How the corporate liquidity process affects the value of the firm

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Abstract

We study the simplest discrete-time finite-maturity model in which default arises when the firm is not able to pay its debt obligation using the current cash-flow plus the corporate liquidity. An important distinction is made between liquidity and solvency of the firm. The corporate financial policy is simultaneously defined by the dividend rate (or policy), the coupon and the principal of the bond. In our model, the dividend rate both affects the default probability and the bondholders' recovery rate. When the corporate financial policy implies no credit risk, we find the famous Modigliani-Miller propositions. When the corporate financial policy implies credit risk, we show that the value of the firm is a decreasing piecewise function of the dividend rate. As a consequence, zero is an optimal dividend rate: the value of the firm is not invariant with respect to the dividend rate, as suggested by Modigliani-Miller (1961). However, shareholders may not always have the incentives to implement this optimal dividend rate. We show that when the value of the assets is low, shareholders have an incentive to deviate from this optimal dividend rate, and we also study the resulting agency costs of dividend rate. We finally compare the resulting quantities of our model to the base case suggested by Huang and Huang (2003).

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1 Introduction

What is the impact of corporate liquidity on a firm's value? Is the value of the firm invariant with respect to the dividend policy, as suggested fifty years ago, by Modigliani and Miller (1961)? Does the liquidative value of the assets affects the incentives of shareholders to choose the dividend policy? What is the impact of the volatility of the cash-flow on dividend policy?

This paper attempts to provide answers to these questions through a simple model in which we make a clear distinction between liquidity and solvency of the firm, which in turn allows us to properly introduce dividend policy as a fraction of the net result. Roughly speaking, *solvency* is the ability of the firm to redeem bondholders if the productive assets were to be sold, while *liquidity* is the ability of the firm to pay the current coupon to bondholders using current cash-flow. A firm may be illiquid but solvent, while another may be liquid but insolvent. The first generates low cash-flow but the liquidative value of its assets is high (e.g., an industrial firm) while the second firm generates high cash-flow but the liquidative value of its assets is low (e.g., a commercial firm).

In the structural approach to credit risk modeling, it is generally assumed that the state variable V follows some stochastic process, typically a geometric Brownian motion. One generally interprets V to represent either:

1. the (liquidative) value of the productive assets (e.g Merton (1974)),
2. the expected value of the future cash-flow (e.g. Fries et *al.*).

In the first interpretation, the scrapping value of the assets is the only economic variable. As a consequence, default comes from a "solvency trigger". In standard structural models such as Merton (1974), Black & Cox (1976), Briys and Devarenne (1997), Leland (1994), Leland and Toft (1998), Fan and Sundaresan (2002), firms default when the liquidative value of the assets V falls below some threshold H which may be constant (Leland 1994; Fan and Sundaresan 2002); deterministic Black and Cox (1976); or stochastic Briys and Devarenne 1997). The problem with this approach is that it does not really allow default to come from a "liquidity trigger" but more importantly, as pointed out by Leland (1994) and Goldstein *et al.* (2001) among others, it is not clear whether or not V remains a traded asset when the firm becomes levered[1].

In the second interpretation, the value of the assets is related to the current cash-flow (e.g., the EBIT as Goldstein *et al* (2001) or Fries *et al* (1997)) by assuming that V is the expected infinite sum of the (discounted)

[1] This is important because equity and debt are supposed to be contingent claims on V. If V is not a traded asset, the way equity and debt is computed are misleading.

cash-flows. In such a framework, V becomes a multiple of the current cash-flow. The problem with this approach, as pointed out by Fan and Sundaresan (2002), is that dividend policy is always very specific: when positive, all residual cash-flow (i.e., the net result) is distributed to shareholders as dividends. On the contrary, in this paper, we shall assume that only a fraction of the net result is distributed to shareholders as dividends, while the undistributed part is placed on some bank account which capitalizes at the risk-free rate (i.e., is placed on a zero net-present value project). This bank account constitutes the corporate liquidity. Rubinstein (1976) (see also Brennan (1971)) analyses such a model but in a risk-free debt framework. As a consequence, the value of the firm is invariant with respect to the dividend policy. However, when the debt is not risk-free, the corporate liquidity can constitutes a "reservoir" (Kalay (1982), Lease *et al.* (1999) chapter 6) that can be used by shareholders to avoid default when the current cash-flow is insufficient to meet debt obligations. Taking into account this reservoir is the subject of this paper. Related to our approach are the two following (independent) literatures.

The first, mainly empirical, examines the economic determinants of corporate cash holdings (e.g., Opler *et al.* (1999), Faulkender and Wang (2006), Lie (2005), Dittmar *et al* (2003)). As pointed out by Lie (2005), firms are likely to incur liquidity shortage if:

- the accumulated liquidities are too low (or the access to capital markets is difficult),
- the coupon of the bond is high,
- the future cash-flow is low,
- the volatility of the cash-flow is high.

In their well-known paper on corporate cash holdings, Opler *et al* (1999) find empirical evidence that firms that have the greatest access to capital markets (e.g., those with investment grade rating) tend to hold a lower ratio of cash. When the access to capital markets is difficult, it is interesting for these firms to accumulate corporate liquidity to reduce the likelihood of default when the firm does not generate sufficient cash-flow to service debt payments.

The second, mainly theoretical, initiated by Radner and Shepp (1996), Jeanblanc and Shiryaev 1995; (see also Asmussen and Taksar (1997), Decamps and Villeneuve (2007), Hojgaard and Taksar 1999, Rochet and Villeneuve (2004)) studies the optimal dividend policy of the firm. The problem is to maximize the value of the firm with respect to the dividend policy when it is implicitly assumed that access to capital markets is costly. Default is declared the first time a random process, *controlled* by the dividend policy, hits a given barrier, typically zero. However, the liquidity process generated by retained earnings is never taken into account. Asvanumt *et al.* (2007) and Anderson *et al.* (2005) are exceptions since they also consider the accumulated cash reserve.

In this paper, we study the simplest (discrete-time) model in which default arises when the firm is not able to pay its debt obligations using current cash-flow *plus* the accumulated corporate liquidities. When the firm defaults, there is no Chapter 11 possibility (as e.g in Broadie *et al.* 2007 or Francois and Morellec 2004) nor can the firm looks for external financing (as e.g., in Leland (1994), Fries *et al.* (1997) among others): the firm is immediately liquidated. To be (at least partially) redeemed, bondholders are allowed to seize the current cash-flow, the corporate liquidities (if any), and the proceeds of the assets sale. One of the interesting features of our model is that the dividend policy both controls:

- default risk.
- recovery rate risk.

To provide a complete analysis of our model, we first consider the no taxes, no-default risk case. As expected from Modigliani-Miller (1958, 1961) and Stiglitz (1974), we show that the corporate financial policy is irrelevant. Although the result is not new, we believe that our proof is valuable since we present a direct computational proof of this irrelevance result with respect to *both* the leverage policy (coupon and nominal) and the dividend policy. Moreover, this invariance result holds for any probability measure. Under the pricing measure $\mathbb{Q}$, we provide an explicit computation of the value of the firm. When there is default risk, we show that the value of the firm is invariant with respect to the leverage policy but not to the dividend policy. As was suggested by Black (1976), we show that the maximization of total firm value implies no distribution of dividends. Of course, shareholders may not have the incentive to implement this particular dividend policy. We thus study their optimal dividend policy (given some leverage policy) and the resulting agency cost due to deviation from the optimal dividend policy. Finally, we compare the prediction of our model with the "target data" collected in Huang and Huang (2003).

2 Assumptions and discussion

Here are the basic assumptions of our simple model.

1. Markets are arbitrage free,
2. Information is symmetric,

3. Investment policy is fixed: productive assets allow the firm to produce one unit of output and the firm sells it at the market price P.

4. When default (payment) occurs, the (levered) firm cannot raise external funds nor can it rely on Chapter 11: it is liquidated (Chapter 7).

5. There exists a bond indenture that stipulates that creditors have an (unconditional) right to receive payment of the nominal (i.e., principal) and the accrued interest after liquidation.

6. No wealth expropriation of bondholders is possible.

Some comments are in order. We assume that markets are arbitrage free[2]. This ensures the existence of a risk-neutral (probability) measure, i.e., the measure under which the (discounted) underlying is a martingale. Although assumption 2 is in some sense implied by assumption 1, we want to point out that we offer a symmetric information model. It says that the various parameters of the stochastic process of price are known by the stakeholders (i.e., shareholders, bondholders, etc.) so that increasing dividends to signal say higher profitability has no sense[3]. Assumption 3 says that investment policy is fixed (as in Modigliani-Miller 1958, and almost all structural models) and is not subject to change after debt issuance. In particular, there is no growth option as e.g., in Decamps and Villeneuve (2007). It also says that as opposed to many structural models, the stochastic variable is not the liquidative value of assets, but typically an element of the EBIT (Earnings Before Interest and Taxes), as in Fries *et al.* (1997), Goldstein *et al.* (2001) among others. This modeling approach allows us to explicitly compute the net result and to properly define the dividend policy. Given a positive net result R, the dividend policy is the fraction α of the net result which is distributed to shareholders as dividends, i.e., αR. The complement $(1-\alpha)R$, which capitalizes at the risk free rate, constitutes the *corporate liquidity*. This corporate liquidity is a sort of "reservoir" that can be used to avoid default when the current EBIT is lower than the coupon. In many structural models (e.g., Leland (1994, Fries *el al.* (1997)), there is no dilution cost so that shareholders can reinject cash whenever they want. As a consequence, as noted in Asvanumt *et al.* (2007), the amount of cash is irrelevant. We take here the opposite direction since we assume that dilution is infinitely costly[4], so that if the firm cannot pay the current coupon, it defaults. This is what means assumption 4. Assumption 5 is self-evident. Note that few papers analyse the case in which default does not imply liquidation (see e.g. Francois and Morellec (2004) and Broadie *et al.* (2007a) p.

[2]This assumption is very close to the assumption of *perfect market* in economics.

[3]See e.g., Tirole, 2006, chapter 6 for asymmetric information models.

[4]Few papers assume positive but finite dilution cost, e.g., Asvanumt *el al.* (2007).

1353 for a discussion). Assumption 6 means that shareholders can not alter dividend policy just after the issuance of the bond if the bond is priced using a given dividend policy. If shareholders change dramatically the dividend policy following debt issue, this creates a wealth transfer from bondholders to shareholders (Kalay (1982), Long *et al.* (1994)), since it changes the magnitude of the reservoir. This assumption is consistent with the empirical result of Long *et al.* (1994), who find no evidence that firms alter dividend policy following debt issue.

3 The model

3.1 EBIT, dividends and liquidity processes

The considered time interval $[0, T]$ is discretized in J equally spaced bins of size δt (i.e. $J = [T/\delta t]$). To simplify the notations, all time-dependent variables will be indexed by j rather than $j\,\delta t$. In this paper, r will denote the risk free rate.

Let $(\Omega, \{\mathcal{F}_j\}, \mathcal{F}, \mathbb{P})$ be our filtered probability space, where $\mathbb{P}$ is the "historical" probability measure. We choose a binomial stochastic process (see e.g., Nowalkla and Chambers (1995) for a discussion of the various approaches) to support the evolution of the price P_j in which the parameters u and d are given by:

$$u = e^{\mu\,\delta t + \sigma\sqrt{\delta t}} \quad (1)$$

$$d = e^{\mu\,\delta t - \sigma\sqrt{\delta t}} \quad (2)$$

To avoid arbitrage, one must also have $d < e^{r\delta t} < u$. In numerical computation, we shall typically choose $\mu = 0$, which gives the constraint $\sigma > r\sqrt{\delta t}$. Let z_j be a Bernoulli random variable such that

$$\mathbb{P}(z_j = +1) = 0.5 \quad (3)$$

The price dynamics

$$P_j = P_{j-1}\, e^{\mu\,\delta t + z_j\,\sigma\sqrt{\delta t}} \quad (4)$$

is thus defined through the realizations of i.i.d. Bernoulli random variables $z_1, \ldots, z_J$. A sample path ω is thus given by a realization of $(z_1, z_2, ...z_J)$. As a consequence, the set of all sample path Ω is finite. Throughought the paper, $(\mathcal{F}_j, 1 \leq j \leq J)$ will denote the natural filtration associated with the z_j, which means that at time j, the vector $(z_1, z_2, ...z_j)$, and thus $(P_1, P_2, ...P_j)$, are measurable with respect to $\mathcal{F}_j$, i.e., known. We assume that there are no production costs[5] so that the EBIT (Earning Before Interests and Taxes) is equal to the price P_j. As a consequence, the firm is not subject to *business risk* since P_j is always non negative. As e.g., Andrade and Kaplan (1998), Fries et *al*. (1997), we thus focus on firms that are not economically distressed, but can be financially. Let $\overline{C}$ be the annual coupon. Between time j and $j+1$, the coupon paid is proportional to the time step δt so that:

$$C = \overline{C}\ \delta t \tag{5}$$

When the current EBIT (i.e., P_j) is not sufficient to pay the coupon, we assume that the firm is not subject to corporate taxation. Let τ_c be the corporate tax income and R_j be the net income at time j. It follows that:

$$R_j = (P_j - C)(1 - \tau_c\, \mathbf{1}_{P_j > C}) \tag{6}$$

where $\mathbf{1}_{P_j > C}$ is an indicator function which is equal to 1 when $P_j \geq C$ and 0 otherwise. Let

$$\alpha_j = \alpha \ \text{ for all } j \tag{7}$$

be the constant dividend policy[6] and D_j be the dividend at time j. When $R_j > 0$, $D_j = \alpha R_j$, and $(1-\alpha)R_j$ is placed on a bank account which capitalizes at the risk-free rate r. As we stated earlier, this bank account constitutes a *cash reserve* (i.e., corporate liquidity), which may be used by shareholders to avoid default when the current EBIT P_j is lower than the coupon C. When $R_j < 0$, $D_j = 0$ and the firm uses a fraction of its corporate liquidity to pay C if they are high enough. Otherwise, the firm defaults and is liquidated. The value of the dividend is thus:

$$D_j = \ R_j\, \alpha\ \mathbf{1}_{P_j > C} \tag{8}$$

At time j, the value of corporate liquidity is equal to the capitalized value of L_{j-1} plus the difference between the net result and the dividend. It is the (Markov) stochastic process defined by:

$$L_j = L_{j-1}\, e^{r\,\delta t} + R_j - D_j \tag{9}$$

which can be rewritten as:

$$L_j = L_{j-1}\, e^{r\ \delta t} + (P_j - C)(1 - \tau_c\, \mathbf{1}_{P_j > C})(1 - \alpha\, \mathbf{1}_{P_j > C}) \tag{10}$$

Note that *ceteris paribus*, L_j is a decreasing function of α.

[5] If K were a (constant) unit production cost, the EBIT would be equal to $P_j - K$. The firm might thus have the (real) option to temporarily shut down production. We won't study this possibility here.

[6] This assumption could be relaxed later since it could be interesting for shareholders to postpone dividends to keep the firm alive when both the price and the bank account are low. In this paper, we thus assume a smooth dividend policy.

3.2 Asset value, redeployability and competitivity of the second hand market

As noted in the Standard & Poor's "corporate rating criteria (2006)", collateral can consist of assets that have value *independent* of the business of the firm (e.g., real estate, vehicles), but also assets that have value *directly* linked to the business of the firm (e.g., inventory, production, equipment). To quote Standard and Poor's "corporate rating criteria (2006)" p 65:

"*Knowing the value of the collateral–relative to the amount owed–provides an approximation of just how well a creditor is secured*"

Let V_j be the value of the assets (i.e., the value of the collateral), which may be tangible (e.g. plant and equipment) or intangible (e.g. patents, brand name, etc.). In our model, as in many others, we assume that the value of the assets is a multiple of the EBIT:

$$V_j = m\ P_j \qquad m \geq 0 \tag{11}$$

where $m \geq 0$. In most models (e.g. Goldstein *et al.* (2001), Broadie, Chernov, Sundaresan (2007), Fries *et al.* (1997)), it is assumed that V_j is equal to the expected value of the *infinite* future cash-flows assuming no debt, that is:

$$V_j = \frac{1}{r-\mu} P_j \quad \mu < r \tag{12}$$

where μ is here the risk-neutral drift of the EBIT. However, we argue that the value of m should be related to the "degree of specialization" of the assets (and to the "degree of competitivity" of the second-hand-market for assets when they are specialized) rather than the expected value of the cash-flow assuming a particular business activity.

Consider for example a firm that has issued a bond to buy a building in Paris or in New York to develop a consulting activity in quantitative finance. Assume now that for some reason, the firm cannot pay its debt obligations so that it has to be liquidated. We argue that the value of the assets need not be related to the expected flow of profits. When the firm's assets consist of buildings in Paris or New York, they can clearly be used for many types of activities other than consulting in finance, which means that they are not specific to a particular business activity. To use Williamson's terminology, they are "redeployable" (see e.g., Schleifer and Vishny, 1992 or Maksimovic and Phillips 1998). As a consequence, their value is not related to the expected flow of profit assuming a particular business activity.

On the contrary, when the firm's assets are not "redeployable", which means that they are very specific (e.g, specific real estate, pharmaceutical patents, steel plants, a roller coaster, etc.) they will be sold to another firm which will used them in the same way. As a consequence, the value of these assets will come from the same business, and, as such, have to be related to the expected flow of cash-flow. However, the value of the multiple m will also be related to the existence of potential candidate(s) that may purchase the assets, and this in turn may depend on their capacity (or "financial muscle" to use Tirole's terminology) to buy them. Assume for example that the Paris Eurodisney amusement park defaults and try to liquidate say a roller coaster, whose "market value", i.e., the value of the expected discounted cash-flow, is V. If there is only one interested candidate (another amusement park near Paris, for example Asterix park) who can at most pay half the value of V, the liquidation value will be $0.5V$. Indeed, it could be even less if the potential buyer is aware that the assets have a liquidation value only if they purchased, i.e., he could try to negotiate a lower price. In the particular case in which the second-hand market for used assets exists and is competitive, then, the liquidation price will be close to its market value V.

It follows from this discussion that, *ceteris paribus*, the more redeployable the assets, the higher should be their value. In our model, since we assume that $V = mP$ it may thus be difficult to interpret m as a "redeployability paramter". Assuming that the assets are not redeployable, i.e., they are very specific, m is now related to the "degree of competivity" of the second-hand market for used assets. When m is "low", the market is not competitive (i.e., there are few potential buyers) while when m is "high", the market for used assets is highly competitive and V is very close to an expected discounted value. Since bond and equity depend on this parameter, m has to be estimated at the bond issuance date. It is interesting to note that this problem is very close to the estimation problem of the recovery rate in credit derivatives markets. For example, to compute the mark-to-market of a Credit Default Swap (and thus of a CDO tranche), one has to estimate the recovery rate. It is generally assumed that the recovery rate is fixed at 40%, but this value is quite conventionnal and may be very different from the realized recovery rate, i.e., after the default of the underlying issuer in the (swaps) contract.

So far, as we stated, we shall here consider the case in which $V = mP$, which means that the value of the assets V is *perfectly correlated* to the current cash-flows[7] P. To estimate m at time $t = 0$, one can thus observe the value of the current cash-flow P_0, compute the market value of the assets V_0, and deduce $m = P_0/V_0$. With the development of IAS (IFRS) standards, the accounting value of the assets in balance sheets should be very close to the mark-to-market, that is, to their market value. This way of estimating

[7]Leland (2004) notes that because asset value is a fixed multiple of cash flow, EBIT-based models are virtually identical to asset-value-based models. This is, however, true as long as all the cash-flow is distributed as dividends. When this is not the case, the two types of models are not equivalent. Note also that the multiple m may be thought of as the value of the assets *net* of the bankruptcy costs.

m may be considered very crude but it doesn't seem cruder than the current practice of estimating the recovery rate in credit derivatives markets.

3.3 Corporate financial policy and default threshold

Given the various parameters of our model $(P_0, J, r, m, \sigma, \mu)$, we shall call the triplet

$$\mathbf{x} = (\alpha, C, N) \tag{13}$$

a *corporate financial policy*, where $\alpha \in [0,1]$ is the dividend policy (i.e., dividend rate) and (C, N) is the leverage policy, N is the principal or nominal. Let $j^*(\mathbf{x})$ be the default time, which is obviously a function of the corporate financial policy. Since by assumption shareholders don't re inject funds, default time is such that:

$$j^*(\mathbf{x}) = \inf\left\{1 \le j \le J : \; P_j + L_{j-1}(\mathbf{x})\, e^{r\,\delta t} < C\right\} \tag{14}$$

Note that the bank account depends obviously on the corporate financial policy $\mathbf{x}$. *Ceteris paribus*, the higher α, the lower $L_j(\mathbf{x})$, and the earlier is the default time. One can also say that default happens at the first time j such that:

$$P_j < C_j^{def}(\mathbf{x}) \tag{15}$$

where $C_j^{def}(\mathbf{x}) = (L_{j-1}(\mathbf{x})\, e^{r\,\delta t} - C$ can be interpreted as an *endogenous stochastic default threshold*. Given a leverage policy, the threshold is endogenous since α is chosen (optimally or not) by shareholders, and is stochastic because $L_{j-1}(\mathbf{x})\, e^{r\,\delta t}$ is not known at time $t = 0$. The default threshold decreases when the current net income is positive and increases when the current net income is negative i.e., when the corporate liquidity is used to pay some fraction of the coupon C. When the default threshold is negative at a given time j, since the current price is always positive, default cannot occurs at time j.

Let $\mathbf{x}(\alpha) = (\alpha, C, N)$ be a corporate financial policy in which only the dividend policy may vary. Consider now $\alpha' \le \alpha$. It is easy to see that this implies for all j and all ω that :

$$C_j^{def}(\mathbf{x}(\alpha'), \omega) \le C_j^{def}(\mathbf{x}(\alpha), \omega) \tag{16}$$

$$j^*(\mathbf{x}(\alpha'), \omega) \ge j^*(\mathbf{x}(\alpha), \omega) \tag{17}$$

Let $j^*(\mathbf{x}(\alpha), \omega)$ be a default time under dividend policy α in state ω. By equation (15), it follows that $P_{j^*(\mathbf{x}(\alpha),\omega)} < C_{j^*}^{def}(\mathbf{x}(\alpha), \omega)$. Given equation

(16), for dividend policy $\alpha' < \alpha$, it may be the case that $j^*(\mathbf{x}(\alpha), \omega)$ is not anymore a default time since the default threshold is lower than for α'. If, along ω, the price monotonically decreases after time $j^*(\mathbf{x}(\alpha), \omega)$, then, $P_{j^*(\mathbf{x}(\alpha'),\omega)} < P_{j^*(\mathbf{x}(\alpha),\omega)}$. As a consequence, a lower dividend policy may imply a lower price in default, and thus a lower recovery for bondholders.

Note importantly that equation (14) implies that if there exists a subset of sample paths such that $j^*(\mathbf{x}) > J$ while $R_J(1 - \alpha \mathbf{1}_{P_J > C}) + L_{J-1}(\mathbf{x})\, e^{r\,\delta t} + V_J < N$, default has not been declared while the nominal of the bond is not fully repaid. As a consequence, we shall call equation (14) a liquidity-triggering default mechanism since no solvency constraints are involved[8].

Lemma 1. The set Λ of dividend policies which affects default probabilities is of the form $\{0, \alpha_1, \alpha_2 ... \alpha_H, 1\}$, with $0 < \alpha_h < 1$ for all $h = 1, 2..H$.

Proof : see the Appendix

This means that for all $\alpha \in]\alpha_{h-1}, \alpha_h]$, the various probabilities are *invariant* with respect to $\omega \in \Omega$. The reason is quite simple: along each sample path, default is defined by an inequality (see equation (14)). As a consequence, for fixed dividend policy α, if we decrease α by an amount $\delta\alpha$ small enough, then, for each sample path, the various inequalities will still hold. We shall denote

$$\mathcal{P} = \Lambda \times \mathbb{R}^{2+} \tag{18}$$

to be the set of corporate financial policies.

A number of papers (e.g., Radner and Shepp 1996; Jeanblanc and Shiryaev 1995; Hojgaard *et al.* 1999; Decamps and Villeneuve 2007 to quote a few papers) analyze the general stochastic optimal control problem in which dividend policy partially controls a stochastic process $X = (X_t)_{t \geq 0}$ which is stopped at the first time t for which $X_t \leq 0$. The problem is then to find the optimal (term structure of) dividend policy that maximizes the value of the firm. We depart from these (more mathematically involved) papers in that we explicitly consider a firm which has to choose *simultaneously* the optimal (constant) dividend policy α and a leverage policy (C, N). This allows us to obtain results that confirm (or not) the classic ones of Modigliani-Miller but also to study whether or not shareholders have an incentive to choose the corporate financial policy that maximizes the total firm value.

[8]As in many structural models, one could easily define a solvency-triggering default mechanism, since default could also be declared (via a bond covenant) when the scrapping value of the assets falls below some threshold, e.g. a fraction of the nominal N.

3.4 Liquidation and bondholders' recovery rate

At a default (i.e., liquidation) time $j^*(\mathbf{x}) \leq J$, by assumption, bondholders have an unconditional right to seize everything of worth, that is, the assets of the balance sheet, until they are fully repaid. Since the current coupon C has not been paid, by assumption, what is due to the bondholders is the nominal plus accrued interests, that is $N + C$. Let

$$Z_j(\mathbf{x}) = L_{j-1}(\mathbf{x})\, e^{r\,\delta t} + P_j + V_j \tag{19}$$

be the total value of the assets in the balance sheet, i.e., the value of the bank account plus the value of the current cash-flow, plus the value of the productive assets. The following table gives the balance of the firm at a given default time. Note importantly that due to *limited liability* of shareholders, the value of the stock is the maximum between $Z_{j^*}(\mathbf{x}) - (N + C)$ and zero.

Balance sheet at default time

Assets	Liabilities and Equity
Cash : $L_{j^*(\mathbf{x})-1}\, e^{r\,\delta t} + P_{j^*(\mathbf{x})}$	Debt: $\min\{Z_{j^*(\mathbf{x})}, N + C\}$
Fixed assets: $V_{j^*(\mathbf{x})} = mP_{j^*(\mathbf{x})}$	Stock: $\max\{Z_{j^*(\mathbf{x})} - (N + C); 0\}$
$Z_{j^*(\mathbf{x})}$	$Z_{j^*(\mathbf{x})}$

The bondholders recovery rate in default is equal to:

$$R^{ec}_{j^*(\mathbf{x})} = \min\left\{\frac{Z_{j^*(\mathbf{x})}}{N + C}; 1\right\} \tag{20}$$

and is a function of the corporate financial policy $\mathbf{x}$. Note that the recovery rate is obviously stochastic.

3.5 Equivalent martingale measure, know-how, and market incompleteness

In "classical" structural models of credit risk (e.g., Merton 1974; Leland 1994; Briys and De Varenne 1997; Fan and Sundaresan 2001), the value of the assets V coincides with the value of the unlevered firm. As a consequence, under the risk-neutral measure $\mathbb{Q}$, the expected rate of return of V (i.e., capital gain) must be equal to the risk free-rate r, that is

$$\mathbb{E}^{\mathbb{Q}}\left(\frac{\Delta V_t}{V_t}\Big|\mathcal{F}_t\right) = r\delta t \tag{21}$$

where $\Delta V = V_{t+\delta t} - V_t$, and where $\mathcal{F}_t$ is the filtration at time t of the stochastic process $V = (V_t)_{t\geq 0}$. In stationary EBIT-based models of credit risk (e.g., Broadie, Chernov, Sundaresan (2007), Fries et *al*. (1997), Goldstein et *al*. (2001)), since there is a constant dividend yield $\frac{P_t}{V_t} = r - \mu = \kappa > 0$ (see equation 12), under the risk-neutral measure $\mathbb{Q}$, the overall expected rate of return of the firm must be equal to the risk free-rate r:

$$\mathbb{E}^{\mathbb{Q}}\left(\frac{\Delta V_t}{V}\Big|\mathcal{F}_t\right) + \kappa\delta t = r\delta t \Longleftrightarrow \mathbb{E}^{\mathbb{Q}}\left(\frac{\Delta V_t}{V_t}\Big|\mathcal{F}_t\right) = (r-\kappa)\delta t \qquad (22)$$

Whether or not there are dividends, markets are assumed to be arbitrage free and complete, which implies that the value of the levered firm is perfectly replicable with the underlying, that is, with V. We believe that this "perfect replicability" is rather counter factual since the very reason for the existence of a firm is the development and the use of some "know-how" (e.g., technical skill, research and development, etc.) which, by definition, is not (or not totally) replicable. In this paper, as in many structural models, we assume that the productive assets (tangible or intangible) are traded and such that under the (pricing) measure $\mathbb{Q}$, its discounted value is a martingale. Since $V = mP$, it follows that under measure $\mathbb{Q}$, the discounted price is also a martingale:

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-r(k-j)\delta t}P_k(\omega)\Big|\mathcal{F}_j\right] = P_j \quad k \geq j \qquad (23)$$

which means that (under $\mathbb{Q}$) the probability of a given sample path $\omega \in \Omega$ is

$$\mathbb{Q}(\omega) = q^{\mathrm{Card}(up)}(1-q)^{\mathrm{Card}(down)} \qquad (24)$$

where Card(up) (resp Card(down)) is the number of time j such that $z_j = 1$ (resp $z_j = -1$) and where

$$\mathbb{Q}(P_j = uP_{j-1}|P_{j-1}) = \frac{e^{r\delta t} - d}{u - d} = \mathbb{Q}(z_j = +1) = q \qquad (25)$$

When a given investor (i.e., a firm) buys V, without know-how, he can just expect its capital gain to be equal to the risk-free rate r under $\mathbb{Q}$ since he is NOT able to generate the flow of EBIT $\{P_j\}_{j=1}^{J}$. If, for some reason, this firm becomes levered, since there is no flow of EBIT, it can issue a zero-coupon bond of maturity say J and nominal K. As a consequence, the value of the securities are given by Merton (1974) model. The value of equity $E_0(V_0, K)$ is a (european) call option on V while the value of the risky bond $B_0(V_0, K)$ is a risk-free bond minus a (european) put option. The value of the firm is thus equal to V_0 no matter what the nominal K is, i.e., the value of this "mertonian" firm is $V_0 = E_0(V_0, K) + B_0(V_0, K)$ for all $K \geq 0$.

Things are quite different for the particular firm we shall consider here. We assume that our firm (and only our firm) has some "know-how" which is such that, combined with the productive assets, allows her to generate a flow of EBIT $\{P_j\}_{j=1}^{J}$, and thus a flow of dividends $\{D_j(\alpha)\}_{j=1}^{J}$. Since the expected capital gain of V is equal to r under $\mathbb{Q}$, due to the dividend yield, this implies that the overall rate of return of our firm is *higher* than the risk-free rate r. However, this dividend yield is not replicable by mertonian firms since they don't have the "know-how". For example, the "know-how" may come from the success of their R & D efforts intensity[9], and as a consequence, our firm has say the exclusive right to exploit some patent during some (finite) time, which allow her to receive a flow of dividends in addition to the expected capital gain. From the other Mertonian firms' point of view, markets are arbitrage free but not complete since dividends generated due the know-how are not replicable. What we do is indeed very close to the "classic" capital budgeting approach, in which a project is accepted if its NPV (Net Present Value) is higher than zero. As far as we know, this is the first attempt to decompose the value of the firm into a replicable part, and a non-replicable one.

3.6 Debt value for a given corporate financial policy

The value of debt, denoted $B_0(\mathbf{x})$, is by definition equal to the expected (given P_0) discounted cash-flow under the risk-neutral probability measure. recall that the "min" that occurs in debt valuation comes from the *limited liability* of shareholders. Let $\mathbb{E}_0^{\mathbb{Q}}(...)$ be a notation for $\mathbb{E}^{\mathbb{Q}}(...|\mathcal{F}_0)$, where $\mathbb{Q}$ is the equivalent martingale measure. Recall that $Z_j(\mathbf{x})$ is defined by equation (19).

3.6.1 No credit risk

Given a corporate financial policy $\mathbf{x} \in \mathcal{P}$, let

$$\bar{\Omega}_d(\mathbf{x}) = \{\omega \in \Omega : j^*(\mathbf{x}, \omega) > J\} \tag{26}$$

be the set of sample paths in which default does not occur. We say that there is *no credit risk* if the corporate financial policy $\mathbf{x} \in \mathcal{P}$ (assuming it exists) is such that the two following conditions are met:

- $j^*(\mathbf{x}, \omega) > J \quad \forall\, \omega \in \Omega$
- $R_J(\omega)(1 - \alpha \mathbf{1}_{P_J > C}) + L_{J-1}(\mathbf{x}, \omega)\, e^{r\,\delta t} + P_J(\omega) \geq N \quad \forall\, \omega \in \Omega$

Given a corporate financial policy $\mathbf{x}$, the first condition says that the firm survives on each sample path, while the second one ensures that the full nominal is repaid at maturity on each sample path. No credit risk implies thus that there is no loss on each sample path. As a consequence, the rate of return of the bond is equal to the risk-free rate. Given P_0 and the set of parameters (J, r, m, σ, μ), let

[9] As noted in Lease *et al* (1999), in cosmetic industry, for example, firm value depends critically on the success of their R & D efforts. See also Chan *et al.* (2001).

$$\mathcal{P}^{ND} \subseteq \mathcal{P} \tag{27}$$

be the set of corporate financial such that there is no credit risk. Note that it may be the case that $\mathcal{P}^{ND} = \emptyset$.

3.6.2 Credit risk : default risk and/or recovery rate risk

Given a fixed corporate financial policy $\mathbf{x} \in \mathcal{P}$, let

$$A_k(\mathbf{x}) \equiv A_k = \{\omega \in \Omega : j^*(\mathbf{x}, \omega) = k\} \tag{28}$$

denote the set of sample paths in which default occurs at time k, with $k \leq J$ and let

$$\Omega_d(\mathbf{x}) = \{\omega \in \Omega : j^*(\mathbf{x}, \omega) \leq J\} = \bigcup_{k=1}^{J} A_k(\mathbf{x}) \tag{29}$$

be the set of sample paths in which default occurs at, or prior to maturity. Note that $\Omega_d(\mathbf{x})$ and $\bar{\Omega}_d(\mathbf{x})$ form a partition of Ω for all corporate financial policy. We say that there is (some) *credit risk* if the corporate financial policy $\mathbf{x} \in \mathcal{P}$ is such that at least one condition is met :

1. $\Omega_d(\mathbf{x})$ is not empty. We shall talk about **default risk**.
2. there exists a non empty subset of $\Omega_d(\mathbf{x})$ such that $Z_{j^*(\mathbf{x},\omega)} < N + C$, and/or there exists a subset of sample paths such that $j^*(\mathbf{x}, \omega) > J$ while $R_J(\omega)(1 - \alpha \mathbf{1}_{P_J > C}(\omega)) + L_{J-1}(\mathbf{x}, \omega)\, e^{r\,\delta t} + V_J(\omega) < N$. We shall talk about **recovery rate risk**.

For a given leverage policy, note importantly that dividend policy α both affects default probability and the recovery rate. Let

$$\mathbb{Q}(j^*(\mathbf{x} \leq J) = \mathbb{Q}(\Omega_d(\mathbf{x})) \tag{30}$$

be the risk-neutral default probability. Due to our distinction between recovery rate risk and default risk, the risk-neutral default probability is *independent* of m.

3.6.3 Debt value

The value of the debt is equal to

$$B_0(\mathbf{x}) = \mathbb{E}_0^{\mathbb{Q}} \left[\sum_{j=1}^{J-1} e^{-(r\,j\;\delta t)}\; C\, \mathbf{1}_{j^*(\mathbf{x},\omega) > j} \right] +$$

$$\mathbb{E}_0^{\mathbb{Q}} e^{-(r\,J\,\delta t)} \left[C + \min\{N, R_J(1 - \alpha \mathbf{1}_{P_J > C}) + L_{J-1}(\mathbf{x})\, e^{r\,\delta t} + V_J\} \mathbf{1}_{j^*(\mathbf{x},\omega) > J} \right]$$

$$+\mathbb{E}_0^{\mathbb{Q}}\left[e^{-(rj^*(\mathbf{x},\omega)\delta t)}\min\left\{Z_{j^*(\mathbf{x},\omega)};N+C\right\}\mathbf{1}_{j^*(\mathbf{x},\omega)\leq J}\right] \quad (31)$$

In general, different corporate financial policies will give different debt values.

3.6.4 Credit spread

Once the value of the debt $B(\mathbf{x})$ is known, we can can thus imply the yield of the bond and thus the spread. Let $Y_J \equiv Y(\alpha, C, J, N)$ be the rate of return (at the issuance date) of the risky bond of maturity J. By definition, Y_J is the solution of the following polynomial equation:

$$B_0(\mathbf{x}) = \sum_{j=1}^{J-1} Ce^{-(Y_J j\delta t)} + (C+N)e^{-(Y_J J\delta t)} \quad (32)$$

Given Y_J, the spread is thus:

$$S_J = Y_J - r \quad J \in \mathcal{J} \quad (33)$$

where $\mathcal{J}$ is the set of maturity.

Fundamentally, the spread remunerates the *expected loss* due to credit event. The higher is the expected loss, the higher is the spread. Note importantly that in our model, default risk does not necessarily imply a positive spread. When the bond is issued at par and when there is no recovery rate risk (e.g. because m is high or because α is low), whether or not there is default risk, the expected loss due to default is zero. As a consequence, the spread is zero. On the contrary, if there is no default risk but if there exists a subset of sample paths for which the nominal cannot be fully repaid at maturity, then, the expected loss due to recovery rate risk is positive. As a consequence, the spread is positive[10].

3.7 Equity value for a given corporate financial policy

As usual, the value of equity $E_0(\mathbf{x})$ is equal to the expected (given P_0) discounted cash-flow under the risk-neutral probability measure. Note that

[10]When the bond is not issued at par, things may be more complicated. Assume for example that the bond is issued over the par. In that case, default risk alone (i.e., without recovery rate risk) may imply positive spread. To see this, assume that $B_0(\mathbf{x}) = 1.5N$, with $N = 10$ and $C = 1$. It may be the case that at time 1, default is declared so that bondholders recuperates exactly $N + C = 11$. However, the loss is obviously positive. Indeed, in that context, the spread remunerates *default time risk*. When the bond is issued over the par, this could give an incentive to shareholders to declare the default prematurely.

the “max” that occurs in equity valuation comes from the limited liability of shareholders. The value of equity is equal to:

$$E_0(\mathbf{x}) = \mathbb{E}_0^{\mathbb{Q}}\left[\sum_{j=1}^{J-1} e^{-(r\,j\,\delta t)} D_j(\omega)\mathbf{1}_{j^*(\mathbf{x},\omega)>j}\right] +$$

$$\mathbb{E}_0^{\mathbb{Q}} e^{-(r\,J\,\delta t)}\left[D_J + \max\left\{(R_J(1-\alpha\mathbf{1}_{P_J>C}) + L_{J-1}(\mathbf{x})\,e^{r\,\delta t} + V_J - N; 0\right\}\mathbf{1}_{j^*(\mathbf{x},\omega)>J}\right]$$

$$\mathbb{E}_0^{\mathbb{Q}}\left[e^{-rj^*(\mathbf{x},\omega)\delta t}\max\left\{Z_{j^*(\mathbf{x},\omega)} - (N+C); 0\right\}\mathbf{1}_{j^*(\mathbf{x},\omega)\leq J}\right] \tag{34}$$

where we recall that $D_j = \underbrace{(P_j - C)(1-\tau_c\,\mathbf{1}_{P_j>C})}_{=R_j}(\alpha\,\mathbf{1}_{P_j>C})$.

3.8 Value of the firm and optimal corporate financial policy

The value of the firm is equal to the value of the debt plus the value of equity:

$$W_0(\mathbf{x}) = E_0(\mathbf{x}) + B_0(\mathbf{x}) \tag{35}$$

Note that the value of the firm may be simplified using the following equality

$$\max\{Z - A; 0\} + \min\{Z; A\} = Z \quad \forall A \geq 0 \tag{36}$$

Given a set of exogenous parameters, the optimal corporate financial policy of the firm denoted $\mathbf{x}^* \in \mathcal{P}$, where $\mathbf{x}^* = (\alpha^*, C^*, N^*)$, is solution of the following problem:

$$\max_{\mathbf{x}\in\mathcal{P}} W_0(\mathbf{x}) = E_0(\mathbf{x}) + B_0(\mathbf{x}) \tag{37}$$

We show in appendix how to compute (recursively) the value of the debt and equity for a given $\mathbf{x}$. Note importantly that since in practice most bonds are issued “around” the par value, we shall look for optimal corporate financial policy such that $B_0(\mathbf{x}^*) \approx N$.

4 Analytical and numerical results

In this section, we shall present some analytical results on this model, that will be complemented by numerical ones. We first consider the case in which the corporate financial policy is such that there is no-default risk. As we shall see, in the no-default risk no-taxes case, corporate financial policy is irrelevant. This is the famous Modigliani-Miller theorem. When there is now some default risk, we show the corporate financial policy is indeed relevant. In particular, there exists an optimal dividend policy.

4.1 No credit risk

In this paragraph, we show that in the no-taxes case, when $\mathbf{x} \in \mathcal{P}^{ND}$, the value of the firm is invariant with respect to $\mathbf{x}$.

Proposition 1a (Irrelevance of corporate financial policy)
Assume a fixed investment policy and no taxes. If $\mathbf{x} \in \mathcal{P}^{ND}$, then, the value of the firm is equal to

$$W_0(\mathbf{x}) = \mathbb{E}^{\mathbb{Q}} \left[\left(\sum_{j=1}^{J} e^{-rj\delta t} P_j(\omega) \right) + e^{-rJ\delta t} V_J(\omega) \middle| \mathcal{F}_0 \right] \tag{38}$$

Proof. See the Appendix.

The meaning of this proposition is simple. When there is no credit risk and no taxes, shareholders are indifferent between receiving all the net result in dividends at each time $j = 1, 2...J$ or receiving at maturity the capitalized value of the bank account. As a consequence, dividend policy is irrelevant. Different leverage policies lead to a different *split* of the EBIT at each time between bondholders and equity holders, but have no impact on the firm's value. As a consequence, leverage policy is irrelevant. It thus follows that the value of the firm is *invariant* with respect to the corporate financial policy and this invariance result does not depend on the probability measure. It holds in particular under the historical probability measure $\mathbb{P}$. In fact, the choice of some probability measure only affects the *absolute* value of the firm.

This invariance result was first discovered by Modigliani and Miller (1961) in a specific framework, in which they consider a specific dividend policy where all the residual cash-flow is distributed as dividends to shareholders. One of the most (economic) general "irrelevance results" was obtained by Stiglitz (1974). In a general equilibrium setting, Stiglitz (1974) shows, in the no default risk case, that different corporate financial policies leave the opportunity set of each agent invariant, so that they have no impact on the firm's value[11]. In the same way, Rubinstein (1976) obtains an irrelevance result in a model in which, as we do in this paper, the undistributed cash is placed on a zero-NPV project. If we assume that the firm is levered in the Rubinstein (1976) model, then, there is (implicitly) no default risk (time) since the

[11] See Gottardi (1995) for more on this general equilibrium approach.

summation (over time) is done for $s = 1...T$ and not for $s = 1...\min(\tau; T)$, where τ is the default time, and T the maturity. Moreover, if there were default risk in the Rubinstein (1976) model, a bond indenture should specify what happens in default.

Proposition 1b (firm value under the martingale measure)
Under the (equivalent) martingale measure $\mathbb{Q}$, the value of the firm is equal to:

$$W_0(\mathbf{x}) = (J + m)P_0 \quad \forall\, \mathbf{x} \in \mathcal{P}^{ND} \tag{39}$$

The proof is now simple. Permuting expectation and summation in equation (38) gives:

$$W_0(\mathbf{x}) = \left(\sum_{j=1}^{J} e^{-rj\delta t} \mathbb{E}_0^{\mathbb{Q}} P_j(\omega) \right) + e^{-rJ\delta t} \mathbb{E}_0^{\mathbb{Q}} V_J(\omega) \tag{40}$$

Using now martingale measure (23), we thus obtain that:

$$W_0(\mathbf{x}) = JP_0 + V_0 = (J + m)P_0 \tag{41}$$

Note that the value of the firm is invariant with respect to r. However, the value of this firm is not invariant with respect to the maturity J. In some sense, the maturity J contributes to determine the "size of the pie".

Numerical illustrations. Let $N = 10, \sigma = 10\%, \mu = 0, m = 10, J = 10, \tau_c = 0, r = 8\%$. When $C = 0.8$, the value of equity is equal to 40.22 and the value of the bond is equal to 9.78 whatever the dividend policy. When $C = 1.8$, the value of equity is equal to 33.6 and the value of the bond is equal to 16.4 whatever the dividend policy. In these two examples, the volatility σ is low, the multiple m is high, and the coupon is low with respect to the initial price. As a consequence, there no credit risk[12]: the only impact of debt is to change the split of the firm's value among the claimants, but not the total value of the firm. However, for higher coupon, it may be the case that the bond becomes risky when dividend policy becomes higher than some threshold.

Proposition 1c (Relevance of coupon choice with tax)
Assume a fixed investment policy and $\tau_c > 0$. If the corporate financial policy $\mathbf{x} \in \mathcal{P}^{ND}$, then, under the equivalent martingale measure $\mathbb{Q}$, the value of the firm is equal to:

[12] In fact, when $\alpha = 1$ and $C = 1.8$, the default probability is around 0.02%, so that the spread is less than 1bp.

$$W_0(\mathbf{x}) = P_0[m+J] - \mathbb{E}_0^{\mathbb{Q}}\left(\sum_{j=1}^{J} e^{-r\delta tj}(P_j(\omega)-C)\tau_c \ \mathbf{1}_{P_j(\omega)>C}\right) \tag{42}$$

where $G_J = \{j = 1, 2...J : P_j > C\}$

Proof. See the Appendix.

When there is corporate income tax (but no default risk), α and N are irrelevant, but the choice of the coupon is relevant since a higher coupon increases the value of the tax-shield. The existence of an optimal financial policy is obvious since $\mathbf{x}$ must belong to $\mathcal{P}^{ND}$ so that C cannot be too high. When $\alpha < 1$, the firm accumulates some cash so that $P_j < C$ does not necessarily imply default. In the particular no-default risk case in which $P_j(\omega) > C$ for all $j = 1, 2...J$ and all ω, then, the value of the levered firm is equal to:

$$W_0(\mathbf{x}) = \underbrace{(1-\tau_c)JP_0 + mP_0}_{\text{Value of the the unlevered firm}} + \underbrace{\sum_{j=1}^{J} e^{-r\delta tj} \ \tau \ C}_{\text{Value of the tax shield}} \tag{43}$$

In this particular case, we are thus back to the "classic" Modigliani-Miller theorem with taxes.

4.2 Credit risk

What is the impact of the corporate liquidity on the firm value when there is credit risk ? When access to capital market is costly, a levered firm could avoid default in bad state (i.e., when the current cash-flow is low) by paying current interests expenses with accumulated corporate liquidities. However, this simple and intuitive idea is not taken into account in EBIT-based (structural) models (e.g., Broadie and Kaya (2007), Broadie, Chernov, Sundaresan (2007), Fries et al (1997), Goldstein et al (2001)) since 100% the net result hen positive) is distributed as dividends to shareholders while empirical data (see Huang and Huang (2003)) suggest that dividend policy is around 6%. In this paper, we shall show that, from the maximisation of the (total) firm value point of view, it is never optimal to distribute dividends, even in the no-taxes case. However, this need not be the optimal dividend policy from shareholders point of view. We shall show that when m is low, shareholders have an incentive to distribute all the net result in dividends. Fan and

Sundaresan (2000) obtain a similar result in a somewhat different model: they show that when the current cash-flow is higher than the coupon (and when there is no bond covenants), it is optimal for shareholders to pay all the residual cash-flows as dividends. However, in their model, when the firm does not distribute all the residual cash-flows as dividends, the gain to shareholders is not so clear.

4.2.1 Optimal dividend policy given a leverage policy

Recall that $\mathbf{x}(\alpha) = (\alpha, C, N)$ is a given corporate financial policy in which the leverage policy (C, N) is fixed but not dividend policy α, and let α^* be solution of the following optimization problem.

$$\max_{\alpha \in [0,1]} W_0(\mathbf{x}(\alpha)) \tag{44}$$

Proposition 2 (optimal dividend policy)
Assume a fixed investment policy and no taxes[13]. Let $\mathcal{D}_W$ be the set of dividend policies that maximises $W_0(\mathbf{x}(\alpha))$. Then $0 \in \mathcal{D}_W$.

Proof . See the appendix.

The idea of the proof is quite simple but need to be understood. For a fixed dividend policy α, let $j^*(\alpha, \omega) = k$ be the default time along the sample path ω. At default time k, the price is equal to P_k and the value of the assets is thus $V_k = mP_k$. In general, one can find a $\delta\alpha$ such that dividend policy $\alpha - \delta\alpha$ "unlocks" $j^*(\alpha - \delta\alpha, \omega) = k$, that is, becomes a non-default time. In fact, dividend policy $\alpha - \delta\alpha$ gives birth to a new set of sample paths. One the one hand, since there is more periods, the value of the firm must increases. On the other hand, since a lower dividend policy implies a lower default threshold, it may be the case that the price at which default will be declared under dividend policy $\alpha - \delta\alpha$ will be lower than P_k. As a consequence, the net result would be ambiguous. We show that given V_k, the expected discounted value of the assets under the new dividend policy is at least equal to V_k. As a consequence, lower dividend policy always implies a higher firm value. For fixed leverage policy (C, N), what we've shown is that the value of the firm is a *piece-wise decreasing function* of the dividend policy, and this result is true whether or not there is corporate income tax. Figure (1) shows the behavior of the firm value as a function of the dividend policy. We clearly see that there is an important discontinuity around $\alpha = 37\%$.

[13]In appendix, we sketch the proof of the proposition 2 in the case of corporate income tax. Tax doesn't affect the optimal dividend policy.

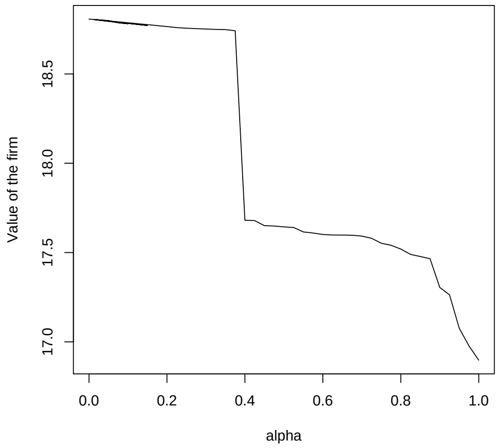

Figure 1: Value of the firm as a function of alpha, vol : 35 %

Proposition 2 shows that the introduction of *credit risk critically affects the Modigliani-Miller irrelevance theorem for dividend policy.* In Stiglitz (1974) seminal paper and in Gottardi (1995), the impact of default risk on corporate financial policy is studied from a general equilibrium theory point of view. However, as noted in Gottardi (1995) p 199-200, no clear answer is provided concerning the validity of the Modigliani-Miller invariance "theorem" for dividend policy when there is credit risk.

DeAngelo and DeAngelo (2006) also suggest that dividend policy is not irrelevant while Black (1976) argues that in a world with taxes, firms should not pay dividend to shareholders. However, their arguments are not related to default risk.

Let $\mathbf{x}^* = (0, C^*, N^*) \equiv (C^*, N^*)$ be the optimal corporate financial policy (see equation (37)). We shall now compute numerically $\mathbf{x}^* \in \mathcal{P}$ while keeping the bond close to its par value.

Numerical illustrations. Let

$$r = 8\%, J = 10, \mu = 0, \tau_c = 0.35, P_0 = 2.5 \tag{45}$$

For each (m, σ), we look for $\mathbf{x}^* \in \mathcal{P}$. Figures 2 and 3 show the value of the firm as a function of α and C. The following tables show the impact on dividend policy on firm value. Note that $\mathbb{Q}$-def is the risk-neutral default probability given by equation (30), and W_0^U is the value of the unlevered firm.

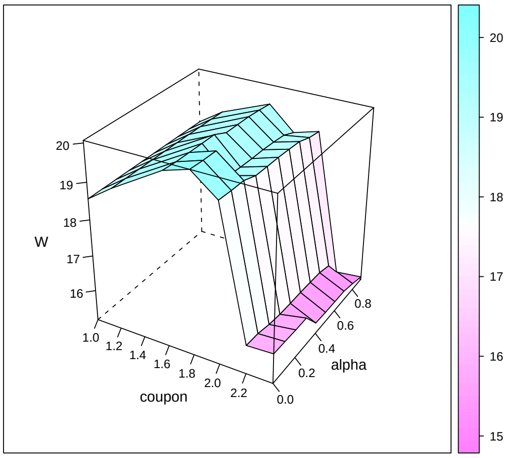

Figure 2: Value of the firm, $m = 0$, vol= 20%

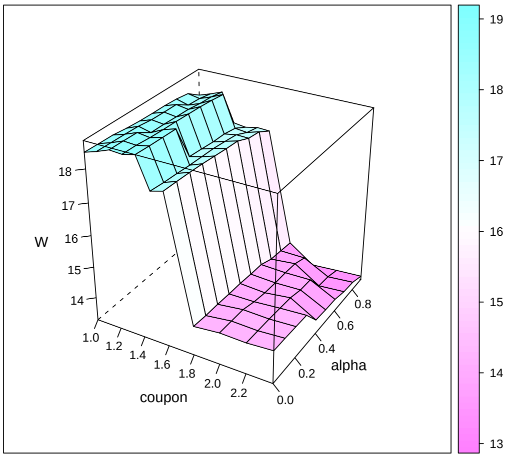

Figure 3: Value of the firm, $m = 0$; vol=35 %

$m=0$, $W_0^U=16.25$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25; 21.3)	21.3	21.3	204 bps	1.234 %
20%	(1.8; 17.4)	17,41	20.06	183 bps	6.350 %
35%	(1.4; 12.5)	12.51	18.8	260 bps	17.28 %

$m=1$, $W_0^U=18.75$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25; 23.6)	23,50	23,80	116 bps	1.234%
20%	(1.8; 18.2)	18.25	22,55	139 bps	6.350 %
35%	(1.4; 13)	12.95	21.29	228 bps	17.28 %

$m=5$, $W_0^U=28.75$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25; 26.55)	26,56	33,80	12 bps	1.234 %
20%	(1.8; 20)	20.03	32.53	60 bps	6.350 %
35%	(1.4; 14.1)	14.09	31.24	148 bps	17.28 %

$m=10$, $W_0^U=41.25$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25; 27)	26,94	46.3	7 bps	1.234 %
20%	(1.8; 21)	20.97	45.02	24 bps	6.350 %
35%	(1.4; 15)	15.07	43.72	86 bps	17.28 %

These numerical results show that

- the value of the firm is a decreasing function of volatility, at least when the volatility is not too high.
- the value of the levered firm is much higher than the value of the unlevered firm (essentially due to tax-shield).
- the optimal leverage policy always implies credit risk.
- when m increases, *ceteris paribus*, the recovery rate increases, and the spread decreases.
- when σ increases, *ceteris paribus*, the default probability also increases.

Note interestingly that when $m=0$, the spread is not a decreasing function of volatility because bondholders may also benefit from a higher volatility since this increases the expected recovery rate.

4.2.2 Agency cost of dividend policy

In practice, since dividend policy is chosen by shareholders (or by managers), we thus have to examine whether or not they have an incentive to implement this first best. We shall call $\alpha^* = 0$ the *first best* dividend policy. Given the optimal leverage policy (C^*, N^*), equityholders maximize equity value with respect to the dividend policy α, that is:

$$\max_{\alpha \in [0,1]} E_0(\mathbf{x}^*(\alpha)) \tag{46}$$

Let $\hat{\alpha}$ be the dividend policy which maximizes equity value. As Mello and Parsons (1992) or Mauer and Ott (2000) did for the agency cost of debt, we define the agency cost of dividend policy[14] given some leverage policy, denoted AC, as the (absolute value) of the loss (in percentage) due to deviation from optimal dividend policy:

$$AC = \frac{|W_0(\mathbf{x}^*(\hat{\alpha})) - W_0(\mathbf{x}^*(0))|}{W_0(\mathbf{x}^*(0))} \geq 0 \tag{47}$$

Proposition 3
Assume a fixed investment. Let $\mathcal{D}_E$ be the set of dividend policies which maximises the value of equity for given leverage policy. If $m = 0$, then, $0 \notin \mathcal{D}_E$.

Proof. See the Appendix.

As noted in Lease *et al.* (1999), when the firm defaults, corporate liquidity (placed on a bank account) is a "common pool" *shared* by all security holders (bondholders, shareholders), while its value is controlled by the dividend policy before default. When the value of the assets is zero (or very low), shareholders can assert with probability one that:

- when default occurs at or prior to maturity, every dollar placed on the bank account will be perceived by bondholders.

[14] La Porta et *al*. (2000) have studies the agency cost of dividend policy from an empirical point of view, but their study is not related to default risk.

- when default has not occurred at maturity, bondholders will be totally repaid before they perceived something.

It follows that corporate liquidity is not a common pool but rather the property of the bondholders. Paying no dividend is thus equivalent to a *wealth transfer* from shareholders to bondholders. As a consequence, they have no incentive to implement the optimal dividend policy.

Based on numerical result, the optimal dividend policy from shareholders point of view is $\hat{\alpha} = 1$ when m is low, indeed lower than 5 in our simulations. The following table show equity and firm's value for the first best dividend policy $\alpha^* = 0$ and equity and firm's value for the optimal dividend $\hat{\alpha} = 1$ seen by shareholders.

$m = 0$					
σ	$\mathbf{x}^*(\alpha)$	$E_0(\mathbf{x}^*(0))$	$E_0(\mathbf{x}^*(1))$	$W_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(1))$
10%	(2.25; 21.3)	0	6.55	21.3	21.28
20%	(1.8; 17.4)	2.65	8.26	20.06	18.66
35%	(1.4; 12.5)	6.3	9.96	18.8	16.9

$m = 1$					
σ	$\mathbf{x}^*(\alpha)$	$E_0(\mathbf{x}^*(0))$	$E_0(\mathbf{x}^*(1))$	$W_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(1))$
10%	(2.25; 23.6)	0.3	6.55	23.8	23.78
20%	(1.8; 18.2)	4.3	8.26	22,55	21.16
35%	(1.4; 13)	8.34	10.41	21.29	19.4

$m = 5$					
σ	$\mathbf{x}^*(\alpha)$	$E_0(\mathbf{x}^*(0))$	$E_0(\mathbf{x}^*(1))$	$W_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(1))$
10%	(2.25; 26.5)	7.23	7.82	33,80	33.78
20%	(1.8; 20)	12.5	12.78	32.52	31.16
35%	(1.4; 14.1)	17.14	17.08	31.24	29.4

The following table gives the agency cost of dividend policy for different value of m and σ.

Agency costs of dividend policy			
	m=0	m=1	m=5
10%	0.1%	0.08%	0.06%
20%	7%	6.2 %	4.2 %
35%	10.1%	8.9 %	No AC

When $m = 0$ and $\sigma = 10\%$, agency costs are very low (0.1%) while shareholders have an incentive to deviate from optimal dividend policy. For the first best dividend policy, the value of equity is zero while when $\hat{\alpha} = 1$, the value of equity is equal to 6.55. As the volatility σ increases, the incentive to choose $\hat{\alpha} = 1$ also increases (since the value of equity increases) while the agency costs become quite important. For example, when $\sigma = 35\%$ and $m = 0$, agency costs represent around 10% of the value of the firm. Note that when m is high (e.g., $m = 10$), numerical simulations reveal that shareholders have no incentive to deviate from the first best dividend policy[15]. Our model yields thus the following empirical prediction:

- when m "low", i.e., when the degree of competivity of the second-hand market for (non redeployable) used assets is low , and when σ is high, dividend policy should be very aggressive and the resulting agency costs are important. If the bond is priced for a low dividend policy, bondholders have an incentive to actively monitor shareholders (i.e., dividend policy) to avoid wealth transfer. We said earlier that m may be thought as an estimation of the value of the assets *net* of the bankruptcy costs. One can also say that when the bankruptcy costs are very high, so that m is low, this gives an incentive to shareholders to choose a very aggressive dividend policy.
- when m is "high", dividend policy should be low.

4.2.3 Huang and Huang (2003) base case

In their paper, Huang and Huang (2003) (see also the Leland (2006) lectures on structural models) consider the predicted quantities of many existing structural models for the following set of parameters:

$$r = 8\%, (C/N) = 8.13\%, \alpha = 6\% \quad (48)$$

The aim of the exercise is to see whether or not our model match the following "target data".

Target data			
Quantities / Rating	A 10Y	Baa (BBB), 10Y	B, 10Y
Leverage ratio	32%	43,3%	65,7%
Spread	123 bps	194 bps	470 bps
Default probability	1,55%	4,39%	44%
Recovery Rate	51%	51%	51%

[15] We have not been able to prove this analytically. For the overall firm, things is "simple" because even one more period increases its value. This is indeed not the case for equity since one more period benefits clearly to bondholders but not to shareholders since they the coupon C may be paid using corporate liquidity.

We have chosen the following base case parameters: $r = 8\%, \alpha = 6\%, C = 1.5, N = 17, (C/N) \approx 8.82\%$, $\mu = 0, \delta t = 1, \tau_c = 35\%, P_0 = 2.5$. We give the result for different values of m and σ. In the tables, R^{ec} is the expectation of the recovery rate (under the martingale measure) conditional on $\omega \in \Omega_d$, i.e., conditional on default event. Note that for $J = 1$, the default probability is zero while the spread is high. In that case, the spread comes from the loss due to recovery rate which is lower than 1.

$m = 8, \sigma = 22\%$						
	B_0	E_0	B_0/W_0	Spread	$\mathbb{Q}$-def	R^{ec}
J=1	16.89	5.24	76.32 %	110 bps	0%	ND
J=4	16.89	11.32	59.8%	62 bps	1.84%	51.7%
7 Y (J=7)	17.19	16.85	5.5 %	26 bps	2.4%	51%
10 Y (J=10)	17,31	22,27	43,36%	20 bps	3 %	50.4%

$m = 8, \sigma = 25\%$						
	B_0	E_0	B_0/W_0	Spread	$\mathbb{Q}$-def	R^{ec}
J=1	16.65	5.45	75.3 %	250 bps	0%	ND
J=4	16.68	11.53	59.1 %	100 bps	2.5%	44.8%
7 Y (J=7)	16.98	17.02	50%	48 bps	4.82 %	48.6%
10 Y (J=10)	17.08	22.39	43.15 %	39 bps	6%	48%

$m = 8, \sigma = 30\%$						
	B_0	E_0	B_0/W_0	Spread	$\mathbb{Q}$-def	R^{ec}
J=1	16.28	5.82	73.6%	478 bps	0%	ND
J=4	16.32	11.93	57.8%	161 bps	8.4 %	50%
7 Y (J=7)	16.39	17.32	48.6 %	112 bps	11.85%	49.45%
10 Y (J=10)	16.52	22.54	42.35%	86 bps	13.8%	48%

$m = 8, \sigma = 40\%$						
	B_0	E_0	B_0/W_0	Spread	$\mathbb{Q}$-def	R^{ec}
J=1	15.47	6.63	70 %	988 bps	0%	ND
J=4	15.04	12.77	54%	393 bps	24.73 %	55%
7 Y (J=7)	15	17.77	45.8%	274 bps	11.85%	49.45%
10 Y (J=10)	15.04	22.55	39.94 %	222 bps	32%	50.7%

$m = 8, \sigma = 45\%$						
	B_0	E_0	B_0/W_0	Spread	$\mathbb{Q}$-def	R^{ec}
J=1	15.06	7.05	68.1%	1200 bps	0%	ND
J=4	14.49	13.3	52.1%	500 bps	27.1 %	49.7%
7 Y (J=7)	14.35	18.32	44%	357 bps	33.4 %	46.21%
10 Y (J=10)	14.37	23.07	38.4%	289 bps	37.7 %	44.7 %

In our model, when m increases, the leverage ratio given by B_0/W_0 decreases since the value of equity increases, but the spread also decreases since the recovery rate increases. On the other hand, when the volatility increases, this increases the spread but also the default probability. As a consequence, our model is unable to match simultaneously *all* the desired quantities such as the the spread, the leverage ratio, and the default probability as in many structural models with no jump component (see Huang and Huang (2003)).

5 Conclusion

In this paper, we have shown how to model the evolution of corporate liquidity in an EBIT-based structural model, and how this corporate liquidity impacts the value of debt, equity, and the total value of the firm. Since the default threshold in our model is both endogenous and stochastic (yet observable), it may be used to compute the default probability for single name credit derivatives (e.g, CDS). We have shown that when there is no credit risk, the value of the firm is invariant with respect to the corporate financial policy. However, when there is credit risk, contrary to what is believed since Modigliani-Miller (1961), we have shown that the value of the firm is not invariant with respect to the corporate financial policy. From the maximization of the firm point of view, given the optimal leverage policy, no distribution of dividends is optimal although shareholders may not always have the incentive to implement this dividend policy.

We leave for further research the empirical test of our model but also the theoretical analysis of the more general model in which the value of the assets is (only) imperfectly correlated with the EBIT and where there is a growth option.

6 Appendix

6.1 Valuation of equity and debt

We shall show how to value debt and equity given a corporate financial policy $\mathbf{x} = (\alpha, C; N)$

Payoff at maturity J

Note that at maturity, $B_J = \mathcal{B}_J$ and $E_J = \mathcal{E}_J$, i.e., the payoff is equal to the value.

- if $P_J > C$ (no default at maturity)

$$\mathcal{E}_J = D_J + \max\{(1-\alpha)R_J + L_{J-1}e^{r\,\delta t} + V_J - N; 0\} \quad (49)$$
$$\mathcal{B}_J = C + \min\{N; (1-\alpha)R_J + L_{J-1}e^{r\,\delta t} + V_J\} \quad (50)$$

- if $P_J < C$ (default at maturity)

$$\mathcal{E}_J = \max\{P_J + L_{J-1}e^{r\,\delta t} + V_J - (N + C); 0\} \quad (51)$$
$$\mathcal{B}_J = \min\{N + C; P_J + L_{J-1}e^{r\,\delta t} + V_J\} \quad (52)$$

When there is no default at maturity, dividend D_J (if ever) is paid to equity holders and the last coupon is paid to debtholders. The nominal is (at least partially) paid with everything of value (if ever), that is, the fraction of the net result which is not distributed, the value of the bank account (if ever) and the liquidative value of the assets. Equation (49) says that the value of equity at maturity is equal to the dividend plus the *maximum* of - the difference between the sum of the everything of value in the firm minus the nominal - and zero. Equation (50) says that the value of the debt is equal to the *minimum* of the nominal and the sum of the everything of value in the firm.

When the net result is negative, the coupon has not been paid and of course no dividend has been paid either. Equation (51) says that the value of equity is equal to the *maximum* of - the difference between the sum of the everything which is worth in the firm minus the nominal plus the last coupon - and zero.

Payoff at a default time $j \leq J$

When the firm defaults at a given time j, then:

$$E_j = \mathcal{E}_j = \max\{P_j + L_{j-1}e^{r\,\delta t} + V_j - (N + C); 0\} \quad (53)$$

$$B_j = \mathcal{B}_j = \min\{N + C; P_j + L_{j-1}e^{r\,\delta t} + V_j\} \quad (54)$$

Note importantly that in a default node, the value is equal to the payoff.

Value at a given non-default time $j \leq J - 1$

At a given time j, the *value* of equity is just the (discounted risk-neutral) expected *payoff*.

$$E_j = e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}(\mathcal{E}_{j+1}|\mathcal{F}_j) = e^{-r\ \delta t}(q\mathcal{E}^u_{j+1} + (1-q)\mathcal{E}^d_{j+1}) \quad (55)$$

$$B_j = e^{-r\delta t}\mathbb{E}^{\mathbb{Q}}(\mathcal{B}_{j+1}|\mathcal{F}_j) = e^{-r\ \delta t}(q\mathcal{B}^u_{j+1} + (1-q)\mathcal{B}^d_{j+1}) \quad (56)$$

where

- for $j < J - 1$

$$\mathcal{E}_{j+1} = \begin{cases} E_{j+1} + D_{j+1} & \text{if } P_{j+1} \geq C \\ E_{j+1} + R_{j+1} > 0 & \text{if } (C - e^{r\ \delta t}L_j) < P_{j+1} < C \\ E_{j+1} \text{ (default)} & \text{otherwise} \end{cases} \quad (57)$$

- for $j = J - 1$, $\mathcal{E}_{j+1}$ is given by equations (49) or (51)

As usual, in the first case, when the coupon can be paid, the payoff of shareholders is equal to the value of equity plus dividends. In the third case, when P_j is lower than the coupon plus the value of the bank account, the firm defaults. In the second case, the net result R_{j+1} is also negative but default is not *a priori* declared. However, when $E_{j+1} + R_{j+1} < 0$, we assume that shareholders declare default since it means that the expected discounted residual cash-flow is lower than the current net result.

At a given time j, the *value* of the debt is just the (discounted risk-neutral) expected *payoff*.

- for $j < J - 1$

$$\mathcal{B}_{j+1} = \begin{cases} B_{j+1} + C & \text{if } (C - e^{r\ \delta t}L_j) < P_{j+1} \\ B_{j+1} \text{ (default)} & \text{if } (C - e^{r\ \delta t}L_j) > P_{j+1} \end{cases} \quad (58)$$

- for $j = J-1$, $\mathcal{E}_{j+1}$ is given by equations (50) and (52).

Working backward until the present date, we thus obtain the value of equity $E(\mathbf{x})$ and the value of the risky debt $B(\mathbf{x})$.

6.2 Proofs

Proof of Lemma 1

Set a dividend policy $0 < \alpha < 1$ and assume that for a given $\mathbf{x}$, $\Omega_d(\mathbf{x})$ is not empty, i.e., there is default risk. Since we focus on dividend policy for a given leverage policy, we shall denote default time as $j^*(\alpha)$ rather than $j^*(\mathbf{x})$. For a given ω, consider the default time $j^*(\alpha,\omega)$.

$$P_{j^*(\alpha,\omega)} + e^{r\delta t} L_{j^*(\alpha,\omega)} < C. \tag{59}$$

The aim is to find a $\delta\alpha \in]0,\alpha]$ such that dividend policy $\alpha - \delta\alpha$ "unlocks" j^*, that is, for dividend policy $\alpha - \delta\alpha$, $j^*(\alpha - \delta\alpha, \omega)$ becomes a *non default time*. If, for $\delta\alpha = \alpha$, the inequality (59) is still true, then, $j^*(0,\omega)$ cannot become a non default time along ω. If the inequality is not true, by continuity of the function $L(\alpha)$, there will exist a smallest $\delta\alpha(\omega) \leq \alpha$ such that:

$$P_{j^*(\alpha-\delta\alpha,\omega)} + e^{r\delta t} L_{j^*(\alpha-\delta\alpha,\omega)} = C. \tag{60}$$

Let $\underline{\delta\alpha} = \inf_{\omega\in\Omega_d} \delta\alpha(\omega) = \delta\alpha(\underline{\omega})$. By construction, given a dividend policy α, one must at least choose an increment of $\underline{\delta\alpha}$ to affect one (or more than one) default probability.

Let $\alpha = 1$ and $1 - \underline{\delta\alpha} = \alpha_H$. We have shown that $\forall\alpha \in]\alpha_H, 1]$, the default probability is invariant with respect to α. Start now with $\alpha = \alpha_H$ and repeat the same way of choosing another $\underline{\delta\alpha}$. Once another $\underline{\delta\alpha}$ is found, we thus have $\alpha_{H-1} = \alpha_H - \underline{\delta\alpha}$. As a consequence, $\forall\alpha \in]\alpha_{H-1}, \alpha_H]$, the default probability is invariant with respect to α. Continuing this procedure, we are going to find a number α_1 such that $\forall\alpha \in [0, \alpha_1]$, the default probability is invariant with respect to α $\square$

Proof of proposition 1 a (no tax)

By iterated conditional expectation theorem, we have:

$$\begin{aligned} B_0(\mathbf{x}) &= \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}(B(\mathbf{x})|\mathcal{F}_J)\Big|\mathcal{F}_0\right] && (61)\\ E_0(\mathbf{x}) &= \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|\mathcal{F}_J)\Big|\mathcal{F}_0\right] && (62) \end{aligned}$$

Since there is no default risk, given $\mathbf{x} \in \mathcal{P}^{ND}$, the default time $j^*(\mathbf{x},\omega) \equiv j^* > J$ $\forall\omega \in \Omega$, where $\omega = (z_1, z_2, ...z_J)$. One can thus write the value of the debt as follows:

$$\mathbb{E}^{\mathbb{Q}}(B(\mathbf{x})/\mathcal{F}_J) = \left[\mathbb{E}^{\mathbb{Q}}\left(\sum_{j=1}^{J} e^{-(r\,j\,\delta t)}C \;+ e^{-r\,J\,\delta t}N\right)\mathbf{1}_{j^*>J}\Big|\mathcal{F}_J\right] \tag{63}$$

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}(B(\mathbf{x})/\mathcal{F}_J) &= \left[\mathbb{E}^{\mathbb{Q}}\left(\sum_{j=1}^{J} e^{-(r\,j\,\delta t)}C \;+ e^{-r\,J\,\delta t}N\right)\Big|\mathcal{F}_J\right]\mathbf{1}_{j^*>J} \\ &= \left(\sum_{j=1}^{J} e^{-(r\,j\,\delta t)}C \;+ e^{-r\,J\,\delta t}N\right) \end{aligned} \tag{64}$$

where the last equality comes from the fact that conditional on $\mathcal{F}_J$, since there is no default risk, $\mathbf{1}_{j^*(\mathbf{x},\omega)>J} = 1 \;\forall\; \omega \in \Omega$.

We use the same methodology for equity. Recalling that there is no corporate tax and that $D_j = (P_j - C)\;\alpha\;\mathbf{1}_{P_j>C}$, it follows that, for a given sample path $\omega \in \Omega$:

$$\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|\mathcal{F}_J) = \left[\sum_{j=1}^{J-1} e^{r\,\delta t}(P_j(\omega) - C)\alpha\;\mathbf{1}_{P_j(\omega)>C}\right] + $$

$$e^{-r\delta tJ}\left[D_J(\omega) + \underbrace{(1-\alpha)R_J(\omega) + L_{J-1}(\mathbf{x},\omega)e^{r\,\delta t} + V_J(\omega) - N}_{>0 \textit{ since there is no default}}\right] \tag{65}$$

Without loss of generality, we've considered the case in which $P_J > C$. We shall now omit the ω and note L_{j-1} rather than $L_{j-1}(\mathbf{x},\omega)$. It is easy to show that the above equation can be rewritten as:

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|\mathcal{F}_J) &= \sum_{j=1}^{J-1} e^{-(rj\delta t)}(P_j - C)\alpha\mathbf{1}_{P_j>C} \\ &+ e^{-rJ\delta t}\left[(P_J - C) + L_{J-1}e^{r\,\delta t} + V_J - N\right]\end{aligned} \tag{66}$$

The value of the cash can be written as follows for all $j \leq J$

$$L_{j-1} = \left[\sum_{k=1}^{j-1} e^{(r\,(j-1-k)\,\delta t)}(P_k - C)(1 - \alpha\;\mathbf{1}_{P_j>C})\right] \tag{67}$$

It follows that:

$$e^{r\,\delta t}L_{J-1} = \left[\sum_{j=1}^{J-1} e^{r\,(J-j)\,\delta t}(P_j - C)(1 - \alpha\;\mathbf{1}_{P_j>C})\right] \tag{68}$$

After simple computations, one can write equation (66) as follows:

$$\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|\mathcal{F}_J) = \left[\sum_{j=1}^{J} e^{-rj\delta t}(P_j - C)\right] + e^{-rJ\delta t}\left[V_J - N\right] \qquad (69)$$

Since $\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})|\mathcal{F}_J) = \mathbb{E}^{\mathbb{Q}}\left[B(\mathbf{x}) + E(\mathbf{x})|\mathcal{F}_J\right]$, it is easy to show that:

$$\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})|\mathcal{F}_J) = \left[\sum_{j=1}^{J} e^{-rj\delta t}P_j\right] + e^{-rJ\delta t}V_J \qquad (70)$$

Up to now, we have shown that the value of the firm $\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})|\mathcal{F}_J)$ is invariant with respect to $\mathbf{x} \in \mathcal{P}^{ND}$. Let $\mathbb{Q}$ be the risk-neutral probability measure.

$$W_0(\mathbf{x}) = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})|\mathcal{F}_J)\Big|\mathcal{F}_0\right] \qquad (71)$$

$$W_0(\mathbf{x}) = \mathbb{E}^{\mathbb{Q}}\left[\left(\sum_{j=1}^{J} e^{-rj\delta t}P_j\right) + e^{-rJ\delta t}V_J\Big|\mathcal{F}_0\right] \qquad (72)$$

$$W_0(\mathbf{x}) = \sum_{\omega\in\Omega}\left[\left(\sum_{j=1}^{J} e^{-rj\delta t}P_j(\omega)\right) + e^{-rJ\delta t}V_J(\omega)\right]\mathbb{Q}(\omega) \qquad (73)$$

where $\mathbb{Q}(\omega)$ is the probability measure of sample path ω knowing $\mathcal{F}_0$.

Proof of Proposition 1 c (corporate income tax)

Since tax doesn't affects debt, we have, as in Proposition 1:

$$\mathbb{E}^{\mathbb{Q}}(B(\mathbf{x})/\mathcal{F}_J) = \left(\sum_{j=1}^{J} e^{-(rj\delta t)}C + e^{-rJ\delta t}N\right) \qquad (74)$$

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|\mathcal{F}_J) &= \left[\sum_{j=1}^{J} e^{-r\, j\delta t}(P_j - C)(1 - \tau\, \mathbf{1}_{P_j>C})\ \alpha\ \mathbf{1}_{P_j>C}\right] + \\ & e^{-r\delta tJ}\left[\underbrace{(1-\alpha\mathbf{1}_{P_J>C})R_J + L_{J-1}e^{r\,\delta t} + V_J - N}_{>0 \text{ since there is no default}}\right]\end{aligned} \qquad (75)$$

Recall that:

$$e^{-r\delta t}L_{J-1} = \left[\sum_{j=1}^{J-1} e^{(r(J-j)\delta t)}(P_j - C)(1-\tau_c\ \mathbf{1}_{P_j>C})(1-\alpha\ \mathbf{1}_{P_j>C})\right] \quad (76)$$

It thus follows that:

$$\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})/\mathcal{F}_J) = e^{-r\ j\delta t}V_j + \sum_{j=1}^{J} e^{-r\ j\delta t}(P_j - C)(1-\tau_c\ \mathbf{1}_{P_j>C}) + e^{-r\ J\delta t}N \quad (77)$$

Summing equations (74) and (77), it follows that:

$$\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})/\mathcal{F}_J) = e^{-r\ j\delta t}V_j + \sum_{j=1}^{J} e^{-r\ j\delta t}(P_j - P_j\tau_c\ \mathbf{1}_{P_j>C} + C\tau_c\ \mathbf{1}_{P_j>C}) \quad (78)$$

Taking now the expectation given $\mathcal{F}_0$, we obtain that:

$$\begin{aligned} W_0(\mathbf{x}) &= mP_0 + JP_0 - \sum_{\omega\in\Omega}\left(\sum_{j=1}^{J} e^{-rj\delta t}(P_j(\omega) - C)\tau_c\ \mathbf{1}_{P_j(\omega)>C}\right)\mathbb{Q}(\omega) \\ &= P_0(m+J) - \mathbb{E}_0^{\mathbb{Q}}\left(\sum_{j=1}^{J} e^{-rj\delta t}(P_j(\omega) - C)\tau_c\ \mathbf{1}_{P_j(\omega)>C}\right) \quad (79) \end{aligned}$$

Note that due to the indicator function, we loose martingality so that we cannot compute this expectation.

Proof of Proposition 2 (no tax)

Lemma A 2.1

Let $W_0(\mathbf{x})|A_k$ be the value at time $t=0$ of the firm conditional $\omega \in A_k$. Then, for a given $\omega \in A_k$

$$[W_0(\mathbf{x})|A_k] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{j=1}^{k} e^{-rj\delta t}P_j(\omega) + e^{-rj\delta t}V_k(\omega)\Big|\mathcal{F}_0\right] \quad (80)$$

Proof

Recall that $A_k = \{\omega \in \Omega : j^*(\mathbf{x},\omega) = k\}$, for $k = 1, 2..J$. We have that

$$
\begin{aligned}
[W_0(\mathbf{x})|A_k] &\equiv \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}} W(\mathbf{x})\mathbf{1}_{A_k}|\mathcal{F}_k)\Big|\mathcal{F}_0\right] && (81)\\
&= \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}} W(\mathbf{x})|A_k\Big|\mathcal{F}_0\right] && (82)\\
&= \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}} E(\mathbf{x})|A_k + \mathbb{E}^{\mathbb{Q}} B(\mathbf{x}))|A_k\Big|\mathcal{F}_0\right] && (83)
\end{aligned}
$$

Note that A_k is measurable with respect to $\mathcal{F}_k$ and that $\mathbf{1}_{A_k} = 1$ conditional on A_k. Fix $\omega \in A_k$ and let $Z_k(\omega) = V_k(\omega) + P_k(\omega) + e^{r\delta t} L_{k-1}(\mathbf{x}, \omega)$.

$$
\begin{aligned}
\mathbb{E}^{\mathbb{Q}}(E(\mathbf{x})|A_k) &= \left[\sum_{j=1}^{k-1} e^{-rj\delta t}(P_j(\omega) - C)\alpha\ \mathbf{1}_{P_j(\omega)>C}\right] \\
&+ e^{-rk\delta t} \max\left[Z_k(\omega) - C) - (N + C); 0\right] && (84)
\end{aligned}
$$

For simplicity, when there is no confusion, we shall omit the notation with ω. Since the coupon is paid until time $k-1$ whenever P_j is higher or lower than C, we have that:

$$
\mathbb{E}^{\mathbb{Q}}(B(\mathbf{x})|A_k) = \sum_{j=1}^{k-1} e^{-rj\delta t} C + e^{-rk\delta t} \min\left[Z_k; (N + C)\right] \tag{85}
$$

Observe that for all $N + C \geq 0$, we have

$$
\min\left[Z_k; (N + C)\right] + \max\left[Z_k - (N + C); 0\right] = Z_k \tag{86}
$$

Recall one more time that:

$$
L_{k-1}(\alpha) = \sum_{j=1}^{k-1} e^{r(k-1-j)\delta t}(P_j - C)\ (1 - \alpha\ \mathbf{1}_{P_j>C}) \tag{87}
$$

Using equations (84) and (85), it follows that:

$$
\mathbb{E}^{\mathbb{Q}}(W(\mathbf{x})|A_k) = \sum_{j=1}^{k} e^{-rj\delta t} P_j + e^{-rj\delta t} V_k \tag{88}
$$

Since

$$
[W_0(\mathbf{x})|A_k] = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}} W(\mathbf{x})|A_k\Big|\mathcal{F}_0\right] \tag{89}
$$

it follows that:

$$
[W_0(\mathbf{x})|A_k] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{j=1}^{k} e^{-rj\delta t} P_j + e^{-rj\delta t} V_k \Big|\mathcal{F}_0\right]\ \square \tag{90}
$$

Note that $W_0(\mathbf{x})$ is just the expectation over the A_k, $k = 1, 2...J$ and $\bar{\Omega}_d$.

If we unlock, as in lemma 1 a sample path ω that so that along ω, time $j^* = k$ is not anymore a default time, it is clear by equation (90) that the first (i.e., left) term in the bracket will increase. However, since a dividend policy $\alpha - \delta\alpha$ implies a default threshold higher than for dividend policy α (see equation ()), it could be that a default time $j^*(\alpha - \delta\alpha) > j^*(\alpha)$, we may also have that $V_{j^*(\alpha-\delta\alpha)} < V_{j^*(\alpha))}$.

Lemma A 2.2 Consider two corporate financial policy $\mathbf{x} = (\alpha, C, N)$ and $\mathbf{x'} = (\alpha - \delta\alpha, C, N)$ for some $\delta\alpha$. Then, $W_0(\mathbf{x'}) \geq W_0(\mathbf{x})$

Proof

In the proof, since C, N are constant, we shall note $W_0(\alpha)$ and $W_0(\alpha - \delta\alpha)$. Recall that we have shown in lemma 1 that there exists a smallest amount $\underline{\delta\alpha} = \inf_{\omega \in \Omega_d} \delta\alpha(\omega) = \delta\alpha(\underline{\omega})$ for which $j^*(\underline{\omega}, \alpha)$ was a default time for dividend policy α but $j^*(\underline{\omega}, \alpha - \delta\alpha(\underline{\omega}))$ is "unlocked" for dividend policy $\alpha - \delta\alpha$, i.e., is not a default time. Set $k \equiv j^*(\underline{\omega}, \alpha)$. By assumption, $j^*(\underline{\omega}, \alpha - \delta\alpha(\underline{\omega})) \geq k + 1$. Indeed, the new dividend policy $\alpha - \delta\alpha$ gives birth to a "new" set of sample paths that "live" after time k that we call $\Omega(\delta\alpha)$. The set $\Omega(\delta\alpha)$ is the "continuation" of some sample paths which were in A_k for dividend policy α.

Set $\underline{\omega} \in \Omega(\delta\alpha)$, and let n, with $n \leq J - k$ be the smallest number of successive "down" after time k for which the firm defaults. Consider the worst case $n = 1$ in which default is declared in "up" and "down". We thus consider the worst scenario in which dividend policy $\alpha - \underline{\delta\alpha}$ allows the firm to default at time $k + 1$ rather than at time k. Seen from time k, if there is one more time before default, there will be one more cash-flow (i.e., more more price), and the value of the firm will increase, as equation (90) clearly shows. The problem concerns V since a lower dividend policy may, as we've seen, imply a lower value of the price at default. Since we have only an "up" and "down" leg, we can use martingality. It thus follows that $\mathbb{E}^{\mathbb{Q}} e^{-r\delta t} V_{k+1} | V_k = V_k$ (see fig (4)). As a consequence, seen from time k, the value of the firm increases when we decrease the dividend policy from α to $\alpha - \underline{\delta\alpha}$. Suppose now that the "up" movement at time $k+1$ is not a default time. As a consequence, one can again look for the smallest number of successive "down" after time $k+1$ for which the firm will default. Assume again (the worst case) this is one period, i.e., at time $k+2$, the firm defaults in "up" and "down". But then, we can once again apply martingality, so that $\mathbb{E}^{\mathbb{Q}} e^{-r\delta t} V_{k+2} | V_{k+1} = V_{k+1}$. Since $V_{k+1} = uV_k$, seen from time k, it thus follows that $\mathbb{E}^{\mathbb{Q}} e^{-r\delta t} V_{k+1} | V_k = V_k$ (see fig (5)). In fact, for default time $k'(\omega) > k$, $\mathbb{E}^{\mathbb{Q}} e^{-r(j(\omega)-k)\delta t} V_{k'(\omega)} | V_k \geq V_k$ for $\omega \in \Omega(\delta\alpha)$, i.e., this is a sub-martingale. The intuition is simple. The set

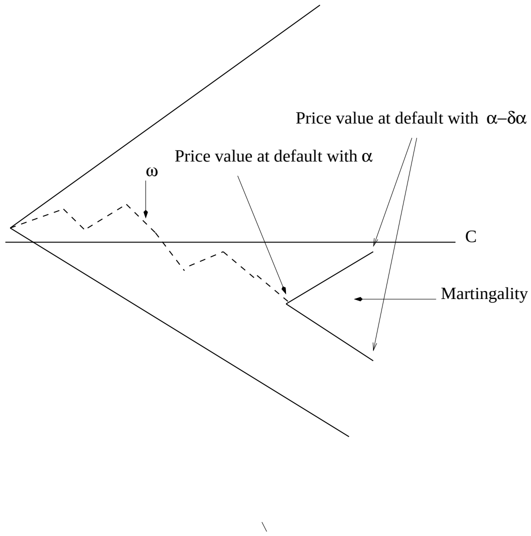

Figure 4:

$\Omega(\delta\alpha)$ contains the sample paths in which the price always increases but not the sample paths in which the price always decreases since there are stopped by default event, hence the sub-martingality.

We have shown that when we "unlock" $\underline{\omega}$, the value of the firm increases. Using the same reasoning, we can iterate this procedure and "unlock" each sample path. Let $\hat{\alpha}$ (possibly 0) be the highest value of α such that, $\forall\ \alpha \leq \hat{\alpha}$, the default probabilities remain invariant. As a consequence, $[0, \hat{\alpha}]$ is optimal. It thus follows that $0 \in \mathcal{D}_W$. $\square$

Sketch of the proof of Proposition 2 (corporate income tax)

Recall that

$$[W_0(\mathbf{x})|A_k] = \mathbb{E}^{\mathbb{Q}}\left[\mathbb{E}^{\mathbb{Q}} W(\mathbf{x})\mathbf{1}_{A_k}|\mathcal{F}_k)\Big|\mathcal{F}_0)\right] \tag{91}$$

Using the same way of reasoning, we obtain that

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}} W(\mathbf{x})\mathbf{1}_{A_k}|\mathcal{F}_k) &= \mathbb{E}^{\mathbb{Q}} W(\mathbf{x})|A_k) \\
&= \sum_{j=1}^{k} e^{-rj\delta t} P_j + e^{-rj\delta t} V_k - \sum_{j=1}^{k-1} e^{-rj\delta t} \tau_c\ P_j\ \mathbf{1}_{P_j > C} \\
&+ \sum_{j=1}^{k-1} e^{-rj\delta t} \tau_c\ C\ \mathbf{1}_{P_j > C}
\end{aligned} \tag{92}$$

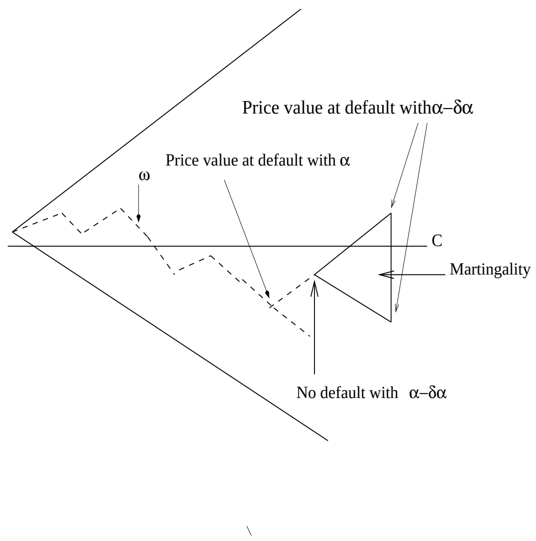

Figure 5:

It is easy to see from the above equation that tax does not change proposition 2.

Proof of proposition 3.

To show that $\alpha = 0$ is not optimal from shareholders point of view, it suffices to show that $E_0(0, C, N) < E_0(\alpha, C, N)$ for some $\alpha > 0$. Let $\mathbf{x} = (0, C, N)$. Note that when $m = 0$, the value of the asset is zero, i.e., $V = 0$. When $m = 0$, on sample path $\omega \in \Omega_d(\mathbf{x})$, the value of equity is 0 since there is no possible recovery for shareholders. However, on sample path $\omega \in \bar{\Omega}_d(\mathbf{x})$, E_J may be positive. Let

$$A(\mathbf{x}) = \{\omega \in \Omega_d(\mathbf{x}) : (1 - \alpha \mathbf{1}_{P_J > C}) R_J(\omega) + L_{J-1}(\mathbf{x}, \omega) e^{r\,\delta t} - N > 0\} \quad (93)$$

If $A(\mathbf{x}) = \emptyset$, then, $E_0(\mathbf{x}) = 0$ for all sample paths. When $\alpha > 0$, since $P_0 \geq C$, there is a positive probability that at least at time 1, $P_1 > C$, in which case, the dividend is positive. As a consequence, $E_0(\mathbf{x}) > 0$

If $A(\mathbf{x}) \neq \emptyset$, then, we need to distinguish as to whether $\omega \in A(\mathbf{x})$ or not. Let $\bar{A}(\mathbf{x}) = \{\omega \in \Omega \backslash A(\mathbf{x})\}$. Since the value of equity $E_0(\mathbf{x})$ is an expectation, one can re-write it as follows:

$$E_0(\mathbf{x}) = \underbrace{[E_0(\mathbf{x}) | A(\mathbf{x})]}_{>0} \; \mathbb{Q}(A(\mathbf{x})) + \underbrace{[E_0(\mathbf{x}) | \bar{A}(\mathbf{x})]}_{=0} \; \mathbb{Q}(\bar{A}(\mathbf{x})) \quad (94)$$

Let $\mathbf{x}' = (\alpha' = \delta\alpha, C, N)$, where $\delta\alpha > 0$. Since the sample paths in the set $A(\mathbf{x})$ is defined by an inequality, by continuity, we can find a positive small enough α' such that $A(\mathbf{x}) = A(\mathbf{x}')$, so that $\mathbb{Q}(A(\mathbf{x}')) = \mathbb{Q}(A(\mathbf{x}))$. As a consequence, it follows that $[E_0(\mathbf{x}')|A(\mathbf{x}')]\ \mathbb{Q}(A(\mathbf{x}')) > [E_0(\mathbf{x})|A(\mathbf{x})]\ \mathbb{Q}(A(\mathbf{x}))$. Since there is now dividends under corporate financial policy $\mathbf{x}'$, it follows that $[E_0(\mathbf{x}')|\bar{A}(\mathbf{x}')]\mathbb{Q}(\bar{A}(\mathbf{x}')) > [E_0(\mathbf{x})|\bar{A}(\mathbf{x})]\mathbb{Q}(\bar{A}(\mathbf{x}))$. Hence, $E_0(\mathbf{x}') > E_0(\mathbf{x})$ $\square$

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