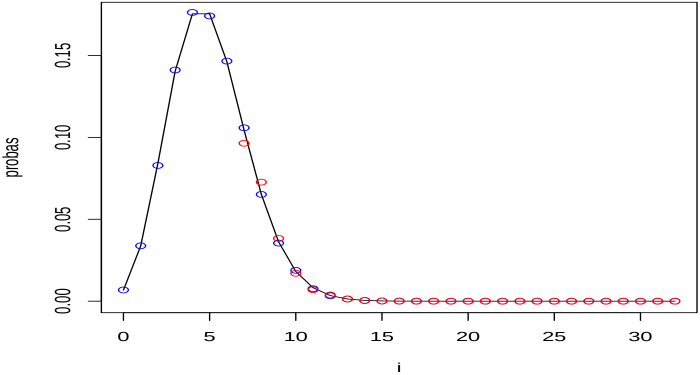


Standard Monte Carlo versus IPS approach.

Numerics

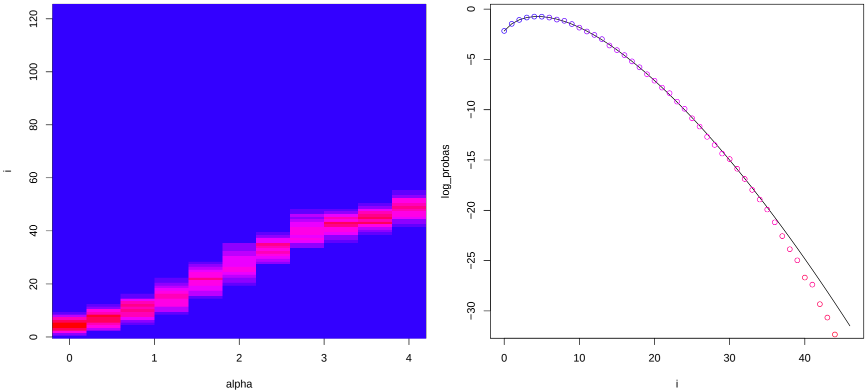
i	$p(i)$	$MC(i)$	$\alpha(i)$	$IPS(i)$	$errIPS(i)$
0	6.737947e-03	6.850000e-03	0	–	1.6630140
1	3.368973e-02	3.381000e-02	0	–	0.3569782
2	8.422434e-02	8.287000e-02	0	–	1.6080120
3	1.403739e-01	1.411600e-01	0	–	0.5600074
4	1.754674e-01	1.762900e-01	0	–	0.4688223
5	1.754674e-01	1.741600e-01	0	–	0.7450786
6	1.462228e-01	1.465800e-01	0	–	0.2442792
7	1.044449e-01	0.10586	1	9.642260e-02	7.6808635
8	6.527804e-02	0.06519	1	7.269477e-02	11.3617485
9	3.626558e-02	0.03548	1	3.832182e-02	5.6699651
10	1.813279e-02	0.01884	2	1.706726e-02	5.8762642
11	8.242177e-03	0.00761	2	7.025229e-03	14.7648865
12	3.434240e-03	0.00331	2	3.798202e-03	10.5980232
13	1.320862e-03	0.00135	2	1.301592e-03	1.4588951
14	4.717363e-04	0.00039	2	4.948383e-04	4.8972237
15	1.572454e-04	0.00014	3	1.315650e-04	16.3314126

Computation of $\mathbb{P}\{N = i\}$.

More Numerics

16	4.913920e-05	0.00007	3	5.158852e-05	4.9844475
17	1.445271e-05	0.00004	3	1.453860e-05	0.5943091
18	4.014640e-06	0	3	3.570278e-06	11.0685371
19	1.056484e-06	0	3	9.457888e-07	10.4777238
20	2.641211e-07	0	3	2.389309e-07	9.5373474
21	6.288597e-08	0	4	6.775490e-08	7.7424802
22	1.429227e-08	0	4	1.578648e-08	10.4547036
23	3.107014e-09	0	4	3.165438e-09	1.8803674
24	6.472947e-10	0	4	6.513996e-10	0.6341628
25	1.294589e-10	0	4	1.149943e-10	11.1731226
26	2.489595e-11	0	4	2.563031e-11	2.9497091
27	4.610361e-12	0	4	2.805776e-12	39.1419521
28	8.232787e-13	0	4	6.662209e-13	19.0771085
29	1.419446e-13	0	5	1.453048e-13	2.3672480
30	2.365743e-14	0	5	2.589671e-14	9.4654196
31	3.815715e-15	0	5	4.039209e-15	5.8571942
32	5.962055e-16	0	5	4.274102e-16	28.3115984
33	9.033417e-17	0	5	5.303823e-17	41.2866355
34	1.328444e-17	0	6	1.845935e-18	86.1045282

Graphics



Losses as function of α (left), and log-probabilibities (right).

Typical example

- $\tilde{X} = (\tilde{X}^1, \cdots, \tilde{X}^d)$ continuous-time d-variate Markov Chain
- Components $\tilde{X}^i_t$ in $\{0, 1, \cdots, \nu\}$ for some fixed integer ν.
- Exclude simultaneous jumps of the $\tilde{X}^i$'s.
- $\tilde{X}_0 = 0 = (0, \cdots, 0)$.
- Particular case $d = 1$ class of $\nu = 125$ names
- Generator matrix is $(\nu + 1)^d \times (\nu + 1)^d$ **TOO LARGE**!

$$(\text{e.g. } (5 + 1)^{25} = 2.843029e + 19)$$

- **Local intensity model** in a pure **Top-Down Approach**
- L is modeled as a Markov point process stopped at level $\nu = 125$
- **Pure birth process** with *local intensity* $\lambda(t, L_t)$
- **Infinitesimal generator**

$$\mathcal{A}_t = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & 0 & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & 0 & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

One Single Group $d = 1$

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One Single Group $d = 1$ (cont.)

- $L_0 = 0$
- L jumps by one at some (increasing) $(0, +\infty)$-valued random times $\tilde{t}_i$
- Loss probabilities $p_i(t) = \mathbb{P}\{L(t) = i\}$, satisfy **forward Kolmogorov equation** (system of ODEs)

$$(\partial_t - \mathcal{A}_t^*)p = 0 \text{ on } (0, T] \qquad p(0) = \delta_0,$$

  $\mathcal{A}^*$ represents the adjoint (transpose) of $\mathcal{A}$
- For (time) homogeneous case

$$p(t) = \exp(t\mathcal{A}^*)\delta_0$$

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Re-Worked Set-up

- $T > 0$ fixed maturity (5yr)
- $\tilde{t}_i$ time of the i^{th} jump of $\tilde{X}$, ($\tilde{t}_0 = 0$)
- Loss process $L_t = V(\tilde{X}_t) = \tilde{X}_t^1 + \cdots + \tilde{X}_t^d$

Introduce

$$n = \inf\{i \in \mathbb{N}\,;\, \tilde{t}_i \geq T\}\ .$$

and assume n is finite almost surely.

Set, for $i \in \mathbb{N}$,

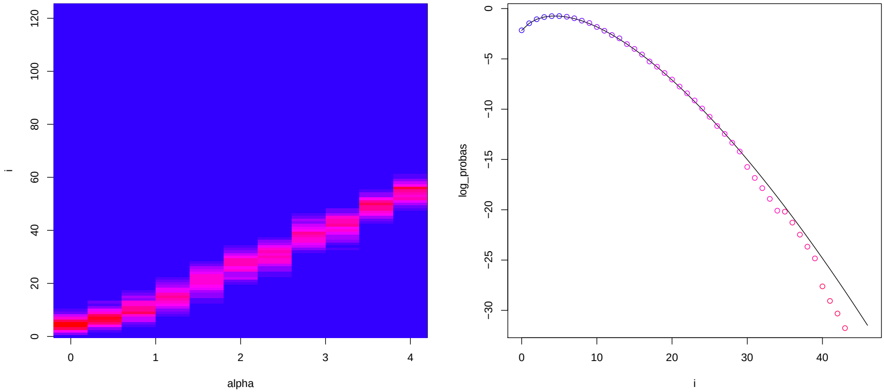
$$t_i = \tilde{t}_{i \wedge n} \wedge T \qquad X_i = \tilde{X}_{t_i}\ .$$

So $t_i = T$ iff $i \geq n$, and for $i \geq n$ we have that $X_i = \tilde{X}_T$ and $V(X_i) = L_T$.

T	$n = d \times \nu$	$\lambda^l(t, \iota)$	μ	α
5y	$125 = 5 \times 25$	$\frac{\nu - i_l}{n}$	11	0.4

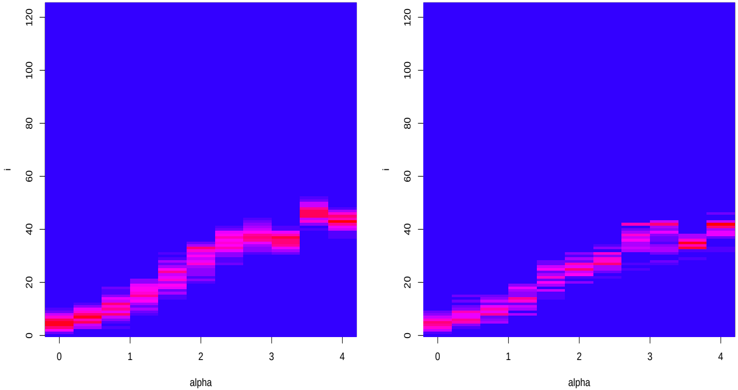
Parameter Values.

IPS Losses



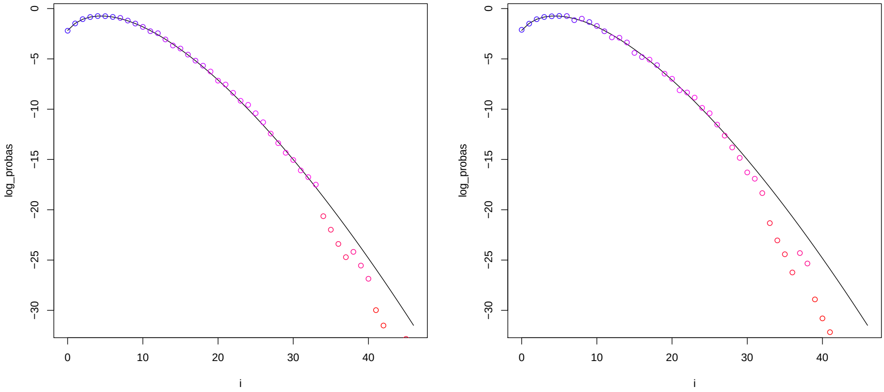
Losses as function of α (left) and log-probabilities (right) for $m = 10^5$.

IPS Losses (cont.)



Losses as function of α for $m = 20000$ (left) and $m = 5000$ (right) samples.

IPS Losses (cont.)



Corresponding log-probabilities ($m = 20000, 5000$).

Simple Importance Sampling Twist

Simple Twist Transformation:

Speed Up Arrivals equivalently **Scale Up the Intensity** by α

- Use (simple form of) Girsanov

$$I = \mathbb{E}^{\mathbb{Q}^\alpha}\left\{ f(X_T)\exp[(\alpha-1)L(T) - (\alpha-1)\int_0^T \lambda(L_s)ds]\right\}$$

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- Analogous formula for the Homogeneous Group Model

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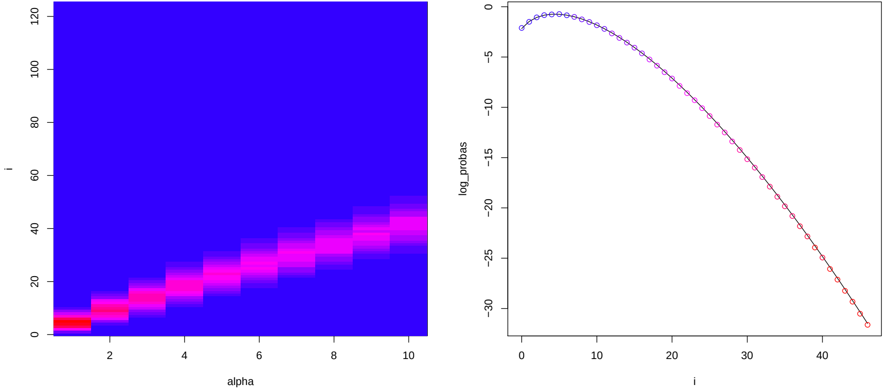
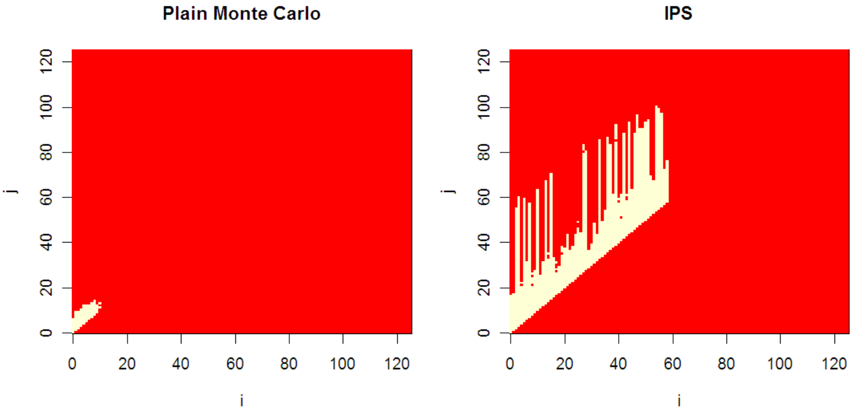
Explicit IS



Explicit IS Losses and Values ($m = 5000$).

IPS Computation of $\mathbb{P}\{L(T_2) = j | L(T_1) = i\}$



Range of (i, j) for which Plain MC (left) and IPS (right) can compute with $m = 5000$ simulated paths

Conclusions

In order to compute small probabilities (e.g. for credit portfolios)

- **Simple MC explicit IP can do wonders when available**
- **When is IPS Competitif?**
  - When explicit/deterministic methods are prohibitive
- **When is IPS a must?**
  - When no obvious **Girsanov** change of measure is explicit
  - When MC simulations are based on a **Black Box** one cannot open

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