PRICING AND TRADING CREDIT DEFAULT SWAPS

Tomasz R. Bielecki, Illinois Institute of Technology, USA
Monique Jeanblanc, Université d'Évry Val d'Essonne, France
Marek Rutkowski, University of New South Wales, Australia

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Dynamics of Prices of Defaultable claims

The **default time** is a strictly positive random variable τ, defined on a probability space $(\Omega, \mathcal{G}, \mathbb{Q})$.
The filtration generated by the jump process $H_t = \mathbb{1}_{\{\tau \leq t\}}$ is denoted by $\mathbb{H}$.
We assume that some auxiliary filtration $\mathbb{F}$ is given, and we write $\mathbb{G} = \mathbb{H} \vee \mathbb{F}$. The filtration $\mathbb{G}$ is referred to as to the **full filtration**.
We assume that any $\mathbb{F}$-martingale is a continuous process.

Survival Process

$G_t = \mathbb{Q}(\tau > t \,|\, \mathcal{F}_t)$ is the **survival process** assumed to satisfy $G_0 = 1$ and $G_t > 0$ for every $t \in \mathbb{R}_+$. We assume that G is continuous.

Then

$$M_t = H_t - \int_0^t (1 - H_u)\lambda_u \, du,$$

is a **$\mathbb{G}$-martingale**, where λ is defined via the Doob-Meyer decomposition of the sub-martingale G.

Defaultable Claims

By a **defaultable claim** maturing at T we mean a quadruple (X, A, Z, τ), where

- X is an $\mathcal{F}_T$-measurable random variable,
- $A = (A_t)_{t\in[0,T]}$ is an $\mathbb{F}$-adapted process of finite variation with $A_0 = 0$,
- $Z = (Z_t)_{t\in[0,T]}$ is an $\mathbb{F}$-predictable process,
- and τ is the default time.

The **total dividend process** $D^X = (D^X_t)_{t\in\mathbb{R}_+}$ of a defaultable claim maturing at T equals, for every $t \in \mathbb{R}_+$,

$$D^X_t = X\mathbb{1}_{\{\tau>T\}}\mathbb{1}_{[T,\infty[}(t) + \int_{]0,t\wedge T]} (1 - H_u)\, dA_u + \int_{]0,t\wedge T]} Z_u\, dH_u.$$

The **reduced total dividend process** D of a defaultable claim maturing at T equals, for every $t \in \mathbb{R}_+$,

$$D_t = \int_{]0,t\wedge T]} (1 - H_u)\, dA_u + \int_{]0,t\wedge T]} Z_u\, dH_u.$$

Price Dynamics of a Defaultable Claim

The **ex-dividend price** process S associated with the dividend process D^X equals, for every $t \in [0,T]$,

$$S_t = B_t \, \mathbb{E}_{\mathbb{Q}^*}\Big(B_T^{-1} X \mathbb{1}_{\{\tau > T\}} + \int_{]t,T]} B_u^{-1} \, dD_u \,\Big|\, \mathcal{G}_t \Big).$$

The **ex-dividend pre-default price** of a defaultable claim is the unique $\mathbb{F}$-adapted process $\widetilde{S}$ such that

$$S_t = \mathbb{1}_{\{t<\tau\}} \widetilde{S}_t$$

The **cumulative price** process S^{cum} associated with the dividend process D^X is

$$S_t^{cum} \quad = \quad B_t \, \mathbb{E}_{\mathbb{Q}^*}\Big(\int_{]0,T]} B_u^{-1} \, dD_u^X \,\Big|\, \mathcal{G}_t \Big) = S_t + B_t \int_{]0,t]} B_u^{-1} \, dD_u.$$

The discounted cumulative price $B^{-1} S^{cum}$ is a $\mathbb{G}$-martingale under $\mathbb{Q}^*$.

Let n be any $\mathbb{F}$-martingale. Then the process $\widehat{n}$ given by

$$\widehat{n}_t = n_{t\wedge\tau} - \int_0^{t\wedge\tau} G_u^{-1} \, d\langle n, \mu \rangle_u$$

is a continuous $\mathbb{G}$-martingale.

- For any $\mathbb{Q}^*$-integrable and $\mathcal{F}_T$-measurable random variable Y we have

$$\mathbb{E}_{\mathbb{Q}^*}(\mathbb{1}_{\{T<\tau\}}Y \,|\, \mathcal{G}_t) = \mathbb{1}_{\{t<\tau\}}\, G_t^{-1}\mathbb{E}_{\mathbb{Q}^*}(G_T Y \,|\, \mathcal{F}_t).$$

- For any $\mathbb{F}$-predictable process R such that $\mathbb{E}_{\mathbb{Q}^*}|R_\tau| < \infty$

$$\mathbb{E}_{\mathbb{Q}^*}(\mathbb{1}_{\{t<\tau\le T\}}R_\tau \,|\, \mathcal{G}_t) = -\mathbb{1}_{\{t<\tau\}}\, \frac{1}{G_t}\, \mathbb{E}_{\mathbb{Q}^*}\left(\int_t^T R_u\, dG_u \,\Big|\, \mathcal{F}_t\right)$$

The dynamics of the cumulative price S^{cum} on $[0, T]$ are

$$dS_t^{cum} = r_t S_t^{cum}\, dt + (Z_t - S_{t-})\, dM_t + G_t^{-1}(B_t\, d\widehat{m}_t - S_t\, d\widehat{\mu}_t)$$

where

$$m_t = \mathbb{E}_{\mathbb{Q}^*}\left(B_T^{-1} G_T X + \int_0^T B_u^{-1} G_u Z_u \lambda_u\, du - \int_0^T B_u^{-1} G_u\, dA_u \,\Big|\, \mathcal{F}_t \right).$$

and μ is the martingale part of the submartingale G.

Proof. We derive the dynamics of the pre-default ex-dividend price $\widetilde{S}$. The price S can be represented as follows

$$S_t = \mathbb{1}_{\{t<\tau\}} \widetilde{S}_t = \mathbb{1}_{\{t<\tau\}} B_t G_t^{-1} Y_t,$$

where Y is defined as follows

$$Y_t = m_t - \int_0^t B_u^{-1} G_u Z_u \lambda_u \, du + \int_0^t B_u^{-1} G_u \, dA_u,$$

An application of Itô's formula leads to the result □

Price Dynamics of a CDS

A **credit default swap** (CDS) with a constant rate κ and **recovery at default** is a defaultable claim $(0, A, Z, \tau)$ where $Z_t = \delta_t$ and $A_t = -\kappa t$ for every $t \in [0, T]$.

The process $\delta : [0, T] \to \mathbb{R}$ represents the **default protection**, and κ is the **CDS rate** (also termed the **spread, premium** or **annuity** of a CDS).

The ex-dividend price of a CDS equals, for any $t \in [0, T]$,

$$S_t(\kappa) = \mathbb{1}_{\{t<\tau\}} \frac{B_t}{G_t} \mathbb{E}_{\mathbb{Q}^*} \left(\int_t^T B_u^{-1} G_u \delta_u \lambda_u \, du - \kappa \int_t^T B_u^{-1} G_u \, du \,\Big|\, \mathcal{F}_t \right).$$

Define the $\mathbb{F}$-martingale n by the formula

$$n_t = \mathbb{E}_{\mathbb{Q}^*} \left(\int_0^T B_u^{-1} G_u \delta_u \lambda_u \, du - \kappa \int_0^T B_u^{-1} G_u \, du \,\Big|\, \mathcal{F}_t \right).$$

Then, the dynamics of the cumulative price $S^{cum}(\kappa)$ are

$$dS_t^{cum}(\kappa) = r_t S_t^{cum}(\kappa) \, dt + \big(\delta_t - S_{t-}(\kappa)\big) \, dM_t + G_t^{-1} \big(B_t \, d\widehat{n}_t - S_t(\kappa) \, d\widehat{\mu}_t\big)$$

Replication of a Defaultable Claim

We now assume that k credit default swaps with maturities $T_i \geq T$, spreads κ_i and protection payments δ^i for $i = 1, \ldots, k$ are traded over the time interval $[0, T]$. The 0th traded asset is the savings account B. Our goal is to examine hedging strategies for a defaultable claim (X, A, Z, τ).

Here, we assume that **immmersion property** holds: any $\mathbb{F}$-martingale is a $\mathbb{G}$-martingale. In that case, G is a non-increasing process.

We consider a trading strategy $\phi = (\phi^0, \ldots, \phi^k)$ where ϕ^0 is $\mathbb{G}$-adapted and $\phi^1, \ldots, \phi^k$ are $\mathbb{G}$-predictable processes. The associated **wealth process** $V(\phi)$ equals, for $t \in [0, T]$,

$$V_t(\phi) = \phi_t^0 B_t + \sum_{i=1}^{k} \phi_t^i S_t^i(\kappa_i)$$

A strategy ϕ is said to be **self-financing** if

$$dV_t(\phi) = \phi_t^0 \, dB_t + \sum_{i=1}^{k} \phi_t^i \, dS_t^{c,i}(\kappa_i)$$

where $S^{c,i}(\kappa_i)$ is the cumulative price process of the ith traded CDS.

We consider a defaultable claim (X, A, Z, τ) such that the price process S for this claim is well defined:

$$S_t^{cum} \;=\; \mathbb{1}_{\{t<\tau\}} \frac{B_t}{G_t} \left(- \int_0^t B_u^{-1} G_u Z_u \lambda_u \, du + \int_0^t B_u^{-1} G_u \, dA_u + m_t \right)$$

with

$$m_t = \mathbb{E}_{\mathbb{Q}^*} \left(B_T^{-1} G_T X + \int_0^T B_u^{-1} G_u Z_u \lambda_u \, du - \int_0^T B_u^{-1} G_u \, dA_u \,\middle|\, \mathcal{F}_t \right).$$

We say that a self-financing strategy $\phi = (\phi^0, \ldots, \phi^k)$ **replicates** a defaultable claim (X, A, Z, τ) if its wealth process $V(\phi)$ is equal to the price S of the claim $t \in [0, T]$.
In particular, the equality $V_{t\wedge\tau}(\phi) = S_{t\wedge\tau}$ holds for every $t \in [0, T]$.

For any $t \in [0, T]$,

$$dV_t(\phi) = r_t V_t(\phi)\, dt + \sum_{i=1}^{k} \phi_t^i \Big(\big(\delta_t^i - \widetilde{S}_t^i(\kappa_i)\big)\, dM_t + (1 - H_t) B_t G_t^{-1}\, dn_t^i \Big)$$

where

$$n_t^i = \mathbb{E}_{\mathbb{Q}^*} \left(\int_0^{T_i} B_u^{-1} G_u \delta_u^i \lambda_u \, du - \kappa_i \int_0^{T_i} B_u^{-1} G_u \, du \,\Big|\, \mathcal{F}_t \right).$$

We assume from now on that the filtration $\mathbb{F}$ is generated by a (possibly multi-dimensional) Brownian motion W under $\mathbb{Q}^*$ and Hypothesis (H) holds (so that W is also a Brownian motion with respect to $\mathbb{G}$).

In view of the predictable representation property of a Brownian motion, there exist $\mathbb{F}$-predictable processes ζ and ζ^i, $i = 1, \ldots, k$ such that $dm_t = \zeta_t \, dW_t$ and $dn^i_t = \zeta^i_t \, dW_t$

Assume that there exist $\mathbb{F}$-predictable processes $\phi^1, \ldots, \phi^k$ such that, for any $t \in [0, T]$,

$$\sum_{i=1}^{k} \phi_t^i \big(\delta_t^i - \widetilde{S}_t^i(\kappa_i)\big) = Z_t - \widetilde{S}_t, \quad \sum_{i=1}^{k} \phi_t^i \zeta_t^i = \zeta_t.$$

Let

$$dV_t(\phi) = r_t V_t(\phi)\, dt + \sum_{i=1}^{k} \phi_t^i \Big(\big(\delta_t^i - \widetilde{S}_t^i(\kappa_i)\big)\, dM_t + (1 - H_t) B_t G_t^{-1}\, dn_t^i \Big)$$

with the initial condition $V_0(\phi) = S_0^{cum}$. Then the self-financing trading strategy $(B^{-1}(V(\phi) - \phi \cdot S^i), \phi^1, \ldots, \phi^k)$ replicates the defaultable claim (X, A, Z, τ).

Enlargement of filtration formula

Let τ be a unique random time. In general, it is not an honest time. However, it is possible to prove that any $\mathbb{F}$-martingale is a $\mathbb{G} = \mathbb{F} \vee \mathbb{F}$-semi-martingale. If X is a $\mathbb{F}$ martingale, if

$$\mathbb{P}(\tau > u | \mathcal{F}_t) = \int_u^\infty f(v; t) dv$$

then

$$\begin{aligned} X_t &= \widetilde{X}_t + \int_0^{t\wedge\tau} \frac{d\langle X, M^\tau\rangle_s}{S_{s-}} + \int_{t\wedge\tau}^t \varphi(\tau, ds)\,, \\ &= \widetilde{X}_t + \int_0^{t\wedge\tau} \int_s^\infty \eta(dv) \frac{d\left\langle X, f^v \right\rangle_s}{S_{s-}} + \int_{t\wedge\tau}^t \varphi(\tau, ds) \end{aligned}$$

where $\widetilde{X}$ is a $\mathbb{G}$-martingale and

$$\varphi(u, ds) = \frac{d\langle f(u; \cdot), X\rangle_s}{f(u; s)}$$

Hedging of Credit Spreads and Default Correlations

In this section, we assume that $\mathbb{F}$ is a Brownian filtration and that the interest rate is null. Our aim is to obtain the dynamics of a CDS in the simple case where two different credit names are considered.

Joint Survival Process

Hence we assume that we are given two strictly positive random times τ_1 and τ_2. We introduce the **conditional joint survival process** $G(u, v; t)$

$$G(u, v; t) = \mathbb{Q}^*(\tau_1 > u, \tau_2 > v \,|\, \mathcal{F}_t).$$

We write

$$\partial_1 G(u, v; t) = \frac{\partial}{\partial u} G(u, v; t), \quad \partial_{12} G(u, v; t) = \frac{\partial^2}{\partial u \partial v} G(u, v; t).$$

We assume that the density $f(u,v;t) = \partial_{12} G(u,v;t)$ with respect to u and v exists, so that

$$G(u,v;t) = \int_u^\infty \Big(\int_v^\infty f(x,y;t)\,dy \Big)\,dx.$$

For any fixed $(u,v) \in \mathbb{R}_+^2$, the $\mathbb{F}$-martingale $G(u,v;t) = \mathbb{Q}^*(\tau_1 > u, \tau_2 > v \,|\, \mathcal{F}_t)$ admits the integral representation

$$G(u,v;t) = \mathbb{Q}^*(\tau_1 > u, \tau_2 > v) + \int_0^t g(u,v;s)\,dW_s$$

where $g(u,v;s)$ is some $\mathbb{F}$-predictable process (in fact an $\mathbb{F}$-martingale under $\mathbb{Q}^*$).

Let us introduce the filtrations $\mathbb{H}^i, \mathbb{H}, \mathbb{G}^i$ and $\mathbb{G}$ associated with default times by setting

$$\mathcal{H}_t^i = \sigma(H_s^i;\, s \in [0,t]), \quad \mathcal{H}_t = \mathcal{H}_t^1 \vee \mathcal{H}_t^2, \quad \mathcal{G}_t^i = \mathcal{F}_t \vee \mathcal{H}_t^i, \quad \mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t,$$

where $H_t^i = \mathbb{1}_{\{\tau_i \le t\}}$. We assume that the usual conditions of completeness and right-continuity are satisfied by these filtrations.

We assume that immersion hypothesis holds between $\mathbb{F}$ and $\mathbb{G}$. In particular, any $\mathbb{F}$-martingale is also a $\mathbb{G}^i$-martingale for $i = 1, 2$.

In general, there is no reason that any $\mathbb{G}^i$-martingale is a $\mathbb{G}$-martingale. Indeed, when $\mathbb{F}$ is a trivial filtration, denoting by $G_t^{1|2} = \mathbb{Q}^*(\tau_1 > t \,|\, \mathcal{H}_t^2)$ and $G(u,v) = \mathbb{Q}(\tau_1 > u, \tau_2 > v)$,

$$dG_t^{1|2} = \left(\frac{\partial_2 G(t,t)}{\partial_2 G(0,t)} - \frac{G(t,t)}{G(0,t)} \right) dM_t^2 + \left(H_t^2 \partial_1 h(t,\tau_2) + (1 - H_t^2) \frac{\partial_1 G(t,t)}{G(0,t)} \right) dt$$

where M^2 is the $\mathbb{H}^2$-martingale given by

$$M_t^2 = H_t^2 + \int_0^{t \wedge \tau_2} \frac{\partial_2 G(0,s)}{G(0,s)} \, ds$$

and $h(t,s) = \frac{\partial_2 G(t,s)}{\partial_2 G(0,s)}$. Hence immersion hypothesis is not always valid between $\mathbb{H}^2$ and $\mathbb{H}^1 \vee \mathbb{H}^2$, since $\frac{\partial_2 G(t,t)}{\partial_2 G(0,t)} - \frac{G(t,t)}{G(0,t)}$ is not always null.

Valuation of Single-Name CDSs

Let us now examine the valuation of single-name CDSs under the assumption that the interest rate equals zero.

We consider the CDS

- with the constant spread κ_1,
- which delivers $\delta_1(\tau_1)$ at time τ_1 if $\tau_1 < T_1$, where δ_1 is a

deterministic function.

The value $S^1(\kappa_1)$ of this CDS, computed in the filtration $\mathbb{G}$, i.e., taking care on the information on the second default contained in that filtration, is computed in two successive steps.

On the set $t < \tau_{(1)} = \tau_1 \wedge \tau_2$, the ex-dividend price of the CDS equals, on the event $\{t < \tau_{(1)}\}$,

$$S_t^1(\kappa_1) = \widetilde{S}_t^1(\kappa_1) = \frac{1}{G(t,t;t)} \left(- \int_t^{T_1} \delta_1(u) \partial_1 G(u,t;t)\, du - \kappa_1 \int_t^{T_1} G(u,t;t)\, du \right).$$

On the event $\{\tau_2 \le t < \tau_1\}$, we have that

$$S_t^1(\kappa_1) = \frac{1}{\partial_2 G(t,\tau_2;t)} \left(- \int_t^{T_1} \delta_1(u) f(u,\tau_2;t)\, du - \kappa_1 \int_t^{T_1} \partial_2 G(u,\tau_2;t)\, du \right).$$

Price Dynamics of Single-Name CDSs

Let us return to the study of a general case. By applying the Itô-Wentzell theorem, we get

$$\begin{aligned} G(u,t;t) &= G(u,0;0) + \int_0^t g(u,s;s)\,dW_s + \int_0^t \partial_2 G(u,s;s)\,ds \\ G(t,t;t) &= G(0,0;0) + \int_0^t g(s,s;s)\,dW_s + \int_0^t \left(\partial_1 G(s,s;s) + \partial_2 G(s,s;s)\right) ds. \end{aligned}$$

The cumulative price $S^{c,1}(\kappa_1)$ satisfies, on $[0, T \wedge \tau_{(1)}]$,

$$dS^{c,1}_t(\kappa_1) = (\delta_1(t) - \widetilde{S}^1_t(\kappa_1))\, d\widehat{M}^1_t + (S^1_{t|2}(\kappa_1) - \widetilde{S}^1_t(\kappa_1))\, d\widehat{M}^2_t$$
$$- \frac{1}{G(t,t;t)}\Big(\int_t^{T_1} \delta_1(u)\partial_1 g(u,t;t)\, du + \kappa_1 \int_t^{T_1} g(u,t;t)\, du \Big) dW_t\, .$$

Here

$$\widehat{M}^i_t = H^i_{t\wedge\tau_{(1)}} - \int_0^{t\wedge\tau_{(1)}} \widetilde{\lambda}^i_u\, du,$$

is a $\mathbb{G}$-martingale, where $\widetilde{\lambda}^i_t = -\frac{\partial_i G(t,t;t)}{G(t,t;t)}$ is the **pre-default intensity** for the ith name

Replication of a First-to-Default Claim

A **first-to-default claim** with maturity T is a defaultable claim $(X, A, Z, \tau_{(1)})$ where X is an $\mathcal{F}_T$-measurable amount payable at maturity if no default occurs, a continuous process of finite variation $A : [0, T] \to \mathbb{R}$ with $A_0 = 0$ represents the dividend stream up to $\tau_{(1)}$, and $Z = (Z^1, Z^2, \dots, Z^n)$ is the vector of $\mathbb{F}$-predictable, real-valued processes, where $Z^i_{\tau_{(1)}}$ specifies the recovery received at time $\tau_{(1)}$ if the ith name is the first defaulted name, that is, on the event $\{\tau_i = \tau_{(1)} \leq T\}$.

The cumulative price S^{cum} is given by

$$dS_t^{cum} = r_t S_t^{cum}\,dt + \sum_{i=1}^{n}(Z_t^i - S_{t-})\,d\widehat{M}_t^i + (1 - H_t^{(1)})B_t(G_{(1)}(t;t))^{-1}\,dm_t,$$

where the $\mathbb{F}$-martingale m is given by the formula

$$\begin{aligned} m_t &= \mathbb{E}_{\mathbb{Q}^*}\bigg(B_T^{-1}G_{(1)}(T;T)X + \sum_{i=1}^{n}\int_0^T B_u^{-1}G_{(1)}(u;u)Z_u^i\widetilde{\lambda}_u^i\,du \\ &\quad - \int_0^T B_u^{-1}G_{(1)}(u;u)\,dA_u\,\Big|\,\mathcal{F}_t\bigg). \end{aligned}$$

The pre-default ex-dividend price satisfies

$$d\widetilde{S}_t = (r_t + \widetilde{\lambda}_t)\widetilde{S}_t \, dt - \sum_{i=1}^{n} \widetilde{\lambda}_t^i Z_t^i \, dt + dA_t + B_t (G(t,t;t))^{-1} \, dm_t.$$

Since $\mathbb{F}$ is generated by a Brownian motion, there exists an $\mathbb{F}$-predictable process ζ such that

$$d\widetilde{S}_t = (r_t + \widetilde{\lambda}_t)\widetilde{S}_t \, dt - \sum_{i=1}^{n} \widetilde{\lambda}_t^i Z_t^i \, dt + dA_t + B_t (G(t,t;t))^{-1} \zeta_t \, dW_t.$$

We say that a self-financing strategy $\phi = (\phi^0, \phi^1, \dots, \phi^k)$ **replicates** a first-to-default claim $(X, A, Z, \tau_{(1)})$ if its wealth process $V(\phi)$ satisfies the equality $V_{t\wedge\tau_{(1)}}(\phi) = S_{t\wedge\tau_{(1)}}$ for any $t \in [0, T]$.

We have, for any $t \in [0, T]$,

$$dV_t(\phi) = r_t V_t(\phi)\, dt + \sum_{i=1}^{n} \phi_t^i \Big(\big(\delta_t^i - \widetilde{S}_t^i(\kappa_i)\big)\, dM_t^i + \sum_{j=1\,,j\neq i}^{n} \big(S_{t|j}^i - \widetilde{S}_t^i(\kappa_i)\big)\, dM_t^j \\ + (1 - H_t) B_t (G(t,t;t))^{-1}\, dn_t^i \Big)$$

where

$$n_t^i = \mathbb{E}_{\mathbb{Q}^*} \left(\int_0^{T_i} \frac{G(u,u;u)}{B_u} \Big(\delta_u^i \widetilde{\lambda}_u^i + \sum_{j=1,j\neq i}^{n} S_{u|j}^i \widetilde{\lambda}_u^j \Big)\, du - \kappa_i \int_0^{T_i} \frac{G(u,u;u)}{B_u}\, du \,\Big|\, \mathcal{F}_t \right).$$

Let $\widetilde{\phi}_t = (\widetilde{\phi}^1_t, \widetilde{\phi}^2_t, \ldots, \widetilde{\phi}^n_t)$ be a solution to the following equations

$$\widetilde{\phi}^i_t \big(\delta^i_t - \widetilde{S}^i_t(\kappa_i)\big) + \sum_{j=1,\, j\neq i}^{n} \widetilde{\phi}^j_t \big(S^j_{t|i}(\kappa_j) - \widetilde{S}^j_t(\kappa_j)\big) = Z^i_t - \widetilde{S}_t$$

and $\sum_{i=1}^n \widetilde{\phi}^i_t \zeta^i_t = \zeta_t$. Let us set $\phi^i_t = \widetilde{\phi}^i(\tau_{(1)} \wedge t)$ for $i = 1, 2, \ldots, n$ and $t \in [0, T]$. Then the self-financing trading strategy $\phi = (B^{-1}(V(\phi) - \phi \cdot S), \ldots, \phi^k)$ replicates the first-to-default claim $(X, A, Z, \tau_{(1)})$.

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