Pricing American currency options in a jump diffusion model

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Abstract

In this article the problem of the American option valuation in a jump diffusion setting is tackeld. The perpetual case is first considered. Without possible discontinuities (i.e. with negative jumps in the call case), known results concerning the currency option value as well the exercise boundary are obtained with a martingale approach. With possible discontinuities of the underlying process at the exercise boundary (i.e. with positive jumps in the call case) original results are derived by relying on first passage time and overshoot associated with a Lévy process. For finite life American currency calls, the formula given by Bates (1991) or Zhang (1995), is rederived in the context of a negative jump size. It is basically an extension of the Barone-Adesi and Whaley (1987) approach. This formula is tested. It is shown that Bates (1996) model generates good results when the process is continuous at the exercise boundary. However, with possible discontinuities results generated by Bates' (1996) model are less accurate.

Keywords : American options, perpetual options, exercise boundary, incomplete markets, jump diffusion model, Laplace transform, stopping times, Lévy exponent, overshoot.

1 Introduction

Several articles have already focused on the valuation of European options when the underlying value follows a jump diffusion process. Merton (1976) was the first to obtain a closed form solution. More recently Duffie et al. (2000) also considered this problem in a larger setting. The problem of the American option valuation, specially useful for real options, is more complex. It was tackled by Scott (1997) and Bates (1996) in a more general context (jumps, stochastic volatility,...). The former author obtained an accurate European option value by using the Fourier transform, but the early exercise premium was obtained by a straightforward application of the Barone-Adesi and Whaley (1987) formula,

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in which the possible jumps are not taken into account. The latter author also obtained in this context a European option formula. In order to derive the early exercise premium, he relies on Bates (1991) in which possible jumps are now taken into accounts. Basically he obtained an extension of the Barone-Adesi and Whaley (1987) approach in a jump diffusion setting.

Pham (1997) considered the American put option valuation in a jump diffusion model (Merton's assumptions) and related this optimal stopping problem to a parabolic integrodifferential free boundary problem. By extending the Riesz decomposition obtained by Carr, Jarrow and Myneni (1992) in a diffusion model, he derived a decomposition of the American put price as the sum of its corresponding European price and the early exercise premium. But the latter term rests on the identification of the exercise boundary. In the same context, Zhang (1994) relies on variational inequalities and shows how to use numerical methods (finite difference methods) in order to price the American put. Zhang (1995) sets this pricing problem into a free boundary problem, and by using the Mac Millan's (1983) approximation, she obtains a price for the perpetual put,and an approximation of the finite maturity put price. These results are obtained only when jumps are positive, i.e. without discontinuities of the underlying process at the exercise boundary. Mastroeni and Matzeu (1995 and 1996) obtain an extension of Zhang (1994) results in a multidimensional state space. Mulinacci (1996), Mordecki (1999) and (2002) and Gerber and Shiu (1998) (and (1999) only in the presence of jumps) also considered the American option pricing problem.

By relying on a martingale approach, we also extend in our study, the Barone-Adesi and Whaley (1987) approach and obtain approximations of the American premium and of the critical stock price (see Bunch and Johnson (2000) without jumps) which fit the assumptions concerning the dynamics of the underlying value (basically Bates Model). We show that this extension generates good results when jumps are negative (for currency call). However, when jumps are positive, the quality of the results depends on the size of the jumps. Indeed the use of the Barone-Adesi and Whaley extension is more delicate, because of possible discontinuities in the process at the exercise boundary. Therefore, a new approach is developed in this context for perpetual options.

We first consider the valuation of a perpetual currency call option, and obtain the exercise boundary in two cases: only negative jumps and then only positive ones. In the first case, analytical solutions for the price are based on the computation of the Laplace transform of the first passage time of the process at the exercise boundary, which is obtained using of the stopping theorem and of Lévy's exponent. In the second case, the valuation problem is more difficult to tackle, the process being possibly discontinuous at the critical boundary. The overshoot at the exercise boundary is introduced. Our original results are derived by relying on Bertoin's book (1996) on Lévy's processes. Indeed, Laplace transforms involving the first passage time and the overshoot at the exercise boundary are used.

As in Zhang (1995) or Bates (1991), we then show how an approximation of the finite maturity option price can be obtained by relying on the perpetual maturity case. When the jump is positive, the approximation is less accurate because of possible discontinuities at the exercise boundary.

This article is organized as follows. In section 2 the perpetual option is considered. In section 3, an approximation of the finite maturity American option value is derived. In section 4, the accuracy of the approximation formula is tested, by comparing it to a numerical approach.

2 The valuation of the perpetual option

Let S_t, σ^2, r, δ, λ, ϕ, T, K, $C_A(S_t, T-t)$, $P_A(S_t, T-t)$, $C_E(S_t, T-t)$, $P_E(S_t, T-t)$ denote the underlying spot foreign exchange rate, the variance of the foreign currency rate of return, the domestic risk-free rate and the foreign interest rate (or the rate of dividend for an index option), the intensity and the size of the jump, the maturity, the strike price of the option, the American and European call and put prices, respectively. Let us assume perfect markets, constant r, δ, σ, λ, and ϕ, a jump diffusion process for the currency rate and a zero price of the market jump risk, following Merton (1976). We assume that the dynamics of the risk adjusted process $(S_t, t \geq 0)$ are defined as follows:

$$\begin{aligned} dS_t/S_{t^-} &= (r - \delta - \lambda\phi)dt + \sigma dW_t + \phi dN_t \\ &= (r - \delta)dt + \sigma dW_t + \phi dM_t \end{aligned} \tag{1}$$

where $(W_t, t \geq 0)$ is a Wiener process, $(N_t, t \geq 0)$ a Poisson process with intensity λ and M the compensated martingale $M_t = N_t - \lambda t$. In order that S remains non-negative, the coefficient ϕ must be chosen strictly greater than -1. The jumps of S occur when the Poisson process jumps and $\Delta S_t = S_t - S_{t-} = S_{t-}(1+\phi)\Delta N_t$. Hence, the jump is strictly positive ($S_t \geq S_{t-}$) for $\phi > 0$, and the jump is strictly negative if $-1 < \phi < 0$. The solution of (1) is $S_t = xe^{X_t}$ where

$$X_t = (r - \delta - \lambda\phi - \frac{\sigma^2}{2})t + \sigma W_t + \ln(1+\phi)N_t \,.$$

The independence of the increments of X implies that $(\exp(kX_t - tg(k)), t \geq 0)$ is a martingale, where $g(k)$ is the Lévy exponent defined by

$$g(k) = bk + \frac{1}{2}\sigma^2 k(k-1) + \lambda((1+\phi)^k - 1 - k\phi)$$

with $b = r - \delta$. (See e.g. Bertoin (1996) for details) The function $k \to g(k)$ is convex, goes to plus infinity as $|k|$ goes to infinity, and $g(0) = 0$, therefore the equation $g(k) = u$ admits, for $u \geq 0$ two solutions of opposite sign. We denote by $g^{-1}(u)$ the *positive* solution and by $g^{-1,n}(u)$ the negative one. Note that for $\delta \neq 0$, the equality $g(1) = r - \delta < r$ leads to $g^{-1}(r) > 1$.

Jump diffusion models induce incompleteness of the market. Therefore there is an infinity of viable prices for options corresponding to an infinity of equivalent martingale measures that can be characterized by identifying the jump risk market price. By assuming that the jump risk is non priced, a unique American pricing formula can be derived (see Merton in the context of European Options). When maturity tends to infinity, the exercise boundary converges to a yet unknown value. The call option value is given by :

$$C_A(x) = C_A(x, \infty) = \sup_\tau E((S_\tau - K)e^{-r\tau}) \tag{2}$$

where τ runs over stopping times, or, w.l.g. over hitting times

$$T_L = \inf\{t \geq 0 \,:\, S_t \geq L\}$$

with $L \geq S_0 = x$.

2.1 Negative jumps

First let us assume that the jumps are negative, i.e., $-1 < \phi \leq 0$. The exchange rate process is therefore continuous at the exercise boundary, i.e., $S_{T_L} = L$ and

$$C_A(x) = \max_{L \geq x} \left[(L-K)E(e^{-rT_L})\right] \,.$$

In appendix A, the Laplace transform of T_L is derived : for $L \geq x$

$$E(e^{-rT_L}) = \left(\frac{x}{L}\right)^\rho$$

where $\rho = g^{-1}(r)$ is greater than 1. Hence:

$$C_A(x) = \max_{L \geq x} \left[(L-K)\left(\frac{x}{L}\right)^\rho\right] \tag{3}$$

We can thus derive the value of the exercise boundary as the value where the supremum in the right hand side of the last equation is obtained :

$$L_c = \frac{\rho}{\rho - 1} K \tag{4}$$

The option value is therefore, for $L_c \geq x$

$$C_A(x) = (L_c - K)\left(\frac{x}{L_c}\right)^\rho \tag{5}$$

and $x-K$ for $x < L_c$. Without jumps ($\phi = 0$), we obtain the known formula for the price of a perpetual call in a Black and Scholes framework.. Indeed in this case, g is a polynomial of degree 2, and

$$\rho = \frac{-\mu + \sqrt{\mu^2 + 2r}}{\sigma} \tag{6}$$

with $\mu = \left(r - \delta - \sigma^2/2\right)/\sigma$.

2.2 Positive jumps

Equation (2) can be rewritten as follows in terms of the process X :

$$C_A(x) = \sup_{\ell \geq 0} E(e^{-rT(\ell)}(xe^{X_{T(\ell)}} - K)) \,.$$

where

$$T(\ell) = \inf\{t \geq 0, X_t \geq \ln(L/x) = \ell\} = T_L \,.$$

The jump size being now positive, the process S could be discontinuous at the exercise boundary and therefore S_{T_L} (resp. $X_{T(\ell)}$) is no longer equal to L (resp. ℓ) with probability 1. The overshoot κ is defined in terms of X by:

$$X_{T(\ell)} = \ell + \kappa(\ell) \,.$$

Let us introduce a function f as follows: for $L \geq x$

$$f(x,L) = E(e^{-rT_L}(S_{T_L} - K)) = E(e^{-rT(\ell)}(xe^{X_{T(\ell)}} - K)) = E(e^{-rT(\ell)}(Le^{\kappa(\ell)} - K))$$

i.e.:

$$\begin{aligned} f(x,u) &= E(e^{-rT(\ln(u/x))}(xe^{\ln(u/x)+\kappa(\ln(u/x))} - K)) \\ &= uE(e^{-rT(\ln(u/x))+\kappa(\ln(u/x))}) - KE(e^{-rT(\ln(u/x))}) \,. \end{aligned} \tag{7}$$

By definition of the perpetual exercise boundary L_c:

$$f(x, L_c) = \sup_{L \geq x} f(x,L)$$

hence:

$$\frac{\partial f}{\partial u}(x, L_c) = 0, x < L_c \,. \tag{8}$$

Let us define:

$$\Phi(q,u) = \int_0^u \frac{1}{x} e^{-q\ln(\frac{u}{x})} f(x,u)dx \,. \tag{9}$$

By the change of variables $y = \ln \frac{u}{x}$, we obtain $\Phi(q,u) = \int_0^{+\infty} e^{-qy} f(ue^{-y}, u)dy$ i.e., by relying on equation (7):

$$\Phi(q,u) = u\alpha(q,r) - K\beta(q,r)$$

with:

$$\begin{aligned} \alpha(q,r) &= \int_0^{+\infty} e^{-qy} E(e^{-rT(y)+\kappa(y)})dy \\ \beta(q,r) &= \int_0^{+\infty} e^{-qy} E(e^{-rT(y)})dy \,. \end{aligned}$$

Therefore:

$$\frac{\partial \Phi}{\partial u}(q,u) = \alpha(q,r) \,.$$

Furthermore, equations (8) and (9) lead to:

$$\frac{\partial \Phi}{\partial u}(q, L_c) = \frac{f(L_c, L_c)}{L_c} - \frac{q}{L_c}\Phi(q, L_c)$$

hence:

$$\alpha(q,r) = \frac{L_c - K}{L_c} - \frac{q}{L_c}(L_c\alpha(q,r) - K\beta(q,r)).$$

Now the functions α and β are known in terms of an auxiliary function h (see appendix A) :

$$\alpha(q,r) = \frac{h(r,q) - h(r,-1)}{(q+1)h(r,q)},\ \beta(q,r) = \frac{h(r,q) - h(r,0)}{qh(r,q)} \tag{10}$$

hence from an easy computation we obtain :

$$L_c = \frac{h(r,0)}{h(r,-1)}K \tag{11}$$

The function h is given by :

$$h(u,k) = \frac{u - \hat{g}(k)}{\hat{g}^{-1}(u) - k} \tag{12}$$

where $\hat{g}(k) = g(-k)$.

Now by relying on equations (11) and (12), the exercise boundary is given by:

$$L_c = \frac{r - \hat{g}(0)}{\hat{g}^{-1}(r)} \frac{\hat{g}^{-1}(r) + 1}{r - \hat{g}(-1)} K$$

Using that $\hat{g}(0) = 0$, $\hat{g}(-1) = g(1) = r - \delta$, and $\hat{g}^{-1}(r) = -g^{-1,n}(r)$, we obtain

$$L_c = \frac{r}{\delta} \frac{g^{-1,n}(r) - 1}{g^{-1,n}(r)} K \tag{13}$$

Proposition 1 *The exercise boundary for a perpetual call is*

$$L_c = \frac{r}{\delta} \frac{g^{-1,n}(r) - 1}{g^{-1,n}(r)} K$$

where $g^{-1,n}(r)$ is the negative root of $g(k) = r$

It is straightforward to check that without jump ($\phi = 0$) both formula (4) and (13) coincide. Indeed, in this case, let us check that

$$\frac{r}{\delta} \frac{g_0^{-1,n}(r) - 1}{g_0^{-1,n}(r)} = \frac{g_0^{-1}(r)}{g_0^{-1}(r) - 1} \tag{14}$$

where g_0 is the Lévy exponent in the case $\phi = 0$, so that $g_0^{-1}(r)$ is the positive root of $bk + \frac{1}{2}\sigma^2 k(k-1) = r$ and $g_0^{-1,n}(r)$ is the negative root. Usual relations between the sum, the product of roots and the coefficients lead to the result.

The case of a put option can be solved using the symmetrical relationship between the American call and put boundaries (see appendix E, or Mordecki (2002)):

$$L_p(K, r, \delta, \phi, \lambda) L_c(K, \delta, r, -\frac{\phi}{1+\phi}, \lambda(1+\phi)) = K^2$$

¿From (13), the exercise boundary L_p of the perpetual put in a jump diffusion setting with constant negative jumps can be obtained:

$$L_p = \frac{K^2}{L_c(K, \delta, r, -\frac{\phi}{1+\phi}, \lambda(1+\phi))} = K \frac{r}{\delta} \frac{\gamma^{-1,n}(\delta)}{\gamma^{-1,n}(\delta) - 1} \tag{15}$$

where the function γ is the Lévy exponent of the process Y:

$$Y_t = (\delta - r - \hat{\lambda}\hat{\phi} - \frac{\sigma^2}{2})t + \sigma W_t + \ln(1 + \hat{\phi}) N_t .$$

Here N is a Poisson process with intensity $\hat{\lambda} = \lambda(1+\phi)$ and $\hat{\phi} = -\phi/(1+\phi)$. Hence, the Lévy exponent of Y is

$$\gamma(k) = (\delta - r)k + \frac{1}{2}\sigma^2 k(k-1) + \lambda((1+\phi)^{1-k} - 1 + \phi(k-1)) = g(1-k) - r + \delta$$

The perpetual exercise boundary for the put can also be obtained by relying on the procedure used for the call (See Appendix B for details):

$$L_p = \frac{r}{\delta}\frac{g^{-1}(r)+1}{g^{-1}(r)}K \tag{16}$$

Both formulae are the same if

$$\frac{\gamma^{-1,n}(\delta)}{\gamma^{-1,n}(\delta)-1} = \frac{g^{-1}(r)+1}{g^{-1}(r)}$$

which reduces to

$$\gamma^{-1,n}(\delta) + g^{-1}(r) = 1$$

this last equality is now obvious from $\gamma(k) = g(1-k) - r + \delta$.

Now that the perpetual exercise boundary for call and puts are known, option prices can be derived.

3 American and European

In a jump-diffusion setting, a decomposition of the American option price into the European price and the American premium can also be obtained. We denote by $L_c(T-t)$ the optimal boundary
We do not know how to compute this boundary, however, we shall get the price of the perpetual call.

Proposition 2 *Let us assume that in the risk neutral economy the dynamics of the currency price are given by equation (1) with constant coefficients. The price of the American Currency Call satisfies the following decomposition (where θ stands for the time to maturity $T-t$ and x for the current value of the underlying, i.e. S_t):*

$$\begin{aligned} C_A(x,\theta) &= C_E(x,\theta) + \delta x \sum_{n=0}^{+\infty}\int_0^{\theta} e^{-(\delta+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(d_1(L_c(\theta-v),n;v))dv \\ &\quad - rK\sum_{n=0}^{+\infty}\int_0^{\theta} e^{-(r+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(d_2(L_c(\theta-v),n;v))dv \end{aligned} \tag{17}$$

with:

$$\begin{aligned} d_1(z,n;v) &= \frac{\ln(x/z) + (r-\delta-\lambda\phi+\sigma^2/2)v + n\ln(1+\phi)}{\sigma\sqrt{v}} \\ d_2(z,n;v) &= d_1(z,n;v) - \sigma\sqrt{v} \end{aligned}$$

and the perpetual American call option is therefore given by:

$$\begin{aligned} C_A(x) &= \delta x \sum_{n=0}^{+\infty}\int_0^{+\infty} e^{-(\delta+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(d_1(L_c,n;v))dv \\ &\quad - rK\sum_{n=0}^{+\infty}\int_0^{+\infty} e^{-(r+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(d_2(L_c,n;v))dv \end{aligned} \tag{18}$$

where L_c is given by equation (13).

PROOF: Let t and T be fixed and apply Itô's lemma to the function: $(u,x) \to e^{-r(u-t)}C_A(x,T-u)$ on the interval $[t,T]$:

$$\begin{aligned} e^{-r(T-t)}C_A(S_T,0) &= C_A(S_t,T-t) + \int_t^T e^{-r(u-t)}\mathcal{L}(C_A)(S_u,T-u)du \\ &+\sigma \int_t^T e^{-r(u-t)}S_u \frac{\partial C_A}{\partial x}(S_u,T-u)dW_u \\ &+ \int_t^T e^{-r(u-t)}\left[C_A((1+\phi)S_{u-},T-u) - C_A(S_{u-},T-u)\right]dN_u \end{aligned} \tag{19}$$

where the differential generator $\mathcal{L}$ is defined by:

$$\mathcal{L}(f)(x,t) = \frac{\sigma^2}{2}x^2\frac{\partial^2 f}{\partial x^2}(x,t) + (r-\delta-\lambda\phi)x\frac{\partial f}{\partial x}(x,t) + \frac{\partial f}{\partial t}(x,t) - rf(x,t)$$

In the continuation region the American call price satisfies the P.D.E. given in appendix C and therefore $\mathcal{L}(C_A)(x,T-u)$ is equal to $-\lambda(C_A((1+\phi)x,T-u) - C_A(x,T-u))$.

In the stopping region the American call is equal to its intrinsic value, hence:

$$\begin{aligned} \mathcal{L}(C_A)(x,T-u) &+ \lambda(C_A((1+\phi)x,T-u) - C_A(x,T-u))1_{x<L_c(T-u)} \\ &= +(rK - (\delta+\lambda\phi)x)1_{x\geq L_c(T-u)} \end{aligned}$$

The second integral in the right hand side of equation (19) is a martingale; moreover using that $E(\int_t^T h_u dN_u|\mathcal{F}_t) = E(\int_t^T h_u\lambda du|\mathcal{F}_t)$ for any predictable process h, taking conditional expectation with respect to $\mathcal{F}_t$ of both members of (19), we get after some trivial simplifications

$$C_A(S_t,T-t) = C_E(S_t,T-t) - \int_t^T e^{-r(u-t)}E((rK-\delta S_u)1_{S_u>L_c(T-u)}|\mathcal{F}_t)du$$

hence (See details in Appendix D):

$$\begin{aligned} C_A(S_t,T-t) &= C_E(S_t,T-t) + \sum_{n=0}^{\infty}\frac{\delta\lambda^n}{n!}\int_t^T (u-t)^n e^{-(\delta+\lambda)(u-t)}\mathcal{N}(d_1(L_c(T-u),n,u-t))du \\ &-rK\sum_{n=0}^{\infty}\frac{\lambda^n}{n!}\int_t^T (u-t)^n e^{-(r+\lambda)(u-t)}\mathcal{N}(d_2(L_c(T-u),n,u-t))du \end{aligned}$$

and equation (17) is obtained.

When the maturity T tends to infinity, equation (18) is obtained.

Along the same lines a decomposition for the put can be obtained.

Proposition 3 *Let us assume that in the risk neutral economy the dynamics of the currency price are given by equation (1) with constant coefficients. The price of the American currency put satisfies the following decomposition:*

$$\begin{aligned} P_A(x,\theta) &= P_E(x,\theta) + rK\sum_{n=0}^{+\infty}\int_0^\theta e^{-(r+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(-d_2(L_p(\theta-v),n;v))dv \\ &-\delta x\sum_{n=0}^{+\infty}\int_0^\theta e^{-(\delta+\lambda)v}\frac{(\lambda v)^n}{n!}\mathcal{N}(-d_1(L_p(\theta-v),n;v))dv \end{aligned} \tag{20}$$

with d_1, d_2 are defined in the previous proposition and the perpetual American put option is therefore given by:

$$\begin{aligned} P_A(x) &= rK\sum_{n=0}^{+\infty}\int_0^{+\infty} e^{-(r+\lambda)u}\frac{(\lambda u)^n}{n!}\mathcal{N}(-d_2(L_p,n,u))du \\ &-\delta x\sum_{n=0}^{+\infty}\int_0^{+\infty} e^{-(\delta+\lambda)u}\frac{(\lambda u)^n}{n!}\mathcal{N}(-d_1(L_p,n,u))du \end{aligned}$$

where L_p is given by equation (15).

4 An approximation of the option value

Let us rely on the Barone-Adesi and Whaley (1987) approach, and on Bates (1991) article. Let us assume that the jump size is negative. If the American and European option values satisfied the same linear P.D.E. (in the continuation region), their difference ΔC, the American premium, must also satisfy this P.D.E. in the same region. Let us write :

$$\Delta C(S_0,T) = yh(S_0,y)$$

where:

$$y = 1 - e^{-rT}$$

and where h is a two argument function that has to be determined. In the continuation region h satisfies the following p.d.e. (see also Merton (1976) and appendix C where Ito's lemma is applied to the discounted American Call) obtained by a change of variables:

$$\frac{\sigma^2}{2}x^2\frac{\partial^2 h}{\partial x^2} + (r-\delta)x\frac{\partial h}{\partial x} - \frac{rh}{y} - (1-y)r\frac{\partial h}{\partial y} - \lambda(\frac{\partial h}{\partial x}\lambda x - h((1+\phi)x,y) + h(x,y)) = 0$$

Like the authors, let us now assume that the term with the derivative of h with respect to y is negligeable. Whether or not it is a good approximation is an empirical issue that will be considered in the following section. The following equation has to be solved :

$$\frac{\sigma^2}{2}x^2\frac{\partial^2 h}{\partial x^2} + (r-\delta)x\frac{\partial h}{\partial x} - \frac{rh}{y} - \lambda(\frac{\partial h}{\partial x}\lambda x - h((1+\phi)x,y) + h(x,y)) = 0$$

The perpetual option value satisfies almost the same differential equation. The only difference resides in the fact that y is equal to 1 in the perpetual case, and therefore we have $\frac{r}{y}$ instead of r in the third term of the left hand-side. In section I (see equation 5)) we derived its solution which is for a negative jump:

$$h = \eta x^\rho$$

where η is still unknown, and by relying on appendix A:

$$\rho = g^{-1}(\frac{r}{y})$$

But when S_0 tends to the exercise boundary $L_c(T)$, by continuity of the option value, the following equation is satisfied :

$$L_c(T) - K = C_E(L_c(T),T) + y\eta L_c(T)^\rho \tag{21}$$

and by use of the smooth fit condition the following equation is obtained:

$$1 = \frac{\partial C_E}{\partial x}(L_c(T),T) + y\eta\rho L_c(T)^{\rho-1} \tag{22}$$

Within a jump-diffusion model, this condition was derived by Zhang (1944) in the context of variational inequalities and by Pham (1995) with a free boundary formulation. We thus have a system of two equations (21) and (22) and two unknowns η and $L_c(T)$ that can be solved. $L_c(T)$ is the implicit solution of:

$$L_c(T) = K - C_E(L_c(T),T) + (1 - \frac{\partial C_E}{\partial x}(L_c(T),T))\frac{L_c(T)}{\rho}$$

and the approximation formula is the following :

$$C_A(S_0,T) = C_E(S_0,T) + A(S_0/L_c(T))^\rho \tag{23}$$

if: $S_0 < L_c$

$$C_A(S_0,T) = S_0 - K$$

otherwise, with:

$$A = (1 - \frac{\partial C_E}{\partial x}(L_c(T), T)))\frac{L_c(T)}{\rho}$$

where (see Merton 1976):

$$C_E(S_0, T) = \sum_{n=0}^{+\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} e^{-(\delta+\lambda\phi-\frac{n\ln(1+\phi)}{T})T} C_{BS}(S_0, T, r - \delta - \lambda\phi + \frac{n\ln(1+\phi)}{T}, \sigma) \tag{24}$$

and:

$$C_{BS}(S_0, T, \theta, \sigma) = S_0 N(d_1) - K e^{-\theta T} N(d_2)$$

$$\begin{aligned} d_1 &= \frac{\ln(S_0/K) + (\theta + \frac{\sigma^2}{2})T}{\sigma T} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}$$

The latter terms are given by Black and Scholes (1973).

An approximation for the put prices, in a context where the jump size is positive, can be obtained by relying on equation (**??**).

A decomposition of the American option price into two components,namely the European option price and the American premium was therefore derived. Pham (1995) has extended the Riesz decomposition obtained by Carr- Jarrow- Mynemi (1992) or Jacka (1991) within a diffusion model. He obtains an early exercise premium which rests on the identification of the exercise boundary. In our paper, as in Zhang (1995), simultaneously approximations of the premium and of the exercise boundary are derived. These results were obtained by Bates (1991) in a more general context in which $1 + \phi$ is a log-normal random variable (for the put). This means that his results could be used even with positive jumps for an American call. However, as shown in appendix C, the differential equation which solution is the American option approximation value, takes a specific form just below the exercise boundary. Unfortunately there is no known solution to this equation. Positive jumps generate possible discontinuities in the process at the exercise boundary, and therefore the problem is more difficult to solve (see section I). As shown in following section, Bates approximation gives better results when the size of the jump is negative or small.

5 Simulations

In this section, formula (23) is tested. First we compare the results generated by this formula to those obtained by use of a numerical method (explicit method). Then, we compute what is called the "pseudo American call price" (last column). It is the sum of the term given in equation (24) (Merton formula), which corresponds to the European price of the option, and of the American premium given by Barone Adesi and Whaley (1987), without jumps ($\phi = 0$). We checked whether good approximations for the American call prices were obtained.

In table 1, it turns out that formula (23) generates very accurate results. However, the positive sign of the interest rate differential induces an early exercise premium which is negligeable. In this case, the European price gives already a very good approximation of the American one.

In table 2, the interest rate differential is negative and formula (23) seems to generate pretty accurate results. Furthermore these results are usually better than those generated by the pseudo American valuation. In table 3 and 4 the jump is positive, and its size is pretty high (10%). We observe that Bates extension of Barone-Adesi and Whaley formula is pretty good when the option is out of the money. Otherwise, when the option is in the money the results are not satisfying. Indeed, the probability for the underlying to reach quickly the exercise boundary and to be discontinuous at this level is higher for in the money than for out the money calls. For in the money calls, it is better to use the pseudo American call price. When the size of the jump is smaller, (see table 5 and 6), the size of the biais gets also smaller, and the quality of the results improve.

By comparing table 2 and 4, we observe that out of the money call options have higher prices with positive jumps than with negative ones, the reverse being true for in the money calls. Positive jumps, even if they have a negative effect on the risk adjusted drift have a strong positive effect on the probability of exercise for out the money calls. For these options the positive "jump effect" is more important than the negative "drift effect". For in the money calls, the probability of exercise is already high (with a negative jump) and therefore a positive jump doesn't have a strong positive effect on the probability of exercise. However, due to possible discontinuities in the process at the exercise boundary, with positive jumps, exercise boundaries are smaller with positive jumps than with negative ones. This implies that in the money call values are higher with negative jumps than with positive ones.

6 Conclusion

In this paper, original results concerning the pricing of perpetual American currency options in a jump diffusion framework are obtained. It has been shown that the sign of the jump size is a relevant parameter. Without discontinuouties at the exercise boundary, known results are obtained. With possible discontinuouties, an overshoot is introduced and new results are derived. For finite life American currency calls, the formula given by Bates (1991) or Zhang (1995), is rederived in the context of a negative size jump. It is basically an extension of the Barone-Adesi and Whaley approach (1987). This formula is tested and gives good results. However, if the exchange rate can be discontinuous at the exercise boundary (positive jumps for the underlying value in the call case), the pricing problem, is more difficult to tackle, and one should be very cautious in applying a Barone-Adesi extension.

Appendix A

Pecherskii and Rogozin result

Let us define $T(\ell)$ the first passage time of the process X at $\ell = \ln(L/S_0)$. By applying the optional stopping theorem and by relying on the left continuity of the process at the stopping time $T(\ell)$, we obtain the following formula :

$$E(\exp(-g(k)T(\ell))) = \exp(-k\ell)$$

By inverting the Levy exponent we obtain the Laplace transform :

$$E(\exp(-uT(\ell))) = \exp(-\ell g^{-1}(u))\,.$$

We recall a result of Pecherskii and Rogozin which can be found in Bertoin : for every triple of positive numbers (α, β, q),

$$\int_0^\infty e^{-q\ell} E(e^{-\alpha T(\ell) - \beta\kappa(\ell)})dx = \frac{h(\alpha, q) - h(\alpha, \beta)}{(q-\beta)h(\alpha, q)} \tag{25}$$

where

$$h(\alpha, \beta) = \exp\left(\int_0^\infty dt \int_0^\infty t^{-1}(e^{-t} - e^{-\alpha t - \beta x})P(X_t \in dx)\right) \tag{26}$$

Let $\hat{g}$ the Lévy exponent of the dual process of X, i.e. $\hat{X} = -X$.

$$h(b, k) = \frac{b - \hat{g}(k)}{\hat{g}^{-1}(b) - k} \tag{27}$$

¿From the definition, the Laplace exponent of the dual process is $\hat{g}(k) = g(-k)$.

Appendix B

For a put, we can introduce, the function ψ in place of the function f and the function Ψ in place of Φ

$$\begin{aligned}
\psi(x, u) &= E(Ke^{-rT(\ln(u/x)} - xe^{-rT(\ln(u/x)+\kappa(\ell)})\\
\Psi(q, u) &= \int_u^\infty \frac{dx}{x} e^{q\ln(u/x)}\psi(x, u)\\
&= \int_0^\infty dy e^{-qy}\left(KE(e^{-r\hat{T}(y)}) - uE(e^{-r\hat{T}(y) - \hat{\kappa}(y)})\right)
\end{aligned}$$

where the hat refers to $\hat{X} = -X$, so that $\hat{T}(y) = T(-y), \hat{\kappa}(y) = -\kappa(-y)$. Then, the same kind of computation leads to

$$L_p = \frac{r}{\delta} K \frac{\hat{g}^{-1,n}(r) - 1}{\hat{g}^{-1,n}(r)} = \frac{r}{\delta} K \frac{g^{-1}(r) + 1}{g^{-1}(r)} .$$

Appendix C

By applying generalized Ito's lemma to the discounted American call price, on the interval $[t, T]$, the following equation is obtained :

$$\begin{aligned} e^{-rt} C_A(S_t, T-t) &= C_A(S_0, T) + \int_t^T e^{-ru} \tfrac{\partial C_A}{\partial s}(S_u, T-u) du - \int_t^T r\, e^{-ru} C_A(S_u, T-u) du \\ &+ \int_t^T e^{-ru} \tfrac{\partial C_A}{\partial x}(S_u, T-u) S_u (r - \delta - \lambda\phi) du + \int_t^T e^{-ru} \tfrac{\partial C_A}{\partial x}(S_u, T-u) S_{u^-} \sigma dW_u \\ &+ \tfrac{\sigma^2}{2} \int_t^T e^{-ru} \tfrac{\partial^2 C_A}{\partial x^2}(S_u, T-u) S_u^2 du + \int_t^T e^{-ru} (C_A((1+\phi) S_{u-}, T-u) - C_A(S_{u^-}, T-u)) dN_u \end{aligned}$$

In the risk adjusted economy, the discounted American call price is a martingale in the continuation region. $(\int_t^v \exp(-ru) \frac{\partial C_A}{\partial x}(S_{u^-}, T-u) S_{u^-} \sigma dW_u, t \le v \le T)$ is also a martingale. Therefore, the drift term is equal to zero. Hence, in the continuation region, the American call value satisfies the following differential equation:

$$\begin{aligned} \frac{\sigma^2}{2} x^2 \frac{\partial^2 C_A}{\partial x^2}(x, T-u) &+ (r - \delta - \lambda\phi) x \frac{\partial C_A}{\partial x}(x, T-u) - r C_A(x, T-u) + \frac{\partial C_A}{\partial s}(x, T-u) \\ &+ \lambda (C_A((1+\phi)x, T-u) - C_A(x, T-u)) = 0 \end{aligned}$$

Now, if the jump is positive, and if S_t belongs to the interval :$[\frac{L_c(T-t)}{1+\phi}, L_c(T-t)]$, the value of the American call satisfies the following differential equation :

$$\begin{aligned} \frac{\sigma^2}{2} x^2 \frac{\partial^2 C_A}{\partial x^2}(x, T-u) &+ (r - \delta - \lambda\phi) x \frac{\partial C_A}{\partial x}(x, T-u) - r C_A(x, T-u) + \frac{\partial C_A}{\partial s}(x, T-u) \\ &+ \lambda ((1+\phi)x - K - C_A(x, T-u)) = 0 \end{aligned}$$

because in this case the value of the American option after the jump is equal to the intrinsic value.

Appendix D

Let us compute $E(1_{S_u > L_c(T-u)} | \mathcal{F}_t)$, the computation shall be the same for $E(S_u 1_{S_u > L_c(T-u)} | \mathcal{F}_t)$. ¿From

$$\begin{aligned} S_u &= S_t \exp((r - \delta - \lambda\phi - \sigma^2/2)(u-t) + \sigma(W_u - W_t) + (N_u - N_t)\ln(1+\phi)) , \\ &= S_t \exp(\nu(u-t) + \sigma(W_u - W_t) + (N_u - N_t)\ln(1+\phi)) = S_t Z \end{aligned}$$

where Z is independent of S_t, we obtain $E(1_{S_u > L_c(T-u)} | \mathcal{F}_t) = \Upsilon(S_t)$ where

$$\begin{aligned} \Upsilon(x) &= Q\left(x \exp(\nu(u-t) + \sigma(W_u - W_t + (N_u - N_t)\ln(1+\phi)) > L_c(T-u)\right) \\ &= Q\left(\nu(u-t) + \sigma W_{u-t} + N_{u-t}\ln(1+\phi))\right) > \ln(L_c(T-u)) - \ln x) \\ &= \sum_{n=0}^{\infty} Q(N_{u-t} = n) Q\left(\nu(u-t) + \sigma W_{u-t} + n\ln(1+\phi) > \ln(L_c(T-u)) - \ln x\right) . \end{aligned}$$

Then, the result follows.

Appendix E

The put-call symmetry formulae are well known in the case of continuous processes (See e.g. Detemple (2001)). Mordecki (2001) establishes, from the Wiener-Hopf decomposition a general symmetry relationship for Lévy processes. Here, we present a simple proof in our case. The price of the currency is

$$S_t = S_0 e^{(r-\delta)t} e^{\sigma W_t - \frac{1}{2}\sigma^2 t} e^{-\lambda\phi t + N_t \ln(1+\phi)} = S_0 e^{(r-\delta)t} Z_t$$

We can write

$$E(e^{-rt}(K-S_t)^+) = E\left(e^{-\delta t}Z_t(\frac{KS_0}{S_t}-S_0)^+\right) = \tilde{E}\left(e^{-\delta t}(\frac{KS_0}{S_t}-S_0)^+\right)$$

and, under $\tilde{Q}$ where $d\tilde{Q}|_{\mathcal{F}_t} = Z_t dQ|_{\mathcal{F}_t}$ the process $\widetilde{S} = 1/S$ follows

$$d\widetilde{S}_t = \widetilde{S}_t((\delta - r)dt - \sigma d\tilde{W}_t - \frac{\phi}{1+\phi}d\tilde{M}_t)$$

where $\widetilde{M}_t = N_t - \lambda(1+\phi)t$ is a Q-martingale (hence, N is a Q-Poisson process with intensity $\lambda(1+\phi)$). Hence

$$P(r,\delta,x,K;\sigma,\phi,\lambda) = C(\delta,r,K,x;\sigma,-\frac{\phi}{1+\phi},\lambda(1+\phi))\,.$$

The same method establishes that for $T_L = \inf\{t\,:\,S_t \geq L\}$ and $\widetilde{T}_L = \inf\{t\,:\,\widetilde{S}_t \leq \frac{KS_0}{L}\}$

$$E(e^{-rT_L}(K-S_{T_L})^+) = E\left(e^{-\delta\widetilde{T}_L}(S_0 - \widetilde{S}_{\widetilde{T}_L})^+\right)\,.$$

Hence $L_c(K,r,\delta;\phi,\lambda)$ and $L_p(S_0,\delta,r;-\frac{\phi}{1+\phi},\lambda(1+\phi))$ (where the first argument is the strike) satisfy $L_p = \frac{KS_0}{L_c}$ and the symmetry formula follows.

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TABLE I

THEORICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, $\phi =$ -0.1, K = 100, r-$\delta = 0.04$

CALL OPTION PRICES

		EUROPEAN				AMERICAN			
PARAMETERS		NO JUMP		WITH JUMPS		NO JUMP	WITH JUMPS		
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Ap-prox
1)	S_0								
$r = 0.08$	80	0.05	0.08	0.08	0.05	0.05	0.08	0.08	0.08
$\sigma = 0.20$	90	0.85	1.07	1.07	0.85	0.85	1.07	1.07	1.07
$T = 0.25$	100	4.44	4.92	4.91	4.44	4.44	4.91	4.91	4.91
	110	11.66	12.06	12.06	11.66	11.66	12.06	12.06	12.06
	120	20.90	21.08	21.07	20.90	20.90	21.09	21.07	21.07
2)	80	0.05	0.08	0.08	0.05	0.05	0.08	0.08	0.08
$r = 0.12$	90	0.84	1.06	1.06	0.84	0.84	1.06	1.06	1.06
$\sigma = 0.20$	100	4.40	4.86	4.86	4.40	4.40	4.86	4.86	4.86
$T = 0.25$	110	11.55	11.94	11.94	11.55	11.55	11.94	11.94	11.94
	120	20.69	20.88	20.86	20.69	20.69	20.88	20.87	20.87
3)	80	1.29	1.40	1.40	1.29	1.29	1.40	1.40	1.40
$r = 0.08$	90	3.82	4.02	4.01	3.82	3.82	4.02	4.02	4.01
$\sigma = 0.40$	100	8.35	8.60	8.60	8.35	8.35	8.60	8.60	8.60
$T = 0.25$	110	14.80	15.05	15.04	14.79	14.80	15.05	15.05	15.05
	120	22.71	22.93	22.92	22.71	22.72	22.93	22.92	22.92
4)	80	0.41	0.59	0.60	0.41	0.41	0.59	0.59	0.59
$r = 0.08$	90	2.18	2.67	2.65	2.18	2.18	2.67	2.65	2.65
$\sigma = 0.20$	100	6.50	7.21	7.17	6.50	6.50	7.22	7.17	7.16
$T = 0.50$	110	13.42	14.12	14.02	13.42	13.42	14.12	14.03	14.03
	120	22.06	22.45	22.45	22.06	22.06	22.45	22.45	22.45

TABLE II

THEORICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, ϕ = -0.1, K = 100, r-δ = -0.04

CALL OPTION PRICES

PARAMETERS		EUROPEAN NO JUMP		EUROPEAN WITH JUMPS		AMERICAN NO JUMP		AMERICAN WITH JUMPS	
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Ap-prox
1)	S_0								
$r = 0.08$	80	0.03	0.05	0.05	0.03	0.03	0.05	0.05	0.05
$\sigma = 0.20$	90	0.57	0.74	0.74	0.58	0.59	0.75	0.76	0.76
$T = 0.25$	100	3.42	3.87	3.86	3.52	3.52	3.94	3.95	3.97
	110	9.85	10.30	10.28	10.35	10.31	10.66	10.63	10.75
	120	18.62	18.87	18.84	20.00	20.00	20.00	20.00	20.00
2)	80	0.03	0.05	0.05	0.03	0.03	0.05	0.05	0.05
$r = 0.12$	90	0.56	0.74	0.74	0.58	0.59	0.74	0.76	0.75
$\sigma = 0.20$	100	3.39	3.83	3.82	3.50	3.51	3.91	3.93	3.94
$T = 0.25$	110	9.75	10.20	10.18	10.32	10.29	10.62	10.60	10.72
	120	18.43	18.68	18.65	20.00	20.00	20.00	20.00	20.00
3)	80	1.05	1.14	1.14	1.06	1.07	1.15	1.16	1.16
$r = 0.08$	90	3.23	3.41	3.41	3.27	3.28	3.45	3.47	3.47
$\sigma = 0.40$	100	7.29	7.54	7.53	7.40	7.41	7.65	7.65	7.65
$T = 0.25$	110	13.25	13.53	13.50	13.52	13.50	13.77	13.75	13.76
	120	20.73	20.97	20.94	21.29	21.23	21.48	21.42	21.45
4)	80	0.21	0.32	0.32	0.21	0.23	0.32	0.34	0.34
$r = 0.08$	90	1.31	1.69	1.68	1.36	1.39	1.73	1.75	1.75
$\sigma = 0.20$	100	4.46	5.12	5.09	4.71	4.72	5.32	5.33	5.35
$T = 0.50$	110	10.16	10.90	10.82	11.00	10.96	11.51	11.49	11.61
	120	17.85	18.47	18.34	20.00	20.00	20.05	20.03	20.00

TABLE III

THEORICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, $\phi = 0.1$, K = 100, r-δ = 0.04

CALL OPTION PRICES

		EUROPEAN				AMERICAN			
PARAMETERS		NO JUMP		WITH JUMPS		NO JUMP	WITH JUMPS		
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Ap-prox
1)	S_0								
$r = 0.08$	80	0.05	0.14	0.14	0.05	0.05	0.14	0.14	0.14
$\sigma = 0.20$	90	0.85	1.19	1.19	0.85	0.85	1.19	1.19	1.19
$T = 0.25$	100	4.44	4.85	4.85	4.44	4.44	4.85	4.85	4.85
	110	11.66	11.87	11.87	11.66	11.66	11.87	11.87	11.87
	120	20.90	20.95	20.95	20.90	20.90	20.95	20.95	20.95
2)	80	0.05	0.14	0.14	0.05	0.05	0.14	0.14	0.14
$r = 0.12$	90	0.84	1.17	1.18	0.84	0.84	1.17	1.18	1.18
$\sigma = 0.20$	100	4.40	4.80	4.80	4.40	4.40	4.80	4.80	4.80
$T = 0.25$	110	11.55	11.75	11.75	11.55	11.55	11.75	11.75	11.75
	120	20.69	20.74	20.74	20.69	20.69	20.74	20.74	20.75
3)	80	1.29	1.41	1.43	1.29	1.29	1.41	1.43	1.43
$r = 0.08$	90	3.82	4.00	4.03	3.82	3.82	4.00	4.03	4.03
$\sigma = 0.40$	100	8.35	8.54	8.57	8.35	8.35	8.54	8.57	8.57
$T = 0.25$	110	14.80	14.94	14.99	14.80	14.80	14.94	14.99	14.99
	120	22.71	22.80	22.85	22.71	22.72	22.80	22.85	22.85
4)	80	0.41	0.67	0.69	0.41	0.41	0.67	0.69	0.69
$r = 0.08$	90	2.18	2.66	2.72	2.18	2.18	2.66	2.72	2.72
$\sigma = 0.20$	100	6.50	6.98	7.08	6.50	6.50	6.98	7.08	7.08
$T = 0.50$	110	13.42	13.71	13.83	13.42	13.42	13.71	13.83	13.83
	120	22.06	22.12	22.26	22.06	22.06	22.12	22.26	22.26

TABLE IV

THEORICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, $\phi = 0.1$, K = 100, r-δ = -0.04

CALL OPTION PRICES

		EUROPEAN				AMERICAN			
PARAMETERS		NO JUMP		WITH JUMPS		NO JUMP		WITH JUMPS	
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Approx
1)	S_0								
$r = 0.08$	80	0.03	0.10	0.10	0.03	0.03	0.10	0.10	0.10
$\sigma = 0.20$	90	0.57	0.86	0.86	0.58	0.59	0.88	0.87	0.88
$T = 0.25$	100	3.42	3.85	3.85	3.52	3.52	3.96	3.89	3.95
	110	9.85	10.10	10.10	10.36	10.31	10.57	10.42	10.57
	120	18.62	18.69	18.69	20.00	20.00	20.00	20.00	20.00
2)	80	0.03	0.09	0.09	0.03	0.03	0.10	0.10	0.10
$r = 0.12$	90	0.56	0.85	0.85	0.57	0.59	0.87	0.86	0.88
$\sigma = 0.20$	100	3.39	3.81	3.81	3.50	3.51	3.94	3.86	3.93
$T = 0.25$	110	9.75	10.00	10.00	10.33	10.29	10.54	10.38	10.54
	120	18.43	18.51	18.51	20.00	20.00	20.00	20.00	20.00
3)	80	1.05	1.16	1.17	1.05	1.07	1.17	1.18	1.19
$r = 0.08$	90	3.23	3.41	3.43	3.27	3.28	3.45	3.46	3.48
$\sigma = 0.40$	100	7.29	7.48	7.52	7.41	7.41	7.61	7.59	7.64
$T = 0.25$	110	13.25	13.41	13.45	13.53	13.50	13.70	13.64	13.70
	120	20.73	20.83	20.88	21.29	21.23	21.40	21.29	21.38
4)	80	0.21	0.39	0.40	0.21	0.23	0.40	0.41	0.42
$r = 0.08$	90	1.31	1.73	1.77	1.36	1.39	1.80	1.79	1.85
$\sigma = 0.20$	100	4.46	4.98	5.06	4.71	4.72	5.27	5.16	5.32
$T = 0.50$	110	10.16	10.55	10.66	11.00	10.96	11.39	11.12	11.45
	120	17.85	18.01	18.14	20.00	20.00	20.01	20.00	20.00

TABLE V

THEORETICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, $\phi = 0.06$, K = 100, r-δ =- 0.04

CALL OPTION PRICES

		EUROPEAN				AMERICAN			
PARAMETERS		NO JUMP		WITH JUMPS		NO JUMP	WITH JUMPS		
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Ap-prox
1)	S_0								
$r = 0.08$	80	0.03	0.05	0.05	0.03	0.03	0.05	0.05	0.05
$\sigma = 0.20$	90	0.57	0.66	0.67	0.58	0.59	0.68	0.68	0.69
$T = 0.25$	100	3.42	3.57	3.59	3.52	3.52	3.68	3.65	3.69
	110	9.85	9.92	9.95	10.36	10.31	10.43	10.34	10.42
	120	18.62	18.62	18.65	20.00	20.00	20.00	20.00	20.00
2)	80	0.03	0.04	0.05	0.03	0.03	0.05	0.05	0.05
$r = 0.12$	90	0.56	0.66	0.66	0.57	0.59	0.67	0.67	0.69
$\sigma = 0.20$	100	3.39	3.53	3.55	3.50	3.51	3.65	3.63	3.67
$T = 0.25$	110	9.75	9.82	9.85	10.33	10.29	10.40	10.30	10.39
	120	18.43	18.43	18.46	20.00	20.00	20.00	20.00	20.00
3)	80	1.05	1.08	1.09	1.05	1.07	1.09	1.10	1.11
$r = 0.08$	90	3.23	3.28	3.30	3.27	3.28	3.32	3.34	3.36
$\sigma = 0.40$	100	7.29	7.33	7.37	7.41	7.41	7.45	7.47	7.49
$T = 0.25$	110	13.25	13.26	13.32	13.53	13.50	13.56	13.54	13.58
	120	20.73	20.71	20.79	21.29	21.23	21.30	21.24	21.29
4)	80	0.21	0.26	0.27	0.21	0.23	0.27	0.28	0.29
$r = 0.08$	90	1.31	1.45	1.48	1.36	1.39	1.50	1.51	1.55
$\sigma = 0.20$	100	4.46	4.63	4.69	4.71	4.72	4.90	4.84	4.95
$T = 0.50$	110	10.16	10.27	10.36	11.00	10.95	11.12	10.95	11.15
	120	17.85	17.86	17.97	20.00	20.00	20.00	20.00	20.00

TABLE VI

THEORICAL EUROPEAN AND AMERICAN CALL VALUES

Parameters: $\lambda = 1$, $\phi = 0.02$, K = 100, r-δ = -0.04

CALL OPTION PRICES

		EUROPEAN				AMERICAN			
PARAMETERS		NO JUMP		WITH JUMPS		NO JUMP		WITH JUMPS	
		Garm. Kohlh.	Fin. diff. meth.	Merton	Fin. diff. meth.	Barone Adesi	Fin. diff. meth.	Bates	Ap-prox
1)	S_0								
$r = 0.08$	80	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
$\sigma = 0.20$	90	0.57	0.59	0.58	0.58	0.59	0.60	0.60	0.60
$T = 0.25$	100	3.42	3.45	3.44	3.52	3.52	3.55	3.53	3.54
	110	9.85	9.86	9.86	10.36	10.37	10.37	10.30	10.33
	120	18.62	18.62	18.62	20.00	20.00	20.00	20.00	20.00
c 2)	80	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
$r = 0.12$	90	0.56	0.58	0.57	0.57	0.59	0.59	0.59	0.60
$\sigma = 0.20$	100	3.39	3.41	3.41	3.50	3.51	3.53	3.51	3.53
$T = 0.25$	110	9.75	9.76	9.76	10.33	10.29	10.34	10.28	10.30
	120	18.43	18.44	18.44	20.00	20.00	20.00	20.00	20.00
3)	80	1.05	1.05	1.05	1.05	1.07	1.06	1.07	1.07
$r = 0.08$	90	3.23	3.24	3.24	3.27	3.28	3.28	3.29	3.29
$\sigma = 0.40$	100	7.29	7.30	7.30	7.41	7.41	7.42	7.41	7.42
$T = 0.25$	110	13.25	13.26	13.26	13.53	13.50	13.54	13.50	13.51
	120	20.73	20.73	20.73	21.29	21.23	21.30	21.22	21.24
4)	80	0.21	0.22	0.22	0.21	0.23	0.23	0.23	0.23
$r = 0.08$	90	1.31	1.34	1.33	1.36	1.39	1.39	1.38	1.41
$\sigma = 0.20$	100	4.46	4.50	4.49	4.71	4.72	4.75	4.70	4.75
$T = 0.50$	110	10.16	10.19	10.19	11.00	10.95	11.02	10.91	10.98
	120	17.85	17.86	17.86	20.00	20.00	20.00	20.00	20.00