The Whys of the LOIS: Credit Risk and Refinancing Rate Volatility

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Abstract

The 2007 subprime crisis has induced a persistent disconnection between the LIBOR derivative markets of different tenors and the OIS swap market. Commonly proposed explanations for the corresponding spreads are a combination of credit risk and liquidity risk. However in these explanations the meaning of liquidity in particular is either not precisely stated, or it is simply defined as a residual spread after removal of a credit component. In this paper we propose a stylized equilibrium model in which a LIBOR-OIS spread (LOIS) emerges as a consequence of a credit component determined by the skewness of the CDS curve of a representative borrower, and a liquidity component corresponding to the volatility of the cost-of-capital of a representative lender, where this cost-of-capital involves the lender's CDS spread. The credit component is thus in fact a credit skewness component, and the relevant notion of liquidity appears as the optionality, valued by the aforementioned volatility, of dynamically adjusting through time the amount of a rolling OIS loan, as opposed to lending a fixed amount up to the tenor horizon on LIBOR. In diffusive models this results in a square root term structure of the LOIS, with a square root coefficient related to the CDS spread volatility of a representative lender. Consistently with this theoretical analysis, empirical observations reveal a square root term structure of the LOIS with a square root coefficient much in line with the volatility of major banks one year CDS.

Keywords: LIBOR, OIS, LOIS, Multiple-Curve, Credit, Liquidity, Interest Rate Spread, Equilibrium, Fixed-income Moldeling, Funding Cost, Treasury Management, CVA.

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1 Introduction

The interbank loan market has been severely impacted since the 2007 subprime crisis and the ensuing liquidity squeeze. However the reference interbank rates, the LIBOR, collected daily from major banks, are still of primary influence as underlyings to most vanilla interest-rate derivatives like FRA, IRS, cap/floor and swaptions. The resulting situation where an underlying to financial derivatives has become in a sense arbitrarily fixed by a panel of key players in the derivatives market poses insider issues, as illustrated by the recent mock LIBOR affairs. But first of all, it poses a crucial funding issue as, on the one hand, in parallel to the drying up of the interbank loan market, LIBOR got disconnected from OIS rates (see Fig. 1); whilst on the other hand, as more and more trades are collateralized, their effective funding rate is the corresponding collateral repo rate which is typically indexed on OIS. This creates a situation where the price of an interest-rate product, even the simplest flow instrument like a FRA, involves (at least) two curves, an OIS and a LIBOR curve, and the related convexity adjustment (which for some products is found significant, see (Mercurio 2010)). Via the interrelations between CVA and funding this also has some important CVA implications (see (Crépey 2012a; Crépey 2012b; Pallavicini, Perini, and Brigo 2011)).

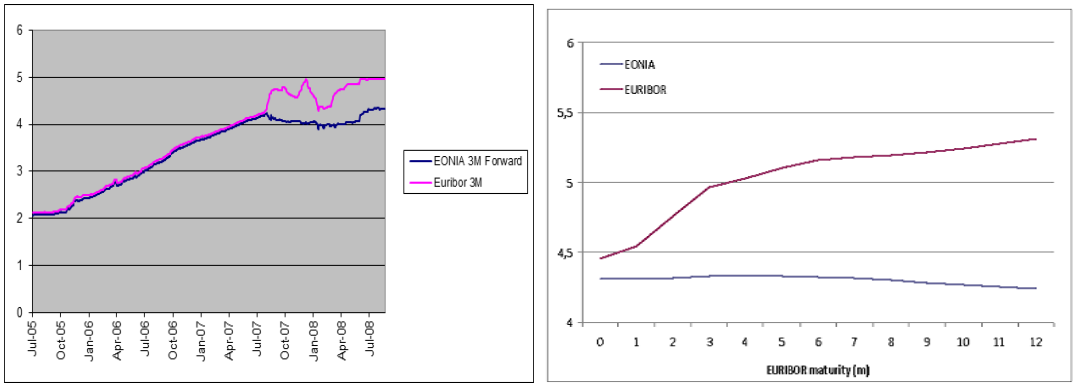


Figure 1: Divergence LIBOR-OIS. *Left*: Sudden divergence that occurred on Aug 6 2007; *Right*: Term structure of LIBOR vs OIS swap rates, Aug 14 2008 (see Formula (62) in Subsection 4.3 of (Crépey, Grbac, and Nguyen 2011) for the definition of "OIS swap rates" which is used in the right panel).

Commonly advanced explanations for the LIBOR-OIS spreads (often called LOIS in the market) are a combination of credit risk and/or liquidity risk. See (Morini 2009; Filipović and Trolle 2011; Crépey, Grbac, and Nguyen 2011; Mercurio 2010; Bianchetti 2010; Fujii, Shimada, and Takahashi 2011; Moreni and Pallavicini 2010) for various multiple-curve models that have been introduced in the literature. A clear tribute to credit risk and liquidity fundamentals is explicit in the first three references. However in these explanations the meaning of liquidity in particular is either not precisely stated, or it is simply defined as a residual spread after removal of a credit component. In this paper we propose a stylized equilibrium model to evaluate at which rate does a bank find it interesting to lend at a given tenor horizon, as opposed to rolling an overnight loan which it can cancel at any moment. In this setup LOIS emerges as a consequence of the skewness of the credit curve of a representative borrower, and of the volatility of the cost-of-capital of a representative lender, where this cost-of-capital includes in particular the CDS spread of the lender.

2 Equilibrium Model

We assume that the funding rate of a lending bank with a short term debt D is given by a random function $\rho_t(D)$. This funding rate mainly depends on the level of the short term debt of the bank as compared to its immediately repositable capital C_t, and on the bank's CDS spread rate S_t. For instance, in a first approximation, we may have

$$\rho_t(D_t + x) \quad = \quad \varphi(t, S_t, (D_t + x)/C_t) \tag{1}$$

in which the time dependence in φ reflects macro-economic global variables such as the evolution of interest rates.

Let $\mathbb{P}$ and $\mathbb{E}$ denote the actuarial probability measure and the related expectation. The problem of the bank lending overnight at rate K is modeled as

$$T\mathcal{U}(K;(N_t)) = \mathbb{E}\left(K\int_0^T N_t dt - \int_0^T \int_0^{N_t} \rho_t(D_t + x)dxdt\right) \longleftarrow \max(N_t) \tag{2}$$

where a stochastic process N_t represents the amount of notional that the bank is willing to lend at the OIS rate K between t and $t + dt$. As opposed to the situation of lending overnight a revisable notional N_t, when lending at the LIBOR of maturity T, a bank cannot modify a notional N which is locked between 0 and T. Moreover, as the composition of the LIBOR panel is updated at regular time intervals, there is more default risk in a LIBOR loan than in an OIS loan (even if the panel of contributors is the same at a given time, as is the case for EONIA with respect to EURIBOR; in case of the Fed rate for US dollar the panel of contributing banks is in fact larger than for the related LIBOR). The related optimization problem of the bank can be modeled as

$$T\mathcal{V}(L;N) = \mathbb{E}\left(LN(T\wedge\tau) - \int_0^{T\wedge\tau}\int_0^N \rho(D_t + x)dxdt - \mathbb{1}_{\tau<T}N\right) \longleftarrow \max N \tag{3}$$

Here a constant N represents the amount of notional that the bank is willing to lend on the LIBOR of maturity T at rate L between 0 and T, and a stylized default time τ of the borrower reflects the deterioration of the average credit quality of the LIBOR contributors during the length of the tenor, since this deterioration only affects the lender when lending LIBOR (see (Filipović and Trolle 2011) for a detailed analysis, and see the comments following (8) below). As we are dealing with short-term debt, we assume no recovery in case of default. Inclusion of a positive recovery rate would not change the end-results of Subsection 3.1 though since the final inputs therein are reformulated in terms of CDS spreads, which incorporate $(1 - \text{rec}^y)$ factors anyway.

The "utility functions" of the bank which are implicit in (2)-(3) are taken in a standard economic equilibrium formalism of Legendre transforms U or V of the OIS and LIBOR cost functions, represented by the integrals over x in the right-hand side of (2)-(3). These utility functions are linear to reflect the general risk-neutral behavior of banks when lending, in which gains and losses are assessed in terms of actuarial expectations. In other word, the choice of banks to lend is less driven by preferences than by an optimization of the cost of capital and credit protection (even if it is not bought, as its cost determines capital adequacy). One can incorporate a concavely distorted utility function to account for risk aversion. Such a distortion would appear in our model as an increased volatility of capital needs and of the corresponding borrowing rate. However, we believe that short term lending decisions are more driven by the estimated cost of capital than by a trade-off between interest

returns and default risk. Default risk obviously matters in the lending decision, but as a "yes/no" answer, based on available information on the borrower at the time of lending. Once a borrower is deemed acceptable, the amount lent and the granted rate depend on the impact on the bank capital and its estimated cost, as well as on the borrower's default risk measured through the cost of protection (credit spread, CDS rate), which either is bought or is accounted for in the capital affected to the loan. This justifies the use of purely linear utility functions.

We also did not introduce a discount factor, as we are dealing with short term debt. Extending this model to longer term debt would require this correction. We stick to the stylized formulation (2)-(3) for tractability issues, and also in order to emphasize that the volatility terms which appear in the end-results (11), (13) and (14), are convexity adjustments reflecting an inherent optionality of LOIS, even without any risk premia.

Letting $U(K) = \max_{(N_t)} \mathcal{U}(K;(N_t))$ and $V(L) = \max_N \mathcal{V}(L;N)$, our approach for explaining the LOIS consists, given a value of K, in solving the equation

$$V(L^*) = U(K) \tag{4}$$

which expresses an equilibrium relation between the utility of lending overnight versus LIBOR for a bank involved in both markets (indifference value at the optimal amounts prescribed by the solution of the corresponding optimization problems). A second optimization problem, left aside in this article, would be in the level of the overnight rate K, depending on the supply/demand of liquidities and on base rates from the central bank.

2.1 Linear Case

For tractability we assume henceforth that the funding rate ρ is linear in D, i.e.

$$\rho_t(D_t + x) = \alpha_t + \beta_t x \tag{5}$$

where $\alpha_t = \rho_t(D_t)$ is the cost of capital of the lending bank at time t, and the coefficient β_t (positive in spirit) represents the marginal additional cost of borrowing one more unit of notional for the bank already indebted at the level D_t. For instance $\alpha_t = 2\%$, $\beta_t = 50$bp means that the last 100 euros that were borrowed by the bank cost an annualized interest charge of 2 euros, whereas the next 100 euros to be borrowed by the bank would cost an annualized interest charge of 2.5 euros. E.g., in a specification also consistent with (1), letting $R_t = D_t/C_t$

$$\rho_t(D_t + x) = S_t + AR_t + BS_t x \tag{6}$$

for constants A and B, so

$$\alpha_t = S_t + AR_t\,,\ \beta_t = BS_t$$

in (5). One then has

$$T\mathcal{U}(K;(N_t)) = \mathbb{E}\int_0^T \left((K-\alpha_t)N_t - \frac{1}{2}\beta_t {N_t}^2\right) dt \tag{7}$$

Denoting by λ_t the intensity of τ, one also has, letting $\gamma_t = \alpha_t + \lambda_t$ and $\ell_t = e^{-\int_0^t \lambda_s ds}$

$$T\mathcal{V}(L;N) = \mathbb{E}\int_0^T \left((L-\gamma_t)N - \frac{1}{2}\beta_t N^2\right) \ell_t dt \tag{8}$$

where a standard credit risk computation was used to get rid of the default indicator functions in (8) (see for instance (Bielecki and Rutkowski 2002)). As explained after (3), the stylized default time τ reflects the deterioration of the average credit quality of the LIBOR contributors during the length of the tenor. The intensity λ_t of τ can thus be proxyed by the differential between the borrower's CDS spread of maturity T and the spread of her Certificate Deposit. Accordingly we call λ_t the credit skewness of the borrower. Formally, $\lambda_t dt$ is the difference between the borrower's default probability over the interval $[t, t+dt]$ measured with information known at $t = 0$ and that known at time t, assuming that the bank applies a lending policy preventing lending to counterparties that are considered too risky. Assuming that the borrower's default intensity is a random process with i.i.d. increments and that the bank lending policy prevents lending when the default intensity is above a given threshold, we can thus see the difference $\lambda_t dt$ as the expectation of the default intensity when it exceeds the threshold (i.e. the conditional expectation conditionally to exceeding the threshold, multiplied by the probability of exceeding it).

Note that a possible mock effect, or incentive for a LIBOR contributor to bias its borrowing rate estimate in order to appear in a better condition than it is in reality, can be included as a spread in the borrower's credit risk skewness component λ.

3 Derivation of the Spreads

Problems (2), (7) and (3), (8) are respectively solved, for given K and L, as follows. The OIS problem (2), (7) is resolved independently at each date t according to

$$u_t(K; N_t) = (K - \alpha_t)\, N_t - \frac{1}{2}\beta_t N_t^2 \ \longleftarrow \ \max\, N_t$$

hence

$$N_t^*(K) = \frac{K - \alpha_t}{\beta_t} \quad \text{and} \quad u_t(K; N_t^*) = \frac{(K - \alpha_t)^2}{2\beta_t}$$

The expected profit of the bank over the period $[0, T]$ is

$$U(K) = \mathcal{U}(K; (N_t^*)) = \mathbb{E}\left(\frac{1}{T}\int_0^T \frac{(K - \alpha_t)^2}{2\beta_t} dt\right)$$

In the Libor problem (3), (8), we must solve

$$T\mathcal{V}(L; N) = N\mathbb{E}\int_0^T (L - \gamma_t)\ell_t dt - \frac{1}{2}N^2\mathbb{E}\int_0^T \beta_t \ell_t dt \ \longleftarrow \ \max\, N$$

hence

$$N^* = \frac{\mathbb{E}\frac{1}{T}\int_0^T (L - \gamma_t)\ell_t dt}{\mathbb{E}\frac{1}{T}\int_0^T \beta_t \ell_t} dt \quad \text{and} \quad V(L) = \mathcal{V}(L; N^*) = \frac{\left(\mathbb{E}\frac{1}{T}\int_0^T (L - \gamma_t)\ell_t dt\right)^2}{2\mathbb{E}\frac{1}{T}\int_0^T \beta_t \ell_t dt} \tag{9}$$

Note that in case $\lambda = 0$, one necessarily has $U(K) \geq V(K)$, since the constant N^* solving the LIBOR maximization problem (3) is a particular strategy (constant process $N_t = N^*$) of the OIS maximization problem (2). As U and V are increasing functions, the indifference pricing equation (4) in turn yields that $L^* \geq K$.

Let $\mathcal{V}_0(K; N)$ be the utility of lending LIBOR in case $\lambda = 0$. When $\lambda > 0$, for each given amount N, one has via λ which is present in γ in (8) that $\mathcal{V}(K; N) \leq \mathcal{V}_0(K; N)$ (up

to the second order impact of the discount factor ℓ), hence the maximum of $\mathcal{V}(K;\cdot)$ in N is below that of $\mathcal{V}_0(K;\cdot)$, i.e. $V(K) \leq V_0(K) \leq U(K)$, the latter inequality resulting from the fact that τ doesn't appear in $\mathcal{U}(K;(N_t))$. One concludes as in case $\lambda = 0$ that $L^* \geq K$ holds again.

The sequel of this paper is devoted to the computation of the spread $L^* - K$ where L^* is, given K, the solution of (4) (assumed to exist; note that V is continuous and increasing in L, so that a solution L^* to (4) can only be uniquely defined). For notational convenience let us introduce the time-space probability measure $\overline{\mathbb{P}}$ product of $\mathbb{P}$ times $\frac{dt}{T}$ over $\Omega \times [0,T]$, as well as another time-space probability measure such that $\frac{d\widehat{\mathbb{P}}}{d\overline{\mathbb{P}}} \propto \ell$. For a process $X = X_t(\omega)$ we denote the corresponding time-space averages and variances by

$$\bar{X} = \overline{\mathbb{E}}X = \mathbb{E}\frac{1}{T}\int_0^T X_t dt\,,\ \ \overline{\sigma}_X^2 = \overline{\mathbb{E}}(X - \bar{X})^2$$
$$\widehat{X} = \widehat{\mathbb{E}}X = \overline{\mathbb{E}}\left[X\frac{\ell}{\overline{\ell}}\right]\,,\ \ \widehat{\sigma}_X^2 = \widehat{\mathbb{E}}(X - \widehat{X})^2.$$

As

$$U(K) = \overline{\mathbb{E}}\left[\frac{(K-\alpha)^2}{2\beta}\right] \quad \text{and} \quad V(L) = \frac{\overline{\ell}^2\,(L-\widehat{\gamma})^2}{2\overline{\mathbb{E}}[\beta\ell]}$$

equalizing $V(L^*) = U(K)$ yields

$$\overline{\ell}^2\,(L^* - \widehat{\gamma})^2 = \overline{\mathbb{E}}[\beta\ell]\overline{\mathbb{E}}\left[\frac{(K-\alpha)^2}{\beta}\right] \tag{10}$$

In which

$$\overline{\mathbb{E}}[\beta\ell]\overline{\mathbb{E}}\left[\frac{(K-\alpha)^2}{\beta}\right] = \overline{\mathbb{E}}\left[(K-\alpha)^2\ell\right] - \overline{\mathbb{C}\mathrm{ov}}\left[\beta\ell, \frac{(K-\alpha)^2}{\beta}\right]$$

So

$$\overline{\ell}^2(L^* - \widehat{\gamma})^2 = \overline{\ell}\widehat{\mathbb{E}}\left[(K-\alpha)^2\right] - \overline{\mathbb{C}\mathrm{ov}}\left[\beta\ell, \frac{(K-\alpha)^2}{\beta}\right]$$

In particular, "at-the-money" when $K = \widehat{\alpha} = \widehat{\gamma} - \widehat{\lambda}$, one has

$$\overline{\ell}(L^* - K - \widehat{\lambda})^2 = \widehat{\sigma}_\alpha^2 - \overline{\mathbb{C}\mathrm{ov}}\left[\beta\frac{\ell}{\overline{\ell}}, \frac{(\alpha - \widehat{\alpha})^2}{\beta}\right]$$

A reasonable guess is that the covariance is negligible in the right-hand-side (in particular this covariance vanishes when λ is zero and β is constant). Our first spread formula follows as[1]

$$L^* - K \approx \widehat{\lambda} + \frac{\widehat{\sigma}_\alpha}{\sqrt{\overline{\ell}}}. \tag{11}$$

Alternatively, Equation (10) in L^* can be written in terms of a third time-space probability measure such that $\frac{d\widetilde{\mathbb{P}}}{d\overline{\mathbb{P}}} \propto \beta^{-1}$ as

$$\overline{\ell}^2\,(L^* - \widehat{\gamma})^2 = \overline{\mathbb{E}}\left[\beta\ell\right]\overline{\beta^{-1}}\widetilde{\mathbb{E}}\left[(K-\alpha)^2\right]$$

[1] Admitting that $L^* - K - \widehat{\lambda} \geq 0$, a natural assumption as LIBOR lending should at least compensate for the credit risk over the tenor horizon period, cf. the comments following Equation (9).

or

$$\overline{\ell}\,(L^* - \widehat{\gamma})^2 = \widehat{\beta\beta^{-1}}\widetilde{\mathbb{E}}\left[(K-\alpha)^2\right] \tag{12}$$

Another case of interest is then when $K = \widetilde{\alpha} = \widetilde{\mathbb{E}}\alpha$, in which case (12) boils down to

$$\overline{\ell}\,(L^* - \widehat{\gamma})^2 = \widehat{\beta\beta^{-1}}\widetilde{\sigma}_\alpha^2.$$

yielding our second spread formula[2]

$$L^* - K = \widehat{\lambda} + \widehat{\alpha} - \widetilde{\alpha} + \sqrt{\frac{\widehat{\beta\beta^{-1}}}{\overline{\ell}}}\widetilde{\sigma}_\alpha \approx \widehat{\lambda} + \sqrt{\frac{\widehat{\beta\beta^{-1}}}{\overline{\ell}}}\widetilde{\sigma}_\alpha \tag{13}$$

as $\widehat{\alpha} - \widetilde{\alpha}$ is negligible with respect to other terms. Also note that in case $\lambda = 0$ and $\overline{\mathbb{P}} = \widehat{\mathbb{P}}$, one has in the last expression that $\overline{\ell} = 1$ and

$$\widehat{\beta\beta^{-1}} \geq 1$$

by the Jensen's inequality, with equality in the constant β case.

3.1 LOIS Formula

In (11) as in (13), the two key drivers of the LOIS are on the one hand a suitable time-space average of the borrower's credit skewness λ (which can be seen as the "intrinsic value" component of the LOIS, and is a borrower's credit component), and on the other hand a suitable time-space standard deviation ("time-value" component of the LOIS, a lender's liquidity component) of the lender's cost of capital $\alpha_t = \rho_t(D_t)$.

Under the specification (6) and assuming here independence of the lender's credit spread S_t and debt-ratio R_t, as well as diffusive properties of the input processes λ_t, S_t and R_t, with corresponding "reference volatilities " denoted by σ_λ, σ_S and σ_R, and with a "reference credit skewness λ of the borrower"denoted by λ^*, one can approximate in (11) and (13)

$$\begin{aligned}
&\sqrt{\overline{\ell}} \approx 1 - \frac{1}{2}\lambda^* T\,, \;\; \sqrt{\widehat{\beta\beta^{-1}}} \approx 1 + \frac{1}{2}\sigma_\lambda^2 T \\
&\widehat{\sigma}_\alpha \approx \widetilde{\sigma}_\alpha \approx (\sigma_S + A\sigma_R)\sqrt{T}
\end{aligned}$$

One then has in (11)

$$\begin{aligned}
L^* - K &\approx \widehat{\lambda} + \frac{\overline{\sigma}_\alpha}{\sqrt{\overline{\ell}}} \\
&\approx \lambda^* + (1 + \frac{1}{2}\lambda^* T)(\sigma_S + A\sigma_R)\sqrt{T}
\end{aligned}$$

and in (13)

$$\begin{aligned}
L^* - K &\approx \widehat{\lambda} + \sqrt{\frac{\widehat{\beta\beta^{-1}}}{\overline{\ell}}}\widetilde{\sigma}_\alpha \\
&\approx \lambda^* + (1 + \frac{1}{2}\sigma_\lambda^2 T + \frac{1}{2}\lambda^* T)(\sigma_S + A\sigma_R)\sqrt{T}
\end{aligned}$$

[2]Admitting non-negativity of $L^* - \widehat{\gamma} = L^* - \widehat{\alpha} - \widehat{\lambda} = L^* - K + \widetilde{\alpha} - \widehat{\alpha} - \widehat{\lambda}v$, cf. the comments following Equation (9).

In both cases

$$L^* - K = \lambda^* + (\sigma_S + A\sigma_R)\sqrt{T} + \mathrm{O}(T^{\frac{3}{2}}) \tag{14}$$

which we call the LOIS formula. The square root term structure in this theoretical formula has a very good fit with the market. Fig. 2 thus shows the fit between a theoretical square root term structure and the empirical LOIS term structure corresponding to the data of the right panel in Fig. 1. The slope (coefficient of $\sqrt{T}$ in the regression) and intercept (constant coefficient) are 0.99 and 0.09. A slope of 0.99 is quite in line with current order of magnitudes of one year CDS spread volatilities on major banks, whereas an intercept of 0.09 is also quite reasonable for the differential between the one year CDS spread and the Certificate Deposit spread of a major bank. The downward bump at one month could be due to the credit-skewness component, which would typically display this kind of shape if banks apply a threshold-based lending policy (see end of Subsection 2.1).

Formula (14) can be used for implying the quantity $A\sigma_R$ "priced" by the market from an observed LOIS $L^* - K$, the borrower's CDS slope, taken as a proxy for λ^*, and the lender's CDS spreads volatility, taken as a proxy for σ_S. The quantity $A\sigma_R$ thus implied through (14) can be compared by a bank to an internal estimate of its "realized $A\sigma_R$", so that the bank can decide whether it should rather lend LIBOR or OIS, much like with going long or short an equity option depending on the relative position of the implied and realized volatilities of the underlying stock.

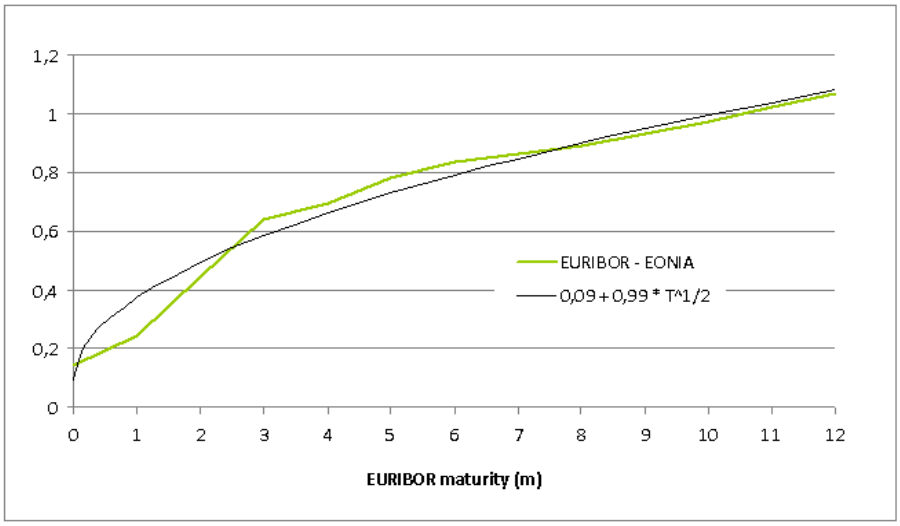


Figure 2: Square root fit of the LOIS corresponding to the data of the right panel in Fig. 1

Conclusion

Since the 2007 subprime crisis, OIS swaps and LIBOR swaps, or equivalently in Euro, EONIA and EURIBOR swaps, diverged suddenly. We show in this paper that, by optimizing their lending, banks are led to apply a spread over the OIS rate when lending at LIBOR. Theory implies that the corresponding LIBOR-OIS spread (LOIS) has two components: One corresponding to the credit skewness of a representative borrower, and one corresponding

to the volatility of the cost-of-capital of a representative lender. In diffusive models this results in a square root term structure of the LOIS, with a square root coefficient related to the CDS spread volatility of a representative lender. This is corroborated by empirical evidence, with an empirical coefficient of a LOIS square root term structure much in line with the volatility of major banks one year CDS.

References

Bianchetti, M. (2010). Two curves, one price. *Risk Magazine*, August 74–80.

Bielecki, T. R. and M. Rutkowski (2002). *Credit Risk: Modeling, Valuation and Hedging*. Springer.

Crépey, S. (2012a). Bilateral Counterparty risk under funding constraints – Part I: Pricing. *Mathematical Finance*. Forthcoming.

Crépey, S. (2012b). Bilateral Counterparty risk under funding constraints – Part II: CVA. *Mathematical Finance*. Forthcoming.

Crépey, S., Z. Grbac, and H. N. Nguyen (2011). A multiple-curve HJM model of interbank risk. Forthcoming in *Mathematics and Financial Economics*.

Filipović, D. and A. B. Trolle (2011). The term structure of interbank risk. SSRN eLibrary.

Fujii, M., Y. Shimada, and A. Takahashi (2011). A market model of interest rates with dynamic basis spreads in the presence of collateral and multiple currencies. *Wilmott Mag.*, July 61–73.

Mercurio, F. (2010). A LIBOR market model with a stochastic basis. *Risk Magazine* (December), 84–89.

Moreni, N. and A. Pallavicini (2010). Parsimonious HJM modelling for multiple yield-curve dynamics. Preprint, arXiv:1011.0828v1.

Morini, M. (2009). Solving the puzzle in the interest rate market. SSRN eLibrary.

Pallavicini, A., D. Perini, and D. Brigo (2011). Funding valuation adjustment: a consistent framework including cva, dva, collateral,netting rules and re-hypothecation. Preprint, arXiv:1521.