A numerical simulation of the human arterial network based on non invasive measurements

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Objectives

- The numerical simulation of blood flow through arteries is a very challenging problem as it involves complex 3D fluid/structure models.
- 1D models have been developpped in order to reduce the computational cost of 3D models.
- The objective here is to show it is possible to fit the parameters of these 1D models by using non invasive experimental measurements.

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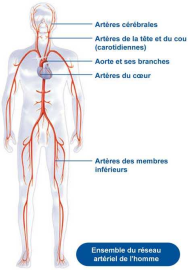

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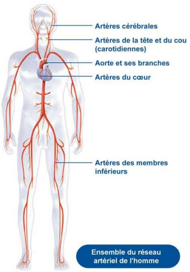

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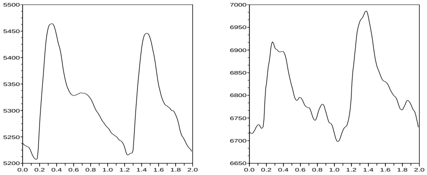
- The pictures below give an example of echotracking measurements of the external iliac and femoral artery diameter for two given patients.
- Patient 1 :



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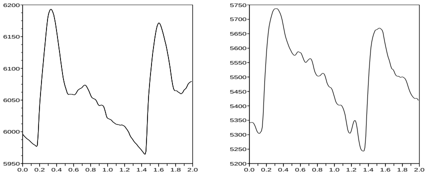

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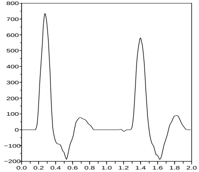

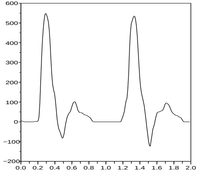

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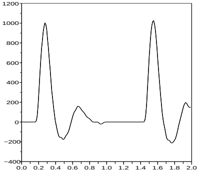

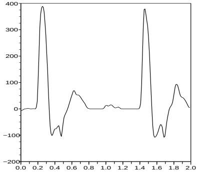

The 1D fluid/structure model : the equations

- After writing the conservation of mass and momentum and assuming a cylindrical symmetry of the artery, $A(t,z)$ and $Q(t,z)$ are found to be solutions of the coupled system :

$$\begin{cases} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial z} = 0 \\ \frac{\partial Q}{\partial t} + \frac{\partial}{\partial z}\left(\frac{Q^2}{A}\right) + \frac{A}{\rho}\frac{\partial p}{\partial z} + K_r \frac{Q}{A} = 0 \end{cases}$$

- To close this system, we assume that the pressure satisfies the following empirical law :

$$P(t,z) - P_{ext} = \beta(A^{\frac{1}{2}} - A_0^{\frac{1}{2}})$$

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The 1D fluid/structure model : the parameters

- The parameter K_r represents the viscous resistance of the flow per unit length of tube and is fixed here equal to 0.75.
- The main value that needs to be estimated is the coefficient β. This value, linked to the compliance of the artery, can be a constant or a function of z. There exists a theoretical value of β issued from a formal averaging of 3D quantities :

$$\beta(z) = \frac{4\sqrt{\pi}h_0 E(z)}{3A_0}$$

  where h_0 and $E(z)$ respectively denote the thickness and the Young modulus of the vessel wall.
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The 1D fluid/structure model : conservative form

- The 1D model can be rewritten in a conservative form :

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial z} = B(U)$$

where $U = (A, Q)^t$ and $F(U) = \begin{pmatrix} Q \\ \frac{Q^2}{A} + \frac{\beta}{3\rho} A^{\frac{3}{2}} \end{pmatrix}$.

- It can be observed that this system is hyperbolic, as the Jacobian $H = \frac{\partial F}{\partial U}$ has always two real and distinct eigenvalues for all the allowable values of U : $\lambda_i = \frac{Q}{A} \pm c$ where

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The 1D model : boundary conditions

- As $\lambda_1 > 0$ and $\lambda_2 < 0$, the system is completed by two appropriate boundary conditions, one at each end for the characteristic variables W_1 and W_2.
- After some algebraic manipulations, it can be seen that a pressure profile can be imposed at the entrance.
- At a bifurcation in a network of arteries, three additional conditions are imposed (conservation of mass and two pressure conditions).

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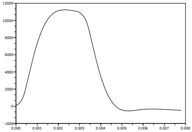
The direct problem : the discretization scheme

- The equations are discretized in time in their conservative form by using a second order Taylor Galerkin scheme :

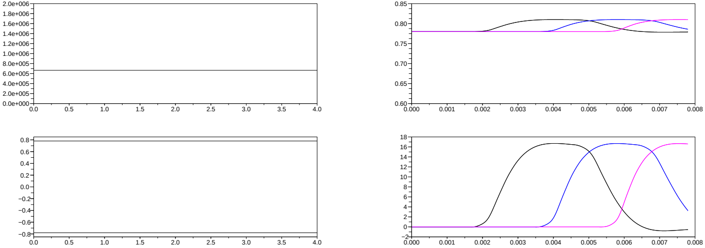
$$U^{n+1} = U^n - \Delta t \frac{\partial}{\partial z}\left(F^n + \frac{\Delta t}{2} \frac{\partial F}{\partial U}^n B^n\right) - \frac{\Delta t^2}{2}\left(\frac{\partial B}{\partial U}^n \frac{\partial F^n}{\partial z} - \frac{\partial}{\partial z}\left(\frac{\partial F}{\partial U}^n \frac{\partial F^n}{\partial z}\right)\right) + \Delta t\left(B^n + \frac{\Delta t}{2} \frac{\partial B}{\partial U}^n B^n\right)$$

- The spatial discretization is then done by using linear finite elements on a subdivision of $[0, L]$.
- At the entrance, a pressure profile is assumed, for instance for the test cases presented here :



The direct problem : a healthy artery

- A first example consists of an artery with a constant compliance and a constant reference cross section.
- The pressure wave is propagating as expected, with a speed proportionnal to β.



- The values of A and Q (left) and p (right) are depicted at three different positions along the artery.

The direct problem : a diseased artery

Blood flow simulation

L. Dumas

Objectives

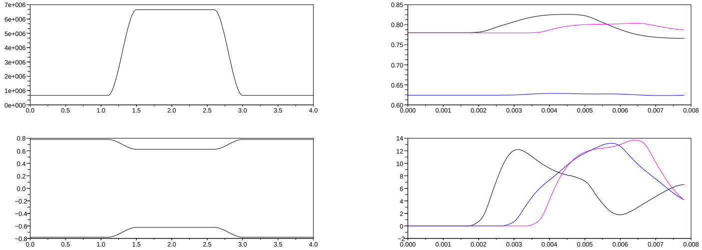
1D model

Direct problem

Inverse problem

Conclusions, perspectives

- A second example concerns an artery with a compliance step at a portion of its length, representing for instance the presence of a plaque :



- In this case, the propagation of the pressure wave is strongly perturbed by the compliance discontinuity.

The inverse problem : the cost function

- In view of constructing an optimal network with respect to experimental values from echotracking, the cost function to minimize has the following form :

$$J(\beta) = \sum_{pts \in \{P_1,\ldots,P_N\}} \sum_{i=1}^{M} (errA(t_i, pts) + errQ(t_i, pts))$$

with

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The inverse problem : a first attempt

- A first attempt to solve the inverse problem has been done by J.F. Gerbeau *et al* (*ESAIM Proc.*, 2005) in the case of a constant value of β.
- The dual problem of the 1D model is constructed and solved in order to compute the gradient of the cost function with respect to β.
- Because of the complexity of the cost function in the general case, a gradient-free approach has been preferred here for the optimization process.

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- They are based on the evolution of a population following the three main Darwinian stochastic principles of selection, crossover and mutation.
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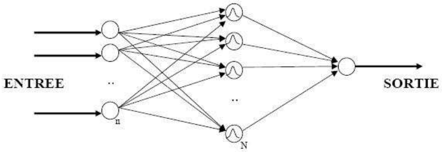
The RBF approximation method

- The RBF approximation method lies on an analogy with a neural network with three layers and N internal neurons :



- The output of the ith internal neuron writes : $y_i = h(||(x_1, ...x_n) - X_i||)$ where h is the radial basis function $r \mapsto h(r)$ and X_i is a network example point.
- The output of the network is a weighted linear combination of the output of the N internal neurons :

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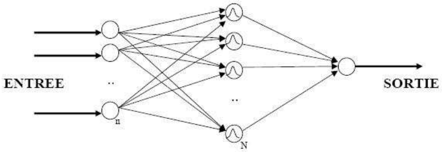

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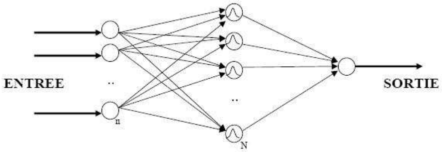

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- In this case, the weights w_i are solutions of the linear system :

$$Aw = z$$

where the general term of the matrix $A \in \mathcal{M}_N(\mathbb{R})$ is equal to : $a_{i,j} = h(||X_i - X_j||)$.
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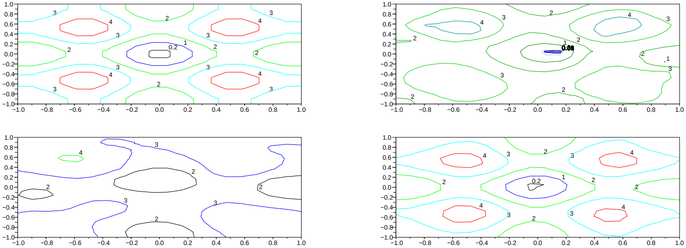
Example of RBF approximation on a test function

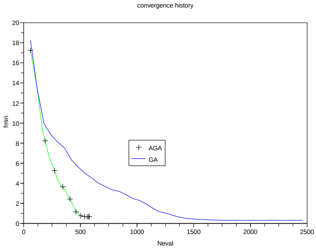
- The n-Rastrigin function :

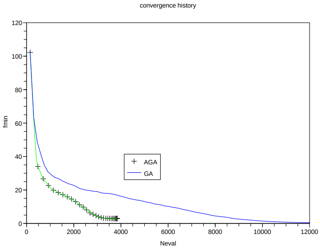
$$Rast(x_1, ..., x_n) = \sum_{i=1}^{n} \left(x_i^2 - \cos(2\pi x_i)\right) + n$$



Evolutionary algorithms with RBF approximation

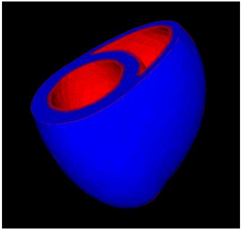
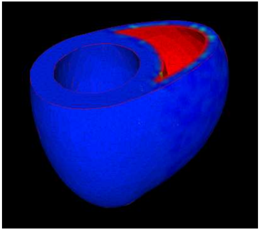
- Example on the Rastrigin function (with $n = 6$ and $n = 20$ respectively) :





Evolutionary algorithms with RBF approximation

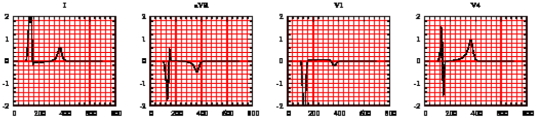
- Example on the cardiac resynchronisation with a pacemaker (joint work with L. El Alaoui).
- The pathological case corresponds to a left bundle branch block :

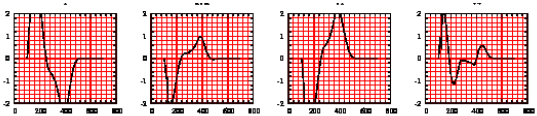
- The cost function J is based on the depolarization time.

Evolutionary algorithms with RBF approximation

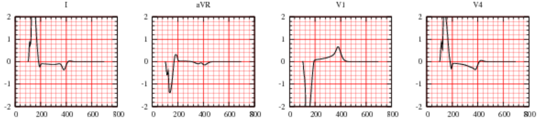
- Healthy case ($J = 0$ms) :



- Pathlogical case ($J = 73$ms) :

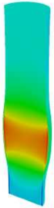


- Pathlogical case with optimal treatment ($J = 34.75$ms) :



The inverse problem : a healthy artery

- The objective of this first inverse problem is to fit the 1D model with the results of an existing 3D computation (done with the LifeV software).



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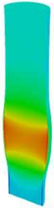

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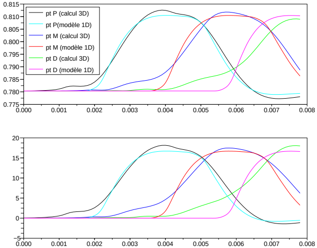

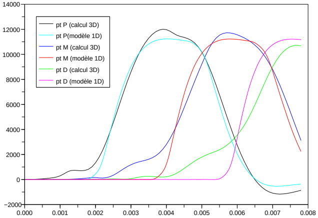

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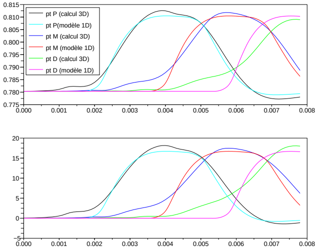

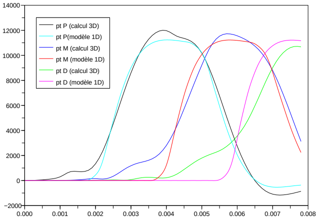

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The inverse problem : a diseased artery

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- In this case, the six unknowns are respectively a_1, a_2, β_1, β_2, S_1 and S_2.
- As a first validation test case, the target results are issued from synthetic data computed with the 1D model.
- The objective is to show that with a reduced number of spatial information, the model can recover the artery geometry and properties.

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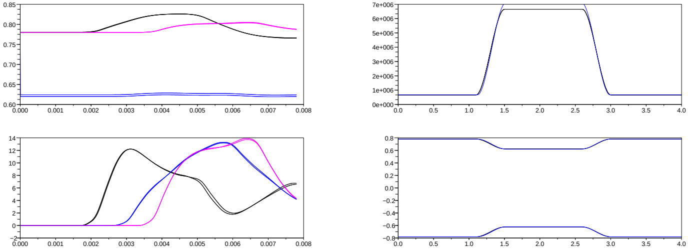
- When the profiles of A and Q are known at three different positions, the optimization process is able to reconstruct a very precise shape of $z \mapsto \beta(z)$ and $z \mapsto A_0(z)$:



- The values of A and Q are also perfectly recovered at the three sections.

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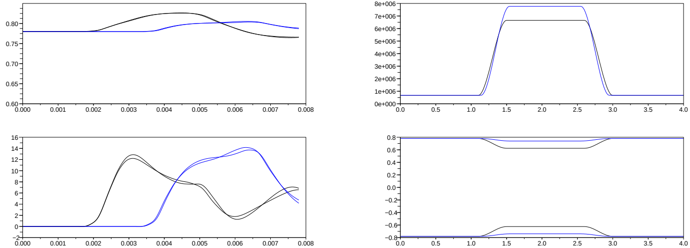
- When the profiles of A and Q are only known at two different positions (outside the plaque area), the optimization process is still able to reconstruct a rather precise shape of $z \mapsto \beta(z)$ and $z \mapsto A_0(z)$:



- The values of A and Q are still perfectly recovered at the two sections.

Conclusions, perspectives

- The inverse problem corresponding to the identification of the main parameters of a 1D model for blood flow has been successfully solved in the case of a pathological artery.
- It has shown in particular that the optimal parameters may be different from those expected from a formal averaging.
- In the future, such model may thus be able to help practitioners for the early diagnosis of atherosclerosis from a non invasive method (echotracking).

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