Stochastic models for electricity markets
Lecture 03 - Structural models of electricity prices

Frontiers in Stochastic Modelling for Finance
Winter School - Università degli Studi di Padova

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Agenda

1. Introduction
2. Structural models
   - The mother of all structural models
   - Stack curve models
3. Conclusion
4. References

Disclaimer

Disclaimer

Any views or opinions presented in this presentation are solely those of the author and do not necessarily represent those of the EDF group.

Introduction

Looking for a power spot price model

- pricing of derivatives on the spot
- asset valuation (strip of hourly fuel spread options)
- hedging
- energy market risk management

Model requirements

- realistic
- robust
- tractable
- consistent

Modeling strategies

Modeling futures prices

pros modeling the real available instruments

cons introduction of many parameters to reconstruct hourly futures prices

Modeling spot prices

1. Exogeneous
   - **pros** tractability
   - **cons** dependancies
2. Equilibrium
   - **pros** dependancies
   - **cons** complexity

Modeling spot prices dynamic

Electricity prices exogeneous dynamics

Deng (00), Benth et al. (03, 07, 09), Burger et al. (04), Kolodnyi (04), Cartea & Figueroa (05), Geman & Roncoroni (06)

Structural models

- Deduces the spot from very simplified equilibrium models to allow realistic dependencies. Observed factors.

Remark

See recent books by G. Swindle (2015) and A. (2015) for more references on electricity prices modeling.

Structural models

Structural models

Idea

- Instead of using non-Gaussian process for the spot, use Gaussian model + non-linear structure on offer and demand.
- Publicly available datas make structural models possible.
- Simplified fundamental economic models to allow forward price computation.

Literature

- Barlow (02), Kanamura & Ohashi(07)
- Cartea & Villaplana (07)
- Pirrong & Jermakayan (09), Cartea, Figueroa & Géman (09), Coulon & Howison (09), Lyle & Helliott (09), A., Campi, Nguyen Huu & Touzi (09),
- A., Campi & Langrené (12), Carmona & Coulon (13)
- Carmona & Coulon (11): survey on structural models.

The mother of all structural models

Barlow (02)

- The spot price S_t is determined by the equilibrium between an increasing offer function $u_t(x)$ and a decreasing demand function $d_t(x)$:

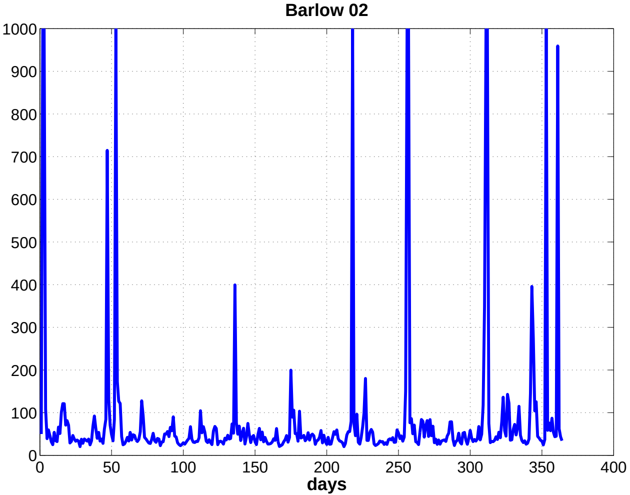
$$u_t(S_t) = d_t(S_t)$$

- Assume constant supply function $u_t = g$ and inelastic demand $d_t(x) = D_t$ with D_t the electricity consumption: $S_t = g^{-1}(D_t)$.
- Basic non-linear form for $g(x) = a - bx^{\alpha}$ with $\alpha < 0$.
- Price is caped if demand exceeds maximum capacity a.
- Demand model: simple OU process with mean-reversion parameter λ, long-run value a_1 and volatility σ_1.

$$S_t = \begin{cases} \left(\frac{a-D_t}{b}\right)^{1/\alpha} & D_t \leq a - \varepsilon b \\ \varepsilon^{1/\alpha} & D_t \geq a - \varepsilon b \end{cases}$$

where ε is determined by the level of the price cap.

Illustration



Forward curve dynamic

Remarks

- Forward flatness
- Mean-reversion still kills forward price volatility

Stack curve models

Model idea

- Imagine an fictitious economy where electricity is produced only out of coal with generation with the same efficiency
- Then, electricity spot price $P_t = h_c S_t^c$, where h_c is the coal heat rate.
- Assume no-arbitrage in the market of coal
- Then, preceeding relation hodls also for forwards

$$F_t^e(T) = h_c F_t^c(T)$$

- and no arbitrage relation between spot and forward can be transported to electricity prices:

$$\begin{array}{ccc} S_t^c & \xrightarrow{g} & P_t \\ \| & & \| \\ F_t^c & \xrightarrow{g} & F_t^0 \end{array}$$

A structurel model based on a simplified stack curve

Inputs

- A., Campi, Nguyen & Touzi (09)
- A., Campi & Langrené (2012)

A structurel model based on a simplified stack curve

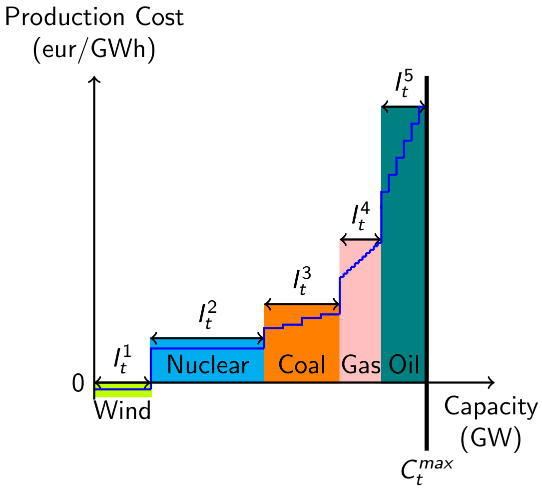
Inputs

n	fuels, $1 \leq i \leq n$
D_t	demand (MW)
C_t^i	capacities (in MW)
S_t^i	fuel prices
h_i	*heat rates* ($h_i S_t^i$ in €/MWh, $\nearrow$ in i)

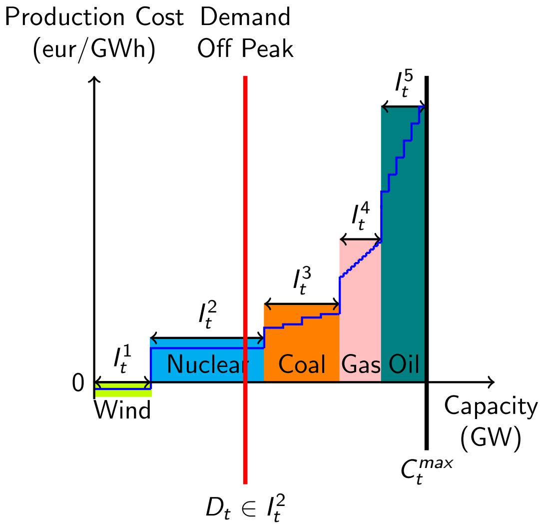
Electricity price (€/MWh)

$$\widehat{P}_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}$$

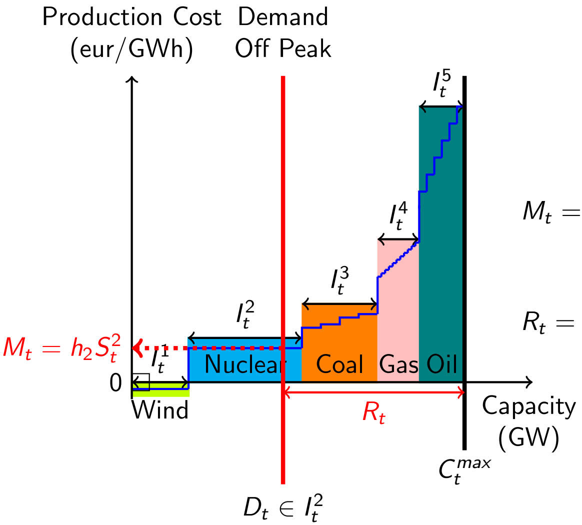
Stack, residual margin & price



Stack, residual margin & price



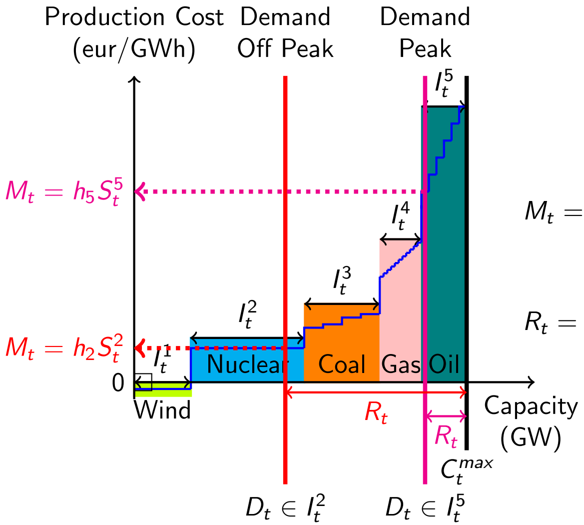
Stack, residual margin & price



$$M_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{D_t \in I_t^i}$$

$$R_t = \sum_{i=1}^{n} C_t^i - D_t$$

Stack, residual margin & price



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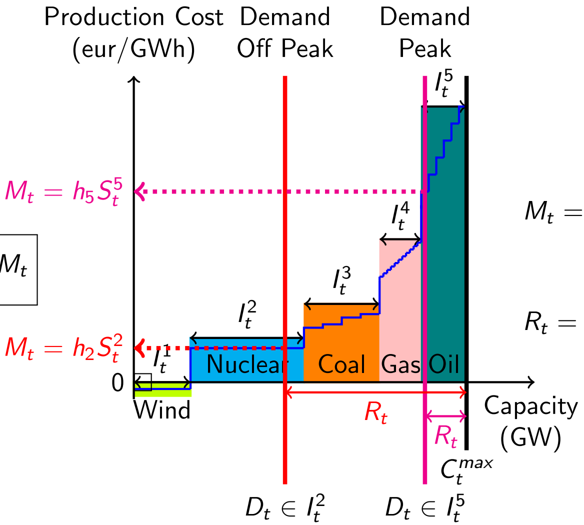
Stack, residual margin & price



$$\boxed{S_t = \frac{\gamma}{R_t^\nu} M_t}$$

$$M_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{D_t \in I_t^i}$$

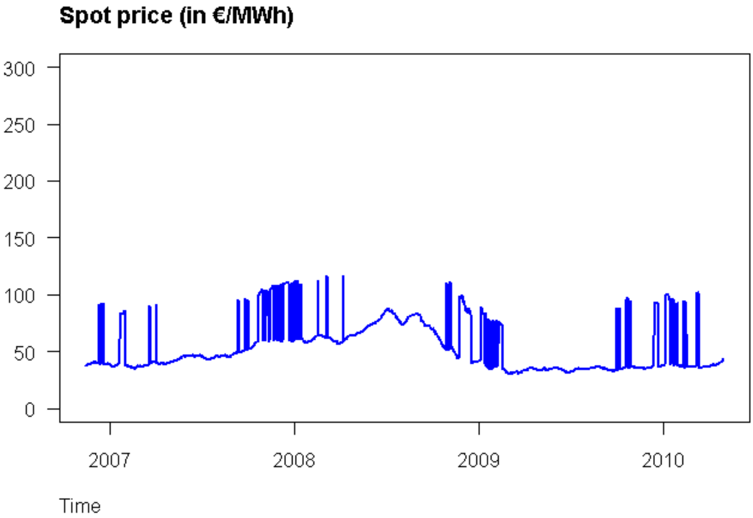
$$R_t = \sum_{i=1}^{n} C_t^i - D_t$$

Model estimation without scarcity effect

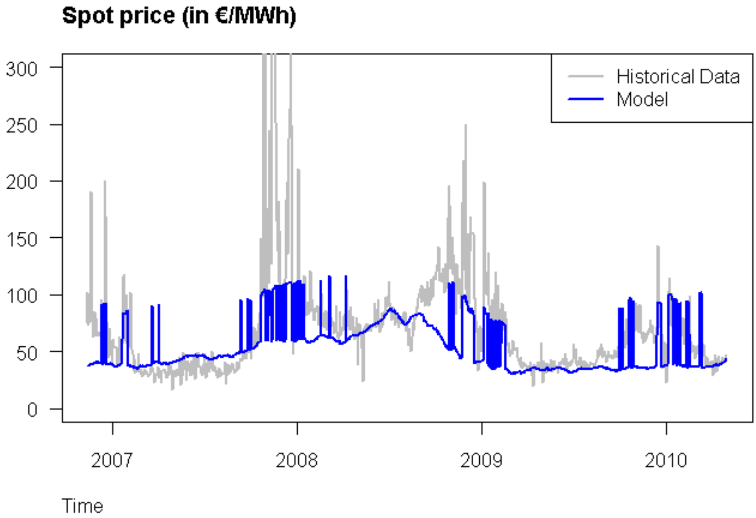
Data

- French power market from Nov. 13th, 2006 to April 30th 2010
- hourly demand provided by RTE website
- Focus on 19th hour of each day
- Either marginal oil or coal
- Model with $n = 2$:
- S^1 and S^2 is daily coal and oil price taking into account spot exchange rate, nominal heat rate, CO2 price and emission rate
- Reduction to residual demand to coal and oil plants
- Average 19th hour price 74 €/MWh, coal including heat rate and CO2 price 47 €/MWh, oil price including CO2 price and heat rate 102 €/MWh

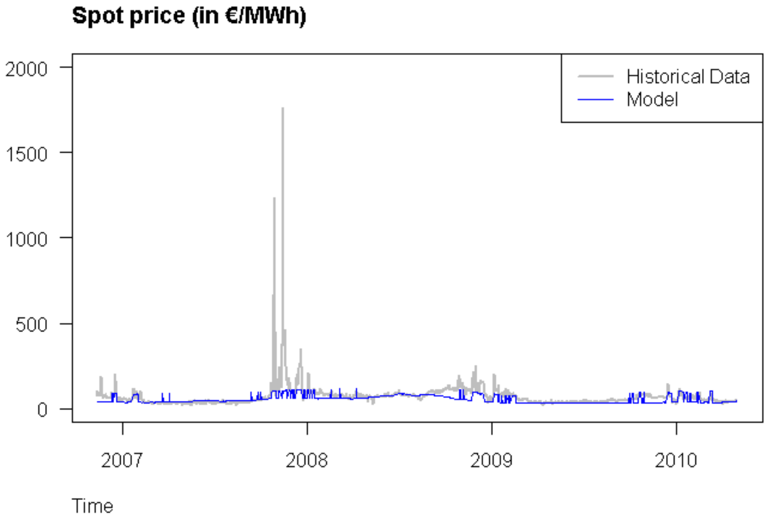
Illustration



SRN Model [10] - illustration



Illustration



Taking into account scarcity

- Marginal fuel cost $\widehat{P}_t := \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\}}$
- Available capacity $\overline{C}_t := \sum_{k=1}^{n} C_t^k$
- Price spikes occur when the electric system is under stress, i.e. $\overline{C}_t - D_t$ is small
- Corresponds to peak-load fixed cost problem recovery...

$y_t := \frac{P_t}{\widehat{P}_t}$ **as a (nonlinear) function of** $x_t := \overline{C}_t - D_t$

Estimated relation

$y_t = \frac{\gamma}{x_t^{\nu}}$
$\gamma = 6.2 + / - 0.06$, $\nu = 1.0 \pm 0.01$ at 95% confidence level.

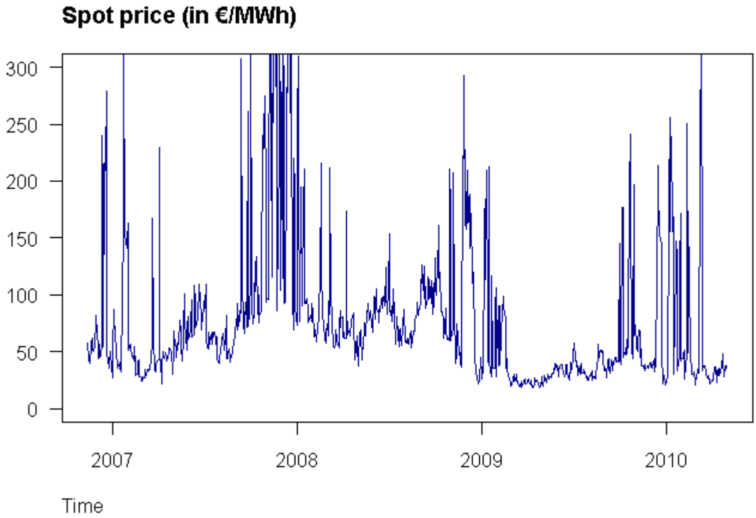
Improved SRN model

$$P_t = g\left(\sum_{k=1}^{n} C_t^k - D_t\right) \times \left(\sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_t^k \leq D_t \leq \sum_{k=1}^{i} C_t^k\right\}}\right)$$

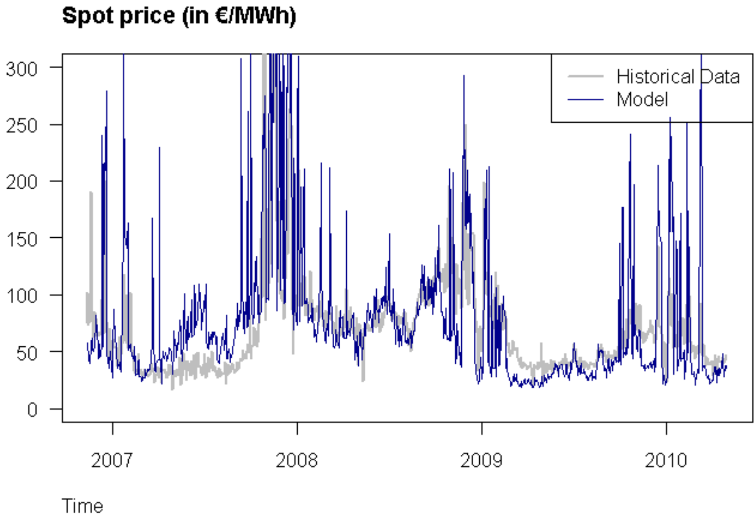
with scarcity function

$$g(x) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbf{1}_{\{x>0\}} + M\mathbf{1}_{\{x\leqslant 0\}}$$

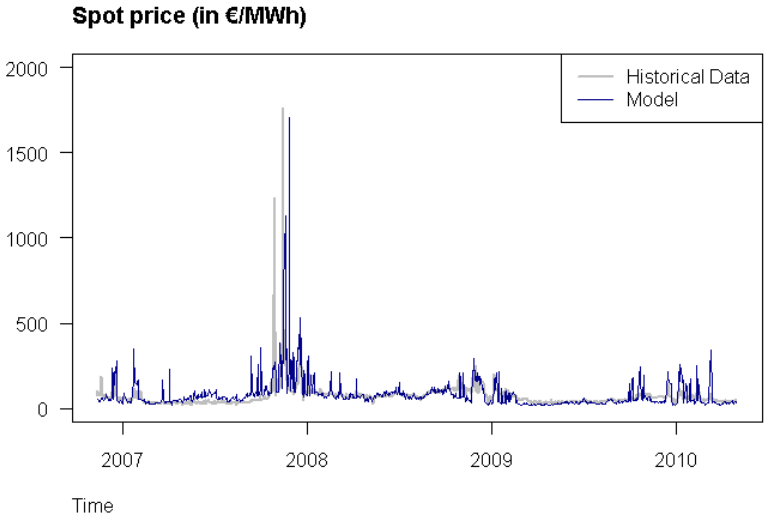
Back-testing



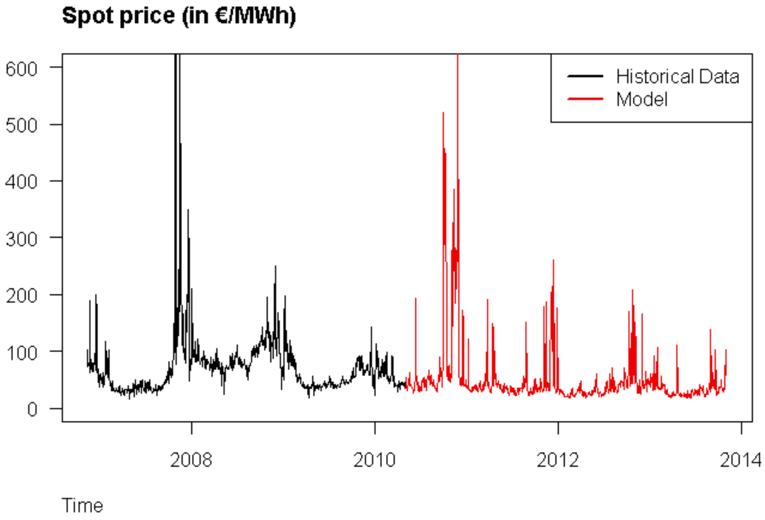
Back-testing



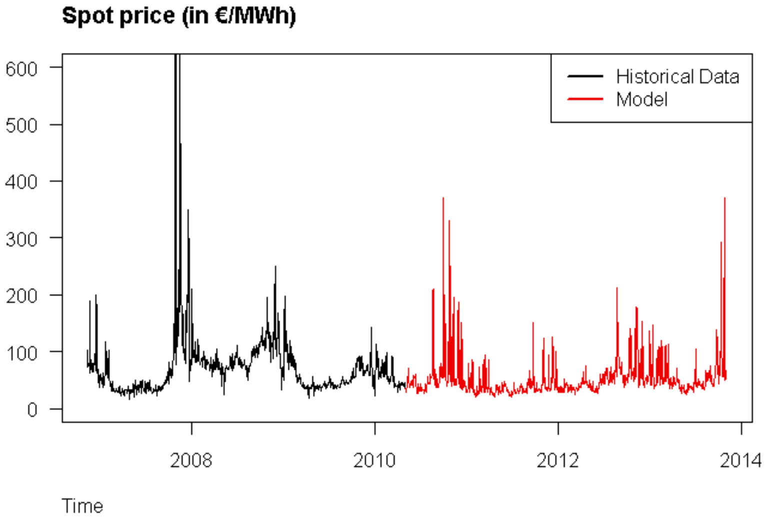
Backtesting



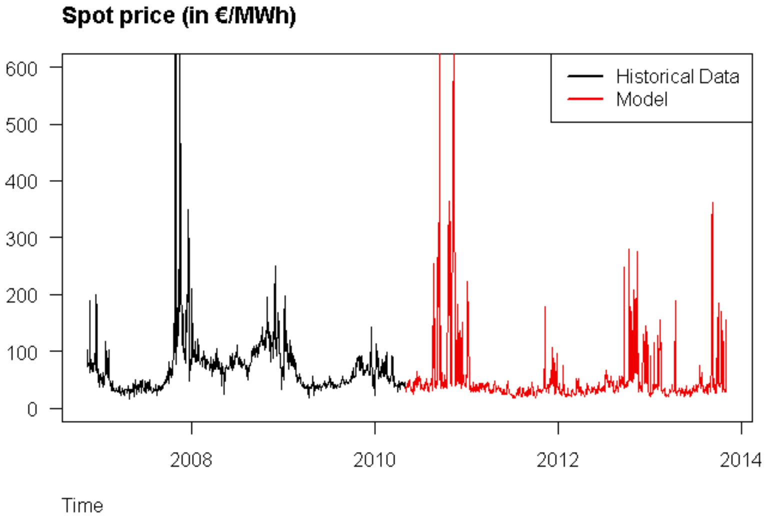
Spot simulations



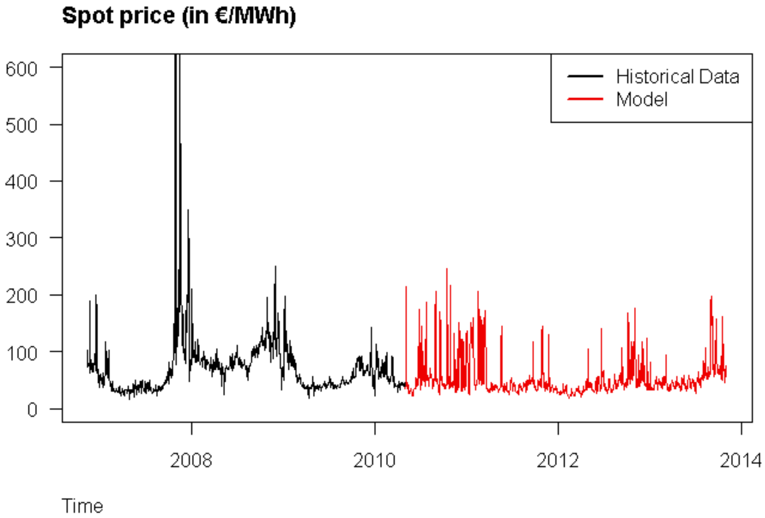
Spot simulations



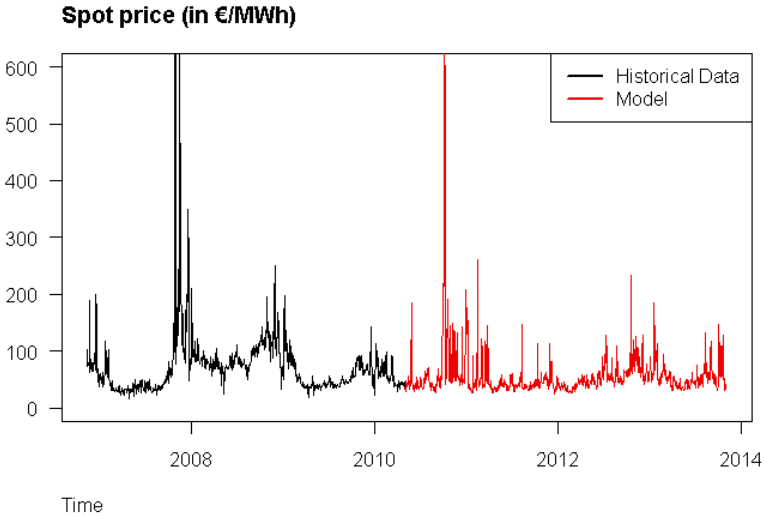
Spot simulations



Spot simulations



Spot simulations



Forward prices

Pricing

- incomplete market
- need for a hedging criterion
- Super-replication, utility indifference or mean-variance
- our choice: Local Risk Minimization

Local Risk Minimization (Pham (00), Schweizer (01))

- valuation: expected discounted payoff under $\widehat{\mathbb{Q}}$
- allows to decompose contingent claim between hedgeable part (fuels) and non-hedgeable part (demand, capacities)
- allows explicit formulas

Futures

Futures prices $F_t^e(T) = \mathbb{E}_t^{\widehat{\mathbb{Q}}}\left[e^{-r(T-t)} P_T\right]$

$$F_t^e(T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t)\, F_t^i(T)$$

with:

$$G_i^T(t, C_t, D_t) = \mathbb{E}_t\left[g\left(\sum_{k=1}^{n} C_T^k - D_T\right) \mathbf{1}_{\left\{\sum_{k=1}^{i-1} C_T^k \le D_T \le \sum_{k=1}^{i} C_T^k\right\}} \right]$$

Futures prices

$$F_t^e(T) = \sum_{i=1}^{n} G_i^T(t, C_t, D_t) \cdot h_i F_t^i(T)$$

- Electricity futures can be considered as a basket of fuel futures. Consistent with observation of cointegration relation between fuel prices and electricity prices.
- The weights of the basket depend on the anticipated fuel marginalities...
- The weights are not the expected fuel marginalities. They also depend on the anticipated tension of the equilibrium.
- The quoted electricity futures price does not depend on the current spot price. □

Fitting observed futures prices

- Does this model provide a good fit for the observed forward price at at given date? for its dynamic?
- Is it possible to have a perfect fit?

Study available in Féron & Daboussi (2015).

Observed futures prices with delivery period θ

$$F_t^e(T, T+\theta) = \frac{1}{\theta} \int_T^{T+\theta} F_t^e(u) du$$

and with only discrete values

$$F_t^e(T, T+\theta) = \frac{1}{\theta} \sum_{T'=T}^{T+\theta} F_t^e(T')$$

Fitting futures price data

Observed futures prices with delivery period θ

$$F_t^e(T, T+\theta) = \frac{1}{\theta} \sum_{i=1}^{n} \underbrace{\left(\sum_{T'=T}^{T+\theta} G_i^{T'}(t, C_t, D_t) \right)}_{\text{stochastic weights}} h_i F_t^i(T, T+\theta)$$

Modelisation of input factors

Demand

$$\begin{aligned} D_t &= f_D(t) + Z_D(t) \\ dZ_D(t) &= -\alpha_D(t) Z_D(t) dt + \beta_D(t) dW_t^D \\ f_D(t) &= d_1 + d_2 cos(2\pi \frac{t-d_3}{l_1}) + d_4 \end{aligned}$$

Capacities

$$\begin{aligned} C_t^i &= f_i(t) + Z_i(t) \\ dZ_i(t) &= -\alpha_i(t) Z_i(t) dt + \beta_i(t) dW_t^i \\ f_D(t) &= c_1^i + c_2^i cos(2\pi \frac{t-c_3^i}{l_1}) + c_4^i + f_i^{evo}(t) \end{aligned}$$

Reconstruction of forward prices

$$F_t^{elec}(T, T+\theta) = \sum_{i=1}^{n} \left(\frac{1}{\theta} \sum_{\tilde{T}=T}^{T+\theta} G_i^{\tilde{T}}(t, C_t, D_t) \right) h_i F_t^i(T, T+\theta)$$

Method

Capacities and demand process estimation
Computation of stochastic weights
Getting fuel prices
$\Rightarrow$ reconstruction of electricity forward prices

Illustration

- French baseload data
- Historical futures prices from EEX
- Period: 2009 - 2012
- Contrat type: 1-year ahead and 1-month ahead
- Three fuel types: nuclear, coal/gas and oil.

1 YAH baseload reconstruction

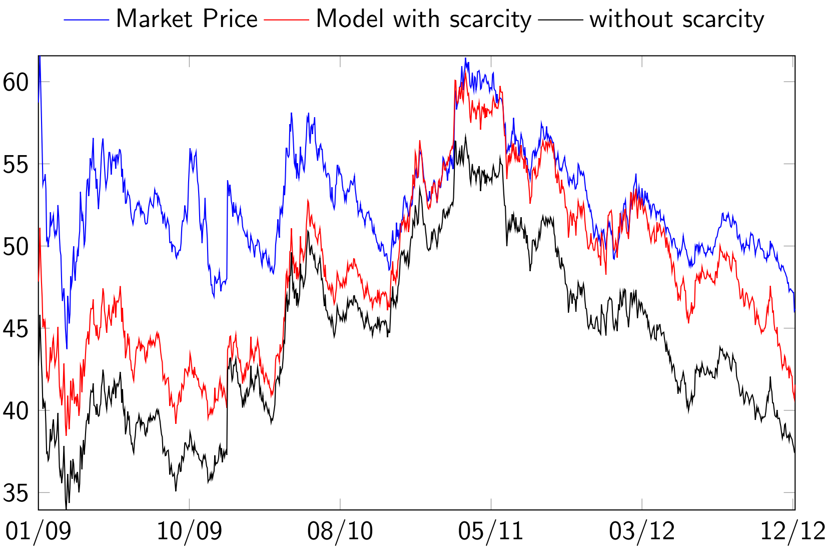
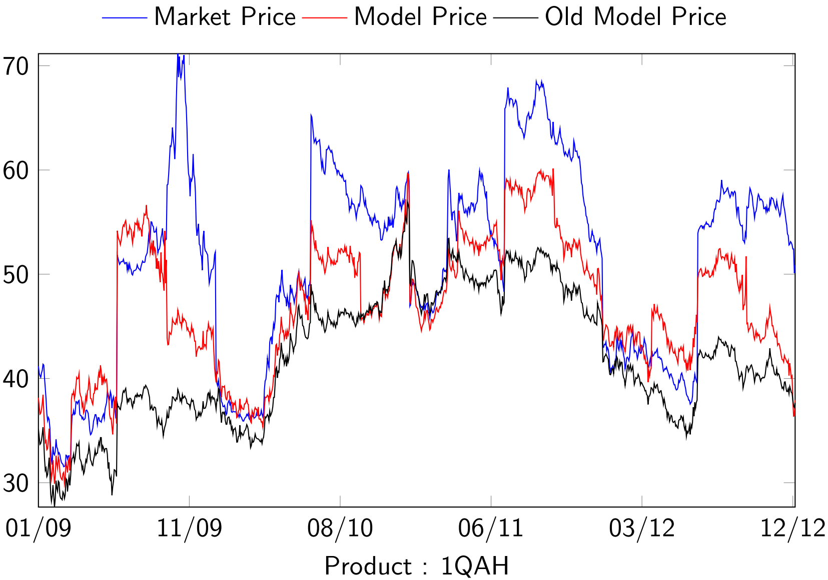
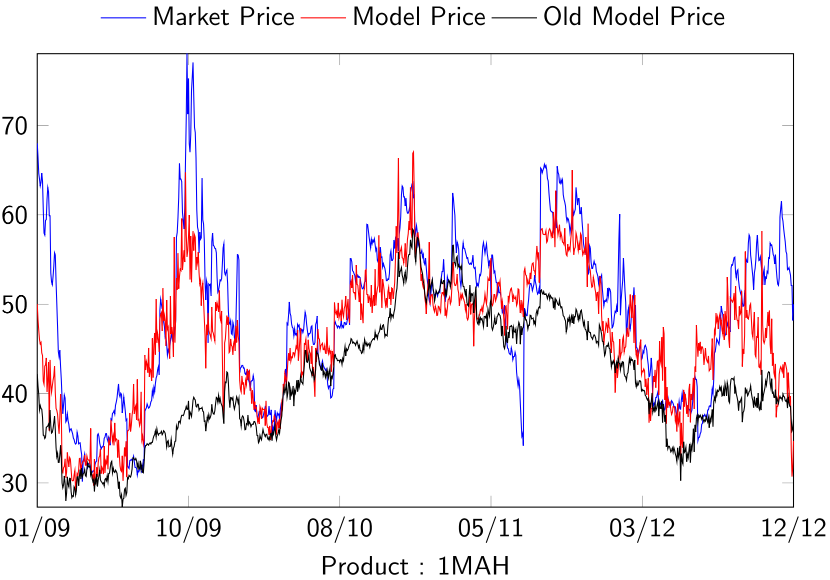


Figure: Reconstruction of the 1YAH during the years 2009-2012

1 QAH baseload reconstruction



1 MAH baseload reconstruction



Reconstruction results

Conclusions

- With historical data and public capacity availability $\Rightarrow$ getting the variations right.
- Without scarcity $\Rightarrow$ important biais.
- With scarcity $\Rightarrow$ less than 5% error since 2011.

Implied capacity

$$F_t^{elec}(T, T+\theta) = \sum_{i=1}^{n} \left(\frac{1}{\theta} \sum_{\tilde{T}=T}^{T+\theta} G_i^{\tilde{T}}(t, C_t, D_t) \right) h_i F_t^i(T, T+\theta)$$

Principle

Constant error.
Monotonic stochastic weights.
Modification of the fuel capacity:

$$C_t^n = C_t^n + \varepsilon_t^n$$

to fit the observed forwards with 1% relative error.

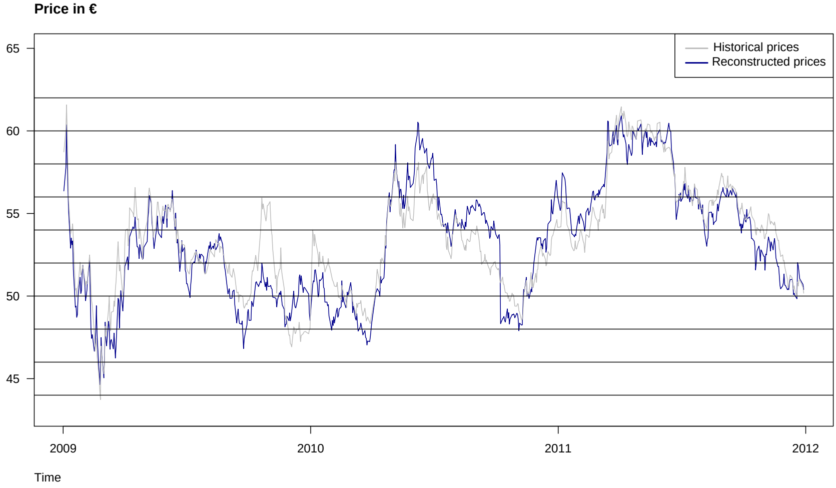
Implied capacity

period	value of ε_t^n(in GW)
2009/01/01 to 2010/10/08	-1.1
2010/10/09 to 2011/04/20	0
2011/04/21 to 2011/09/30	-0.3
2011/10/01 to 2011/12/31	-0.6

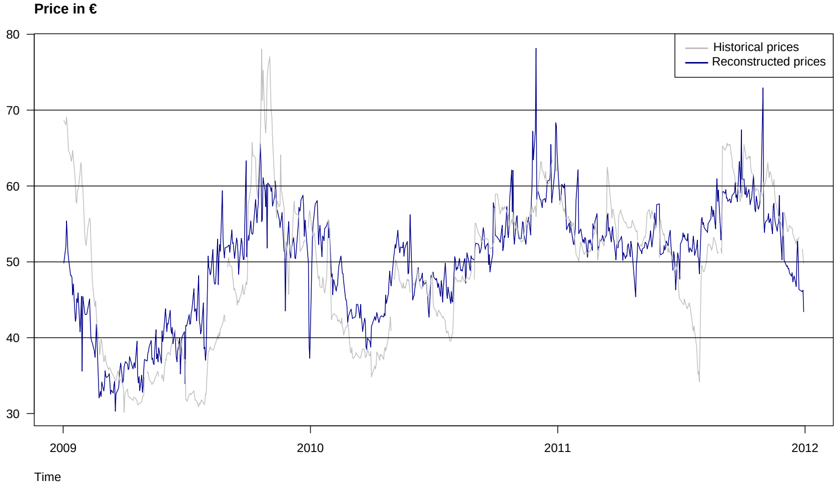
Result

Less than 1 GW of perturbation on capacity to recover forward prices.
Represent less than 1% of the installed capacity.

Implied capacity



1-MAH reconstructed price



Results on calibration

Conclusion

- Possible to fit the forward curve by a small variation of capacities or demand compared to the overall demand.
- The level of perturbation needed decreases with maturity.

Conclusion

(Very) large set of models developped in the last 10 years for electricity price

- realistic spot and forward behaviour.
- possible calibration on observed forward prices.
- Still many alternatives to explore to get numerically efficient models.

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HJM style forward curve model

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