# Air flows and gas exchange in the human lung

## influence of the bronchial wall compliance on the functional efficiency of the lung

Sébastien Martin

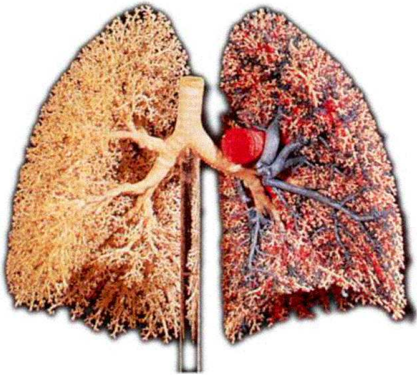
Université Paris-Sud 11 & INRIA - REO Project

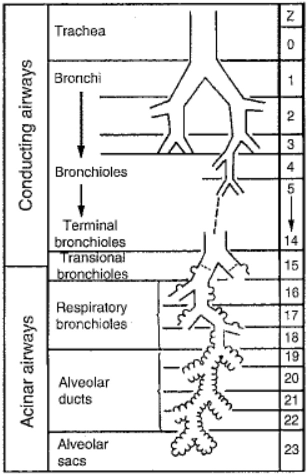
February 19, 2010, La Habana (Cuba)

**Collaborations:**

| | | |
|---|---|---|
| B. Maury | • | Université Paris-Sud 11 |
| T. Similowski | • | Hôpital La Pitié - Salpêtrière |
| C. Straus | • | Hôpital La Pitié - Salpêtrière |

# E. Weibel: "The pathway for oxygen" (1984)





# Outline

*Influence of the airway smooth muscle on the efficiency of the lung as gas exchangers?*

# Outline

# A single-compartment model

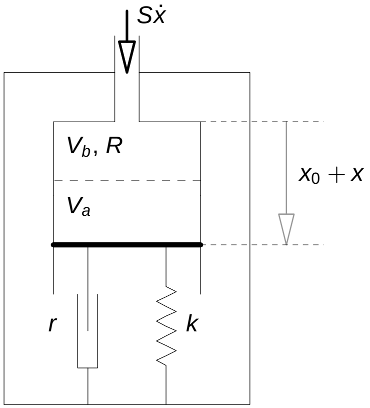



Figure: Single compartment model

- piston displacement $x$
- piston surface $S$
- displaced mass $m$
- alveolar volume $V_a$
- bronchial volume $V_b$
- total volume $V = V_a + V_b$
- spring constant $k$
- airway resistance $R$
- tissue resistance $r$
- external force $F_e$

# A single-compartment model

Equation governing the piston displacement:

$$m\,\ddot{x} + (R + r)S^2\,\dot{x} + k\,x = F_e,$$

with the following data:

| | | | | |
|---|---|---|---|---|
| Piston surface | $S$ | 0.01 | $\mathrm{m}^2$ | Ben Tal (2006) |
| Mass | $m$ | 0.30 | kg | Bates *et al.* (1991) |
| Elastance | $k/S^2$ | 2.50 | $\mathrm{mmHg} \cdot \mathrm{L}^{-1}$ | Begin *et al.* (1975) |
| Airway resistance | $R_0$ | 1.00 | $\mathrm{mmHg} \cdot \mathrm{s} \cdot \mathrm{L}^{-1}$ | West (1979) |
| Tissue resistance | $r$ | 0.25 | $\mathrm{mmHg} \cdot \mathrm{s} \cdot \mathrm{L}^{-1}$ | Weibel (1984) |
| External forces | $F_e$ | **input** | N | |
| Piston displacement | $x$ | **output** | m | |

# Variations of the airway resistance due to the mechanical properties of the bronchial tree

- Conservation of the volume, assuming that the parenchyma tissues have a constant volume:

$$V_a = V_a^0 + (1-\theta)\, Sx, \qquad V_b = V_b^0 + \theta\, Sx.$$

  Values of the bronchial compliance parameter: $\theta \sim 0 - 0.4$

- No modification of the shape of the bronchi during deformation:

$$R(x) = \frac{R_0}{1 + \theta Sx / V_b^0}.$$

- Nonlinear model:

$$m\,\ddot{x} + (R(x) + r)S^2\,\dot{x} + k(x)\,x = F_e.$$

# Validation of the single-compartment model

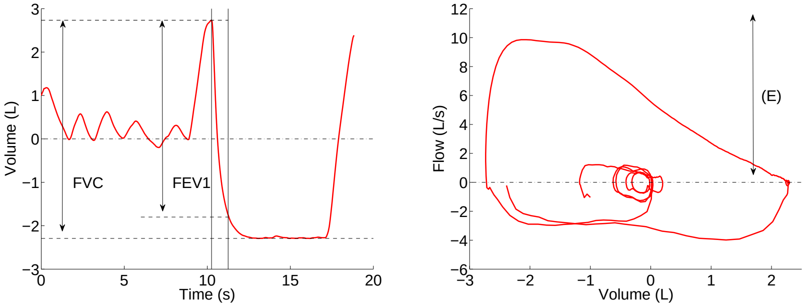



Figure: Experimental results for a normal subject during forced maneuvers.

# Validation of the single-compartment model

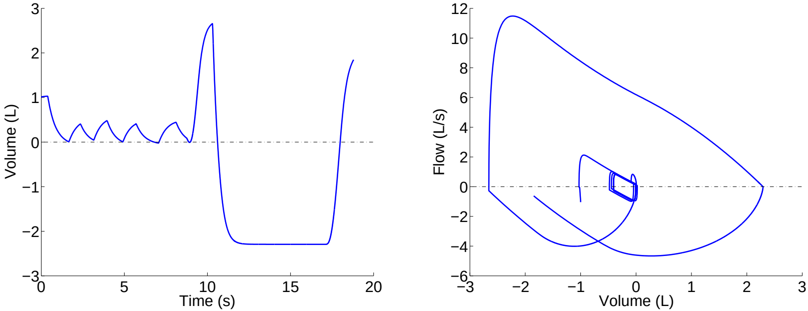



Figure: Numerical results for a normal subject during forced maneuvers.

# Outline

1. Mechanical lung model
2. Gas exchange model
3. Numerical results

# Notion of efficiency: variations of the exchange surface area due to the airway wall compliance

- Alveolar area (under the homothetic deformation assumption):

$$S_a \propto V_a^{2/3} = \left(V_a^0 + (1-\theta)Sx\right)^{2/3}.$$

- Efficiency (average flux of oxygen which diffuses into the bloodstream):

$$\left\langle \dot{V}_{O_2}(\theta) \right\rangle \propto \frac{1}{T} \int_0^T \left(V_a^0 + (1-\theta)S\,x(t)\right)^{2/3} dt.$$

- **Question:** investigate the function $\theta \mapsto \left\langle \dot{V}_{O_2}(\theta) \right\rangle$.

# Oxygen impoverishment

- Flux of oxygen that diffuses into the blood through the alveoli:

$$\dot{q} \propto \left(V_a^0 + (1-\theta)\,Sx\right)^{2/3} c.$$

where $c$ denotes the molar fraction of oxygen in the alveoli.

- Fluxes in the alveoli:
  - (*i*) Alveoli are supplied with fresh air through the bronchial tree:

$$\dot{V}_a\,[\dot{x} \geq 0\,?\,c_0, c].$$

  - (*ii*) At the alveolar membrane, oxygen diffuses into the blood:

$$\dot{q} = \Lambda\,\left(V_a^0 + (1-\theta)Sx\right)^{2/3} c.$$

- Variation of the total quantity of oxygen in the alveoli:

$$\dot{c} = \frac{1}{V_a}\left(\dot{V}_a\,[\dot{x} \geq 0\,?\,c_0 - c, 0] - \dot{q}\right).$$

# Mechanical model and gas exchange model

The lung model reads:

$$\begin{cases} \dot{x} & = & u \\ \dot{u} & = & \dfrac{1}{m}\left(F_e - \left(\dfrac{R_0}{1+\theta Sx/V_b^0} + r\right) S^2 u - k(x)\,x\right) \\ \dot{q} & = & \Lambda\,\left(V_a^0 + (1-\theta)Sx\right)^{2/3} c \\ \dot{c} & = & \dfrac{1}{V_a}\left(\dot{V}_a\,(c_0 - c)\,\mathbf{1}_{\mathbb{R}+}(u) - \dot{q}\right). \end{cases}$$

- Volumes at rest:

$$V_a^0 = 3.0 \cdot 10^{-3}\,\mathrm{m}^3, \quad V_b^0 = 0.5 \cdot 10^{-3}\,\mathrm{m}^3$$

- Gas exchange parameters:

$$\Lambda = 2.5 \cdot 10^{-3}\,\mathrm{m}\cdot\mathrm{s}^{-1}, \quad c_0 = 0.2$$

# Validation of the gas exchange model

- Amplitude of the volume variations with the model:

  $\sim$ 0.5 L **at rest**
  $\sim$ 2.5 L **at exercise**

- Average flux of oxygen which diffuses to the bloodstream

| | Guyton & Hall | Num. Model |
|---|---|---|
| **at rest** | $4 \cdot 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-1}$ | $5 \cdot 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-1}$ |
| **at exercise** | $35 \cdot 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-1}$ | $48 \cdot 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-1}$ |

Table: Average flux of oxygen

# Outline

# Bronchial wall compliance and efficiency

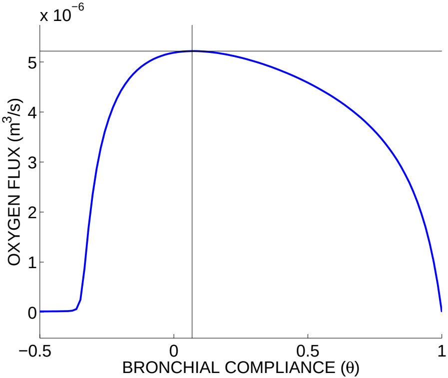



Figure: **Influence of bronchial compliance on the efficiency (at rest).**

# Bronchial wall compliance and efficiency

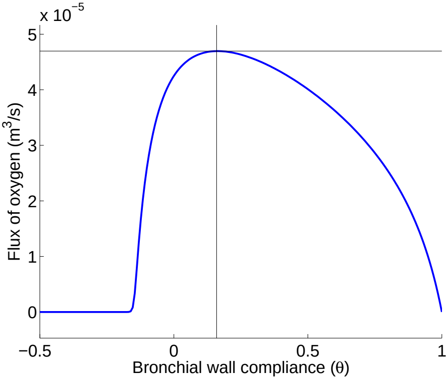



Figure: **Influence of bronchial compliance on the efficiency (at exercise).**

# Bronchial wall compliance and E.P.P.

- Pleural pressure $P_{pl}$ is the pressure surrounding the lung.
- Equal pressure point (E.P.P.): $P_{pl} = P_a$

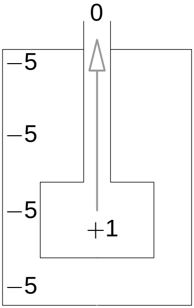



Figure: Passive expiration

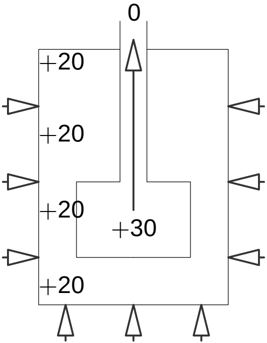



Figure: Forced expiration

# Bronchial wall compliance and E.P.P.

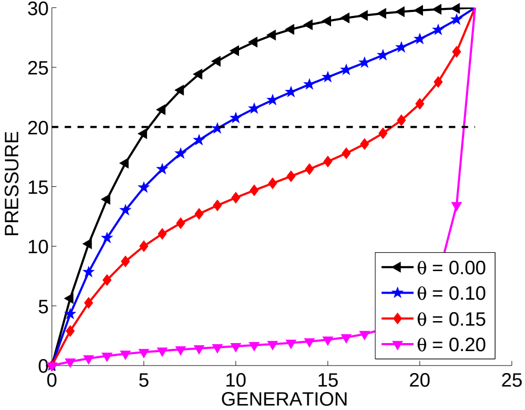



Figure: Pressure in the airway (forced expiration).

# Conclusions

- Hypothesis: structural role of the smooth muscle...
  - ... in the optimization of gas exchange efficiency,
  - ... in the control of collapsus.
- Investigation of screening effects: towards a PDE model.