How the corporate liquidity process affects the value of the firm

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Outline

1. Corporate finance foundations for (credit risk) structural models.
   – Decisions of a firm
   – Solvency of the firm
   – Operating liquidity of the firm

2. Our model : modelling operating liquidity and solvency
   – Classical results of corporate finance
   – Our results

3. Conclusion

Aim of the paper

To provide a structural model in which

– a (clear) distinction is made between solvency and liquidity.

– default is "liquidity driven".

– all quantities are observables (e.g., the value of the assets, the default barrier...).

– can be recast in corporate finance literature.

The basic decisions of the firm

• Long-term decisions (e.g, 10 years). **Investment** in fixed assets and **financing source** (debt versus equity).

Fixed assets : Production $\longrightarrow$ Sales $\longrightarrow$ EBIT

EBIT $=$ Sales - Production costs $\approx$ Operating result

$=$ Income used to pay the claimholders

Three basic claim-holders over the firm's EBIT :

1. Debt-holders receive *interests expenses*
2. Government receive *corporate taxation*
3. Shareholders receive the *net income* R

The basic decisions of the firm

- Short-term decisions (1 year).

– When $R > 0$: **dividends** versus **corporate liquidity**
– When $R < 0$: no dividends, the firm may **defaults**

- At some time $t > 0$, long-term decisions (e.g, 10 years). Increase the production capacity : **growth option** (not considered here but see Asvanumt-Broadie-Sundaresan (2007))

Where do we observe the basic decisions of the firm ?

Answer : in the *financial statements* :

– Balance sheet,
– Profit and Loss statement (income statement).

Company Reports (Consolidated)

Annual Interim Quarterly Nine Monthly

Period Covered	12 Months		12 Months
Report Currency	(EUR)		(EUR)
Profit & Loss	**31Dec07**	**% Change**	**31Dec06**
Revenue	30,427.8m	18.70	25,634.3m
Total Income	30,662.1m	18.60	25,853.1m
Operating Costs	27,649.8m	18.44	23,344.5m
~Depreciation	1,587.2m	16.89	1,357.9m
~Total Staff Costs	7,419.5m	13.29	6,549.2m
Operating Result	3,012.3m	20.08	2,508.6m
Net Int Exp/(Inc)	811m	39.44	581.6m
Interest Payable	1,006.5m	37.18	733.7m
Pretax Profit/Loss	2,326.8m	13.50	2,050.1m
Tax	743.8m	11.45	667.4m
Net Profit/Loss	1,583m	14.49	1,382.7m
Net Attributable	1,461m	15.00	1,270.4m
Common Stk Dividends	714m	16.61	612.3m
Retained Profit/Loss	747m	13.51	658.1m
Balance Sheet-Assets			
Total Current Assets	17,213m	1.94	16,884.8m
~Cash & Equivalents	4,223.8m	-18.06	5,154.8m
~Receivables	11,101.3m	16.82	9,503.1m
~Inventories	702.3m	16.16	604.6m
Fixed Assets	32,331.5m	5.99	30,503m
Total Assets	49,544.5m	4.55	47,387.8m
Balance Sheet-Liabilities			
Total Liabilities	41,347.9m	6.52	38,817.6m
~Current Liabilities	19,099.9m	9.11	17,505.9m
~Long Term Debt	18,640.5m	5.70	17,635m
~Deferred Liabilities	1,067.2m	5.14	1,015m
Minority Interests	571.8m	-23.60	748.4m
Shareholders Equity	7,624.8m	-2.52	7,821.8m

Corporate liquidity

Empirical evidence on cash holdings (Opler *et al* 1999)

– Firms with strong growth opportunities and riskier EBIT hold relatively high ratios of cash.

– Firms that have greatest acces to the capital markets (e.g., firms with high rating) tend to hold lower ratio of cash.

Why do firms hold cash ?

1. To finance its activities and investments

2. To avoid default when EBIT is low.

Financial statement : balance sheet

The balance sheet assets (frequency of publication : 1Y).

1. Fixed assets
   - Tangible fixed assets (building, plant, equipment,...)
   - Intangible assets (patents, brand,...)
2. Current assets
   - Cash (cumulated corporate liquidity)
   - other assets (inventories, accounts receivables)

Solvency : ability to redeem debt-holders with the value of the fixed assets (if they were to be sold) plus the value of the current assets.

Two remarks

– International Financial Reporting Standards (IFRS) : principle of *fair value*

$$\text{book value} \approx \text{market value (MtM)}$$

– Market value of the fixed assets : function of the *competitivness* of the second-hand market for used assets.

Financial statement : profit and loss statement

The (simplified) income statement (frequency of publication : 1 Y).

1. Sales - production costs = EBIT (operating result)
2. - interest expenses C (coupon)
3. - income tax : $\tau_c(\mathrm{EBIT} - C)$
4. Net income :R

$$R = (1 - \tau_c)(\mathrm{EBIT} - \mathrm{C}) = (1 - \tau_{\mathrm{c}})\mathrm{EBIT} - \mathrm{C} + \tau_{\mathrm{c}}\mathrm{C}$$

C is tax-deductible. Tax-shield : $\tau_c C$

Current operating liquidity : ability to pay current interests to debtholders expenses with the current operating result (EBIT).

Modelling perspectives

What to model ?

– The *solvency* of the firm : the value of the fixed assets,

– The *current operating liquidity* of the firm : the value of the EBIT.

Existing literature : structural models

– Asset value based models (e.g, Merton 1974, Leland 1994)

V_t : dynamics of value of the fixed assets.

No liquidity variable : no role for corporate liquidity

– EBIT based models (e.g, Goldstein-Leland-Ju 2001)

V_t : dynamics of the value of the expect. disc. flow of net income

Dividends = net income : no role for corporate liquidity

Main assumptions

1. The firm produces costlessly at each time j one unit of a good sold at the market price P_j (i.e., EBIT=price)
2. The firm has issued *at par* a bond of maturity J, nominal N, and coupon $C = \bar{C}\delta t$,
3. The firm placed a positive fraction of its net income that capitalized at the risk-free rate r : this is the *corporate liquidity*.
4. $(\Omega, \{\mathcal{F}_j\}_{j=1,2..J}, \mathcal{F}, \mathbb{P})$: filtered probability space that supports the price (discrete-time) dynamics :

$$P_j = P_{j-1}\, e^{\mu\,\delta t + z_j\,\sigma\sqrt{\delta t}} \tag{1}$$

   - z_j, $j = 1, 2...J$: i.i.d Bernoulli random variables such that $\mathbb{P}(z_j = +1) = \mathbb{P}(z_j = -1) = 0.5$
   - $(\mathcal{F}_j, 1 \leq j \leq J)$: natural filtration associated to the z_j,
   - ω : realization of $(z_1, z_2, ...z_J)$

The liquidity process

- τ_c : corporate income tax,
- R_j : net income at time j,
- D_j : dividends at time j,
- L_j : corporate liquidity at time j
- $\mathbf{1}_{P_j > C}$: indicator function on the event $P_j \geq C$
- $\mathbf{x} = (\alpha, C, N)$: corporate financial policy
  - $\alpha \in [0, 1]$: constant *dividend policy*,
  - (C, N) : *leverage policy*.

$$R_j = (P_j - C)(1 - \tau_c \, \mathbf{1}_{P_j > C}) \tag{2}$$

$$D_j = R_j \, \alpha \, \mathbf{1}_{P_j > C} \tag{3}$$

$$L_j = L_{j-1} \, e^{r \, \delta t} + (1 - \alpha \mathbf{1}_{P_j > C}) R_j \tag{4}$$

The asset process

Fixed assets may be specialized (e.g., a roller coaster, nuclear reactor or not (e.g., real estate)).

We assume *specialized* fixed assets so that :

$$V_j = m\ P_j \qquad m \geq 0 \tag{5}$$

The value of m is related to the *competitivness* of the second-hand markets for used assets.

– m low : few potential candidate to buy the used assets,
– m high : competitive second-hand market for used assets (V is the expected discouted value of the cash-flow)

How is default declared

In structural models, default is declared at the first time the value of the assets hits a given barrier.

– Constant barrier in Leland (1994).

– Exponentiel barrier in Black and Cox (1976)

Problems.

– The default barrier is not observable.

– Default arises from a “solvency crisis”.

How is default declared

Let $j^*(\mathbf{x})$ be the default (liquidation) time.

$$j^*(\mathbf{x}) = \inf\left\{1 \leq j \leq J:\ P_j + L_{j-1}(\mathbf{x})\, e^{r\,\delta t} < C\right\} \tag{6}$$

Alternative (but equivalent) default) condition

$$j^*(\mathbf{x}) = \inf\left\{1 \leq j \leq J:\ P_j < \underbrace{C - (L_{j-1}(\mathbf{x})\, e^{r\,\delta t})}_{C_j^{def}(\mathbf{x})}\right\} \tag{7}$$

Remarks

– $C_j^{def}(\mathbf{x})$: endogenous stochastic barrier

– Default : declared by a "‘liquidity crisis"’, not by a "‘solvency crisis"’

Balance sheet at a default time

Bond indenture : in a default situation, bondholders have the right to be fully redeemed (i.e., to receive $N + C$) before shareholders receive something.

$Z_{j^*(\mathbf{x})} = L_{j^*(\mathbf{x})-1}\, e^{r\,\delta t} + P_{j^*(\mathbf{x})} + V_{j^*(\mathbf{x})}$: value of the firm at default

Assets	Liabilities and Equity
Cash : $L_{j^*(\mathbf{x})-1}\, e^{r\,\delta t} + P_{j^*(\mathbf{x})}$ Fixed assets : $V_{j^*(\mathbf{x})} = mP_{j^*(\mathbf{x})}$	Debt : $\min\{Z_{j^*(\mathbf{x})}, N + C\}$ Stock : $\max\{Z_{j^*(\mathbf{x})} - (N + C); 0\}$
$Z_{j^*(\mathbf{x})}$	$Z_{j^*(\mathbf{x})}$

max : *limited liability* of shareholders

Bond valuation for fixed financial policy

We say that there is **credit risk** if at least one condition is met :

– *Default risk* : for at least one ω, $j^*(\mathbf{x},\omega) \leq J$,

– *Recovery (rate) risk*
  – bondholders'recovery lower than $N+C$ if $j^*(\mathbf{x},\omega) \leq J$
  – bondholders'recovery lower than N if $j^*(\mathbf{x},\omega) > J$

For a fixed leverage policy (C,N), the dividend policy α affects :

– the *default time*

– the *bondholders' recovery.*

$\mathbb{Q}(j^*(\mathbf{x}) \leq J)$: risk-neutral default probability (independant of m)

Bond valuation for fixed financial policy

$$B_0(\mathbf{x}) = \mathbb{E}_0^{\mathbb{Q}} \left[\sum_{j=1}^{J-1} e^{-(r\, j\ \delta t)}\ C\, \mathbf{1}_{j^*(\mathbf{x},\omega)>j} \right] +$$

$$\mathbb{E}_0^{\mathbb{Q}} e^{-(r\, J\, \delta t)} \left[\left[C + \min\{N, R_J - D_J + L_{J-1}\, e^{r\, \delta t} + V_J\}\right] \mathbf{1}_{j^*(\mathbf{x},\omega)>J} \right]$$

$$+\mathbb{E}_0^{\mathbb{Q}} \left[e^{-(r\, j^*(\mathbf{x},\omega)\, \delta t)} \min\left\{ Z_{j^*(\mathbf{x},\omega)}; N + C \right\} \mathbf{1}_{j^*(\mathbf{x},\omega)\leq J} \right]$$

Once $B_0(\mathbf{x})$ is known, we can compute the credit spread.

Equity valuation for fixed financial policy

$$E_0(\mathbf{x}) = \mathbb{E}_0^{\mathbb{Q}} \left[\sum_{j=1}^{J-1} e^{-(r\, j\, \delta t)} D_j \mathbf{1}_{j^*(\mathbf{x},\omega)>j} \right] +$$

$$\mathbb{E}_0^{\mathbb{Q}} e^{-(r\, J\, \delta t)} \left[D_J + \max\left\{ (R_J - D_J + L_{J-1}\, e^{r\, \delta t} + V_J - N; 0 \right\} \right] \mathbf{1}_{j^*(.)>J}$$

$$\mathbb{E}_0^{\mathbb{Q}} \left[e^{-rj^*(\mathbf{x},\omega)\delta t} \max\left\{ Z_{j^*(\mathbf{x},\omega)} - (N + C); 0 \right\} \mathbf{1}_{j^*(\mathbf{x},\omega)\leq J} \right] \qquad (8)$$

Value of the firm for fixed financial policy

The value of the firm :

$$W_0(\mathbf{x}) = E_0(\mathbf{x}) + B_0(\mathbf{x}) \tag{9}$$

Objective of the firm : maximization of the firm value with respect to the financial policy.

$$\max_{\mathbf{x}} W_0(\mathbf{x}) \quad = \quad E_0(\mathbf{x}) + B_0(\mathbf{x}) \tag{10}$$

$\mathbf{x}^* = (\alpha^*, C^*, N^*)$: solution of (10).

α^* : first best dividend policy.

(C^*, N^*) : first best leverage policy.

Classical results of corporate finance

Modigliani-Miller theorems.

- Proposition 1a : $\tau_c = 0$, $\alpha = 1$, W_0 : invariant with respect to the leverage policy.
- Proposition 1b : $\tau_c > 0$, $\alpha = 1$. W_0 : increases with C (tax-shield).
- Proposition 2 : fixed leverage policy. W_0 : invariant with respect to α.

Results and comments

Rresults

1. When there is no credit risk case, $\tau_c = 0$, $W_0(\mathbf{x})$ is invariant with respect to $\mathbf{x}$
2. When there is credit risk, $\tau_c \geq 0$, for fixed leverage policy, $\alpha^* = 0$ is an optimal dividend policy.

Comments

– No credit risk case : Modigliani-Miller propositions hold.

– Credit risk case : affects critically proposition 2 of Modigliani-Miller.

– Maximizing the corporate liquidity is optimal from the firm point of view.

Optimal financial policy : numerical results

Parameters : $r = 8\%, J = 10, \mu = 0, \tau_c = 0.35, P_0 = 2.5$

$\mathbf{x}^* = (\alpha^* = 0, C^*, N^*)$ maximizes $W_0(\mathbf{x})$

– As C increases, the tax-schield increases, and $W_0(0, C, N)$ increases

– As C increases, default risk increases, and $W_0(0, C, N)$ decreases

Optimal leverage policy :

– Optimal coupon : best tradeoff between tax-shield and default risk.

– Nominal : chosen so that the value of the bond is issued at par. (W_0 is invariant with respect to N)

Optimal financial policy : numerical results

$m = 5$, $W_0^U = 28.75$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25 ; 26.55)	26,56	33,80	12 bps	1.234 %
20%	(1.8 ; 20)	20.03	32.53	60 bps	6.350 %
35%	(1.4 ; 14.1)	14.09	31.24	148 bps	17.28 %

$m = 10$, $W_0^U = 41.25$					
σ	$\mathbf{x}^*(0)$	$B_0(\mathbf{x}^*(0))$	$W_0(\mathbf{x}^*(0))$	Spread	$\mathbb{Q}$-def
10%	(2.25 ; 27)	26,94	46.3	7 bps	1.234 %
20%	(1.8 ; 21)	20.97	45.02	24 bps	6.350 %
35%	(1.4 ; 15)	15.07	43.72	86 bps	17.28 %

Incentives

In practise, shareholoders choose dividend policy.

Do shareholders have an incentive to implement the optimal dividend policy ?

Let $\mathbf{x}^*(\alpha) = (\alpha, C^*, N^*)$: fixed leverage policy

Shareholders choose $\alpha = \hat{\alpha}$ to maximize the value of equity.

Agency cost of dividend policy : cost of deviation from optimal dividend policy

$$AC = \frac{|W_0(\mathbf{x}^*(\hat{\alpha})) - W_0(\mathbf{x}^*(0))|}{W_0(\mathbf{x}^*(0))} \geq 0 \tag{11}$$

Incentive to deviate from optimal dividend policy : depends on m

– m "low" : incentive to deviate. ($\hat{\alpha} = 1$)
– m "high" : no incentive to deviate (numerical result).

Agency costs : numerical illustration

Agency costs of dividend policy			
	m=0	m=1	m=5
10%	0.1%	0.08%	0.06%
20%	7%	6.2 %	4.2 %
35%	10.1%	8.9 %	No AC

Interpretation in terms of "property rights"

Property rights interpretation

Corporate liquidity in default : "common pool" *shared* by all security holders (bondholders, shareholders).

When the value of the assets is very low, shareholders can assert with probability one that :

– when default occurs at or prior to maturity, every dollar placed on the bank account will be perceived by bondholders.
– when default has not occurred at maturity, bondholders will be totally repaid before they perceived something.

Corporate liquidity : not a common pool but (rather) the property of the bondholders.

Paying no dividend : *wealth transfer* from shareholders to bondholders.

No incentive to implement the optimal dividend policy.

Conclusion

- Our model incorporate liquidity and solvency : interesting for financial analysts.
- liquidity and solvency are observables quantities : interesting for CDS (default probability and recovery rate)
- Further work
  - Empirical application (in progress)
  - growth option (Asvanumt-Broadie-Sundaresan (2007) : liquidity may finance growth options)
  - bi-variate model, liquidity and solvency are correlated (solvency and liquidity barriers) (in progress)