Hitting probabilities for systems of non-linear stochastic heat equations in spatial dimension $k \geq 1$

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Abstract. We consider a system of d non-linear stochastic heat equations in spatial dimension $k \geq 1$. The d-dimensional driving noise is white in time and with a spatially homogeneous covariance defined as a Riesz kernel with exponent β, where $0 < \beta < (2 \wedge k)$. The non-linearities appear both as additive drift terms and as multipliers of the noise. Using techniques of Malliavin calculus, we establish an upper bound on the two-point density of the $\mathbb{R}^{2d}$-valued random vector $(u(s,y), u(t,x))$, that, in particular, quantifies how this density degenerates as $(s,y) \to (t,x)$. From this result, we deduce a lower bound on hitting probabilities of the process $\{u(t\,,x)\}_{t \in \mathbb{R}_+, x \in \mathbb{R}^k}$, in terms of Newtonian capacity. We also establish an upper bound on hitting probabilities of the process in terms of Hausdorff measure. These estimates make it possible to show that points are polar when $d > \frac{4+2k}{2-\beta}$ and are not polar when $d < \frac{4+2k}{2-\beta}$ (if $\frac{4+2k}{2-\beta}$ is an integer, then the case $d = \frac{4+2k}{2-\beta}$ is open). We also show that the Hausdorff dimension of the range of the process is $\frac{4+2k}{2-\beta} \wedge d$ a.s. This is joint work with D. Khoshnevisan and E. Nualart.

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