Title :
The nonlinear Schrödinger equation with white noise dispersion.

Abstract :
The following nonlinear Schrödinger equation with white noise dispersion is a model used in the context of optical fibers with random dispersion:

$$\begin{cases} idu + \dot{\beta} o \, \Delta u + |u|^{2\sigma} u = 0, \; x \in \mathbb{R}^d, \\ u(0) = u_0. \end{cases}$$

Here $\dot{\beta}$ is a white noise in time. (In practical $\sigma = 1$.)

We show how Strichartz type estimates can be proved on the linear part of the equation and explain how these can be used to prove existence and uniqueness results for this equation. For general dimension d, we can treat the same exponents as in the classical deterministic theory and obtain global existence in $L^2(\mathbb{R}^d)$ or $H^1(\mathbb{R}^d)$ for $\sigma < 2/d$.

In dimension $d = 1$, we are able to improve the Strichartz estimate and to prove global existence and uniqueness in the critical case $\sigma = 2$. This shows that the white noise dispersion prevents the formation of singularities.

Moreover, we give some results on the derivation of this equation. More precisely, we show that this equation can be obtained as the limit of the following equations

$$\begin{cases} idu^\varepsilon + \dfrac{1}{\varepsilon} m \left(\dfrac{t}{\varepsilon^2} \right) \Delta u^\varepsilon + |u^\varepsilon|^{2\sigma} u^\varepsilon = 0, \\ u^\varepsilon(0) = u_0, \end{cases}$$

where m is a random variable satisfying classical mixing properties.

This is a joint work with Anne de Bouard and Y. Tsutsumi.