Stochastic models for electricity markets
Lecture 04 - Optimal Intraday Electricity Market Trading

Frontiers in Stochastic Modelling for Finance
Winter School - Università degli Studi di Padova

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Agenda

Trading in the intraday market

A bit of context

Development of renewable energy in continental Europe

- windfarm : Germany 31 GW of 177 GW total installed capacity, Spain 22 GW of 105 GW
- solar power : Germany 32 GW, Italy 16 GW of 124 GW of installed capacity.
- source : Department Of Energy, Energy Information Agency

Effects on generation management and trading

- Increasing forecasting error on short time horizon
- root mean square error (RMSE) of the error forecast for the production of a wind farm in six hours can reach 20% of its installed capacity (Giebel et al. (2011))
- Producers / retailers endure inbalance costs. Imbalance : difference between generation (plus purchases) and consumption (plus sales).
- They are penalised for their imbalances because the TSO has to buy energy from someone else to insure the equilibrium of the system.
- Increasing need for producers to find ways to balance their short-term position.
- Development of intraday electricity market

Intraday market

Intraday market

- Operated by a market operator
- Market for next hours where firms
  - exchange power to balance their position
  - minimize the cost of their imbalances
- Exchanged volume in Epex intraday market in Germany has grown from 2 TWh (2008) to 25 TWh (2013).

EPEX Intraday market

- Continuous trading
- Opens at 15 :00 the day before.
- Possibility to buy/sell physical delivery contracts for the 24 periods 0 :00 – 1 :00, ..., 23 :00 – 24 :00.
- Closes 45 minutes before beginning of delivery.

Example of the quotation of a given hour of delivery

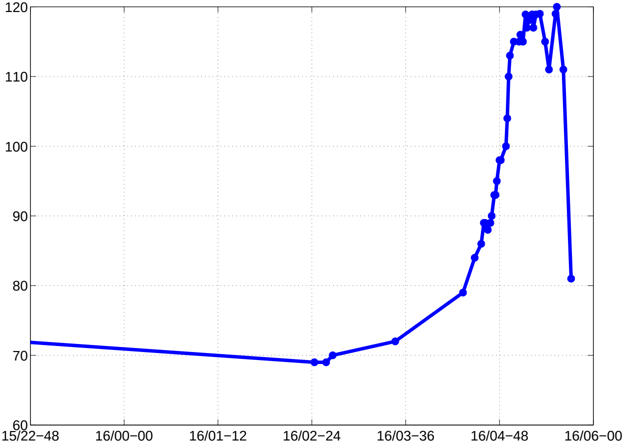

Figure: Epex intraday Germany market december 16th 2010, 7 a.m.

Liquidity and players

EpexSpot - for a given hour (2014)

- Total exchanged volumes can reach 3.000 MW
- Average buy/sell volume $\approx$ 350 MW
- Exchanges are cut in orders $\approx$ 20 MW

EpexSpot - members (2014)

- $\approx$ 100 actors on the French intraday market
- $\approx$ 240 actors on the all Epex intraday markets (France, Germany, Austria, Switzerland)
- Power utilities (EDF, E.On, Enel...), oil companies (BP), agregators (Voltalis), banks and financial institutions (Morgan Stanley, JP Morgan, CitiGroup, Barklays, Merril Lynch Commodities Ltd...)

Problem

Settings

- Demand and renewable production forecasts are frequently updated (every 2 hours for demand, every 4 hours for wind production)
- Shocks in forecast happens.
- Short-term generation management involves many complex constraints of power plants and many different technology costs.
- Mainly, it takes time to mobilize generation.

Questions

- Knowing that the objectif of a power producer is to minimize the total cost of production and trading, what can be the optimal trading strategy ?
- Knowing that shocks may happen, should traders take anticipated precautionnary positions ?
- Dilemna between waiting for a possible better price and closing immediatly an imbalanced position.

Previous works

References

- Henriot (2014) : discrete time model with wind production error forecast as only source of randomness
- Garnier and Madlener (2014) : continuous time model analysing the trade-off between entering into a deal right now and waiting for a better quote. Wind follows a arithmetic Brownian motion and intraday prices a geometric Brownian motion.

This talk

Analysis of the problem with a stylised model simplifying the generation side and yet preserving the main feature of the dilemna, the information structure

- The producer minimises the expected total cost of production using
  - Thermal plants (oil, gas, nuclear) : can be dispatched with anticipation (delay)
- + trading costs
- + Penalization of the imbalance with residual demand.

- Model inspired by Almgren-Chriss (2000) optimal execution model with market impact
- Here, random target, as we only have a forecast of final residual demand
- Residual demand : total final demand minus renewable energy production.

Optimal trading model

Trading

- X_t : net sale/buy position (inventory) on the intraday market with trading rate control $q_t = \dot{X}_t$

$$X_t = X_0 + \int_0^t q_s ds,$$

- Transactions occur at price

$$P_t(\mathbf{q}) = \underbrace{\widehat{P}_t + \int_0^t \nu q_s ds}_{Y_t := \text{observable quoted price}} + \gamma q_t,$$

- $\widehat{P}_t$: unaffected price on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbb{P})$
- ν permanent impact factor, γ : instantaneous impact factor.

Demand, generation and penalization

- D_T residual demand at time T. Continuously updated forecast $(D_t)_{0 \leq t \leq T}$.
- Thermal production ξ is chosen at time $T - h$. Length h is the delay in production.
- cost function $c(\xi) = \frac{\beta}{2}\xi^2$, $\beta > 0$.

Objective function of the agent

Minimize over trading rates $\mathbf{q} \in \mathcal{A}$: $\mathbb{F}$-adapted and generation $\xi \in L^0_+(\mathcal{F}_{T-h})$

$$\mathbf{E}\Big(\underbrace{\int_0^T q_s P_s(\mathbf{q}) ds}_{\text{trading cost}} + \underbrace{\frac{\beta}{2}\xi^2}_{\text{generation cost}} + \underbrace{\frac{\eta}{2}(D_T - X_T - \xi)^2}_{\text{imbalance penalization}} \Big)$$

where

- $C(D_T - X_T, \xi) := \frac{\beta}{2}\xi^2 + \frac{\eta}{2}(D_T - X_T - \xi)^2$ is the cost of holding X_T while the final demand is D_T and the production ξ has been chosen.

Case with no jumps and no delay

- We assume that thermal production can be decided instantaneously. The production decision has to be made at final time T.
- The unaffected price process is

$$\widehat{P}_t = \widehat{P}_0 + \sigma_0 W_t, \text{ thus } dY_t = \nu q_t dt + \sigma_0 dW_t,$$

- the residual demand forecast has dynamics

$$dD_t = \mu dt + \sigma_d dB_t,$$

with $\mu \in \mathbf{R}$, $\sigma_0 > 0, \sigma_d > 0$ and $d < W, B >_t = \rho dt$, $\rho \in [-1, 1]$.

Dynamical formulation

Value function :

$$v(t,x,y,d) = \inf_{\substack{\mathbf{q}\in\mathcal{A}_t \\ \xi\in L^0_+(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi)$$

where

$$J(t,x,y,d;\mathbf{q},\xi) = \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s)ds + C(D_T^{t,d} - X_T^{t,x},\xi)\Big),$$

$\mathcal{A}_t$ being the set of adapted processes $\mathbf{q}$ such that $\mathbf{E}\big(\int_t^T q_s^2 ds\big) < +\infty$.

Note : The production quantity ξ is chosen at time T, after the choice of the whole trajectory $\mathbf{q}$.

Solving the problem

$$v(t,x,y,d) = \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s) ds + \inf_{\xi \in L^0_+(\mathcal{F}_T)} C(D_T^{t,d} - X_T^{t,x}, \xi) \Big)$$

Optimal generation :

$$\begin{aligned} \xi_T^* &= \frac{\eta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x}) \mathbf{1}_{D_T^{t,d} - X_T^{t,x} \geq 0} \\ &=: \hat{\xi}^+(D_T^{t,d} - X_T^{t,x}) \end{aligned}$$

Thus :

$$\begin{aligned} v(t,x,y,d) = \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s) ds + \frac{\beta}{2}\hat{\xi}^+(D_T^{t,d} - X_T^{t,x})^2 \\ + \frac{\eta}{2}(D_T^{t,d} - X_T^{t,x} - \hat{\xi}^+(D_T^{t,d} - X_T^{t,x}))^2 \Big) \end{aligned}$$

We do not expect to get an explicit formula, due to the indicator function.

Auxiliary relaxed problem

Relax the positivity constraint on the production ξ :

$$\tilde{v}(t,x,y,d) = \inf_{\substack{\mathbf{q}\in\mathcal{A}_t \\ \xi\in L^0(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

Optimal generation level :

$$\hat{\xi}(D_T^{t,d} - X_T^{t,x}) = \frac{\eta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x})$$

New value function expression :

$$\tilde{v}(t,x,y,d) = \inf_{\mathbf{q}\in\mathcal{A}_t} \mathbf{E}\Big(\int_t^T q_s(Y_s^{t,y} + \gamma q_s)ds + \frac{1}{2}\frac{\eta\beta}{\eta+\beta}(D_T^{t,d} - X_T^{t,x})^2 \Big).$$

Linear-quadratic problem. Value function is quadratic.
$\Rightarrow$ Straightforward but tedious computations.

Optimal strategy in the auxiliary problem

Optimal trading rate

$$\hat{q}_t = \frac{r(\eta,\beta)(D_t + \mu(T-t) - \widehat{X}_t) - \widehat{Y}_t}{(r(\eta,\beta)+\nu)(T-t) + 2\gamma}$$

where $r(\eta,\beta) := \frac{\eta\beta}{\eta+\beta}$ and $\widehat{X}_t$, $\widehat{Y}_t$ designate the inventory and price trajectory with $\hat{q}_t$ as a control.

Interpretation

- Define the forecast final production seen from time $t \leq s \leq T$:

$$\hat{\xi}_s = \frac{\eta}{\eta+\beta}\left(D_s + \mu(T-s) - \hat{q}_s(T-s) - \widehat{X}_s\right)$$

- The optimal trading rate satisfies :

$$\widehat{Y}_s + \nu \hat{q}_s(T-s) + 2\gamma \hat{q}_s = c'\left(\hat{\xi}_s\right).$$

- The optimal trading strategy consists in making the forecast marginal costs equal to the forecast price.
- Close to the operational strategy.

Interpretation

Consequence

- Suppose that at time 0, the intraday price is equal to the day-ahead spot price and the producer is balanced.
- Thus $\widehat{Y}_0 = c'\left(D_0 + \mu T - \widehat{X}_0\right)$
- Thus the initial trading rate is null $\Rightarrow$ No anticipated precautionnary position is needed.

A nice property of the optimal trading rate in the auxiliary problem

Property

The optimal trading rate is a martingale.

Proof

Itô's lemma applied to $\hat{q}_s = \hat{q}\big(T - s, D_s - \widehat{X}_s, \widehat{Y}_s\big)$ gives :

$$d\hat{q}_s = \big(D_2\hat{q}\big)\sigma_d dB_s + \big(D_3\hat{q}\big)\sigma_0 dW_s.$$

Consequence

- The expected inventory is thus a linear function of time : $\mathbf{E}\big(\widehat{X}_s\big) = X_0 + \hat{q}_0 s$.
- Constant trading rate in Almgren and Chriss (2000).
- Previous "forecasts keeping the same control" are thus pertinent.

Quality of the approximation of the original problem

- The production ξ has to be positive.
- Suggested strategy :
  1. First follow strategy $(\hat{q}_s)_{t \leq s \leq T}$ of the auxiliary problem.
  2. Then choose production level $\hat{\xi}^+$ $(= \hat{\xi}\mathbf{1}_{\hat{\xi} \geq 0})$.
- Denote

$$\mathcal{E}_1(t,x,y,d) = J\big(t,x,y,d;\hat{\mathbf{q}},\hat{\xi}^+\big) - v(t,x,y,d),$$
$$\mathcal{E}_2(t,x,y,d) = v(t,x,y,d) - \tilde{v}(t,x,y,d).$$

- Bounds of $\mathcal{E}_i$ show very low error.

Incorporation of jumps on demand

- Add a compound Poisson process $(N_t^+, N_t^-)_t$ with intensity λ, counting positive and negative jumps, to the residual demand forecast.
- At each jump time t,
  - with probability p^+, $(N_t^+)_t$ has a jump.
  - with probability $p^- = 1 - p^+$, $(N_t^-)_t$ has a jump.
- New dynamics of demand :

$$dD_t = \mu dt + \sigma_d dB_t + \delta^+ dN_t^+ + \delta^- dN_t^-$$

  with $\delta^+ > 0$ and $\delta^- < 0$
- Impact on intraday price :

$$dY_t = \nu q_t dt + \sigma_0 dW_t + \pi^+ dN_t^+ + \pi^- dN_t^-$$

  with $\pi^+ > 0$ and $\pi^- < 0$.

Let $\delta := p^+\delta^+ + p^-\delta^-$ and $\pi := p^+\pi^+ + p^-\pi^-$.

Value function and auxiliary problem

- Value function :

$$v^{(\lambda)}(t,x,y,d) = \inf_{\substack{\mathbf{q}^{(\lambda)} \in \mathcal{A}_t \\ \xi \in L^0_+(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

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- Auxiliary problem, relaxing the constraint of positivity of the production :

$$\tilde{v}^{(\lambda)}(t,x,y,d) = \inf_{\substack{\mathbf{q}^{(\lambda)} \in \mathcal{A}_t \\ \xi \in L^0(\mathcal{F}_T)}} J(t,x,y,d;\mathbf{q},\xi).$$

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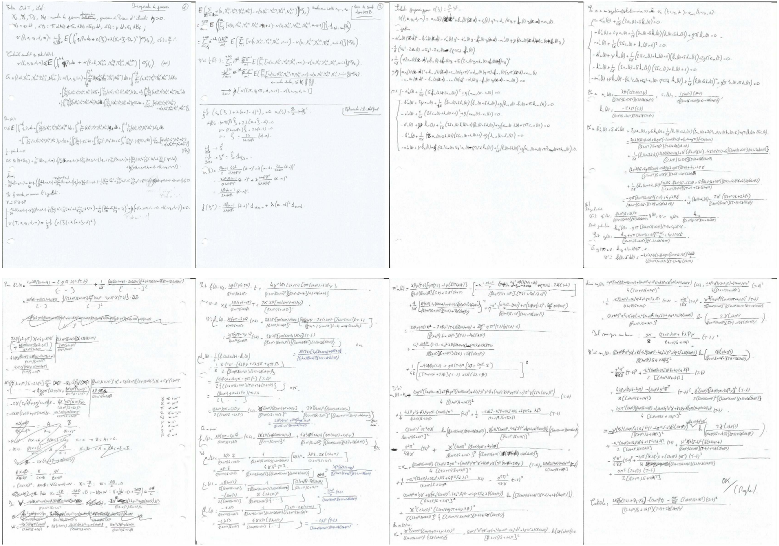
- Tedious ?

Pierre Gruet's handmade computation



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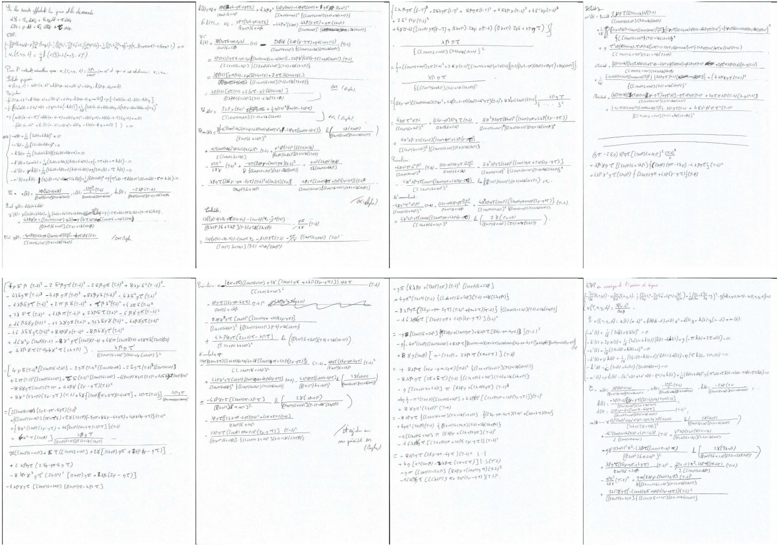


Optimal control in the auxiliary problem

Optimal trading rate

$$\hat{q}_s^{(\lambda)} = \hat{q}_s^{(0)} + \lambda \frac{r(\eta,\beta)\delta(T-s) + \frac{\pi}{4\gamma}(r(\eta,\beta)+\nu)(T-s)^2}{(r(\eta,\beta)+\nu)(T-s)+2\gamma}$$

Property

$$\overline{q}_s^{(\lambda)} := \left(\hat{q}_s^{(\lambda)} + \frac{\lambda\pi}{2\gamma}(s-t)\right)_{t\leq s\leq T}$$

is a martingale :

- if $\pi > 0$, then $(q_s^{(\lambda)})_s$ is a supermartingale.
- if $\pi < 0$, then $(q_s^{(\lambda)})_s$ is a submartingale.

Interpretation

- Expected demand at final time T seen at time t is

$$\overline{D}_t := D_t + \mu(T-t) + \lambda\delta(T-t),$$

- Expectation of $\widehat{X}_T$ seen at time t is

$$\overline{X}_t := X_t + \hat{q}_t^{(\lambda)}(T-t) - \frac{\lambda\pi}{4\gamma}(T-t)^2$$

- Expectation of the final "price plus marginal cost of getting $\hat{q}_T^{(\lambda)}$" is

$$\overline{Y}_t := Y_t + \lambda\pi(T-t) + \nu\Big(\hat{q}_t^{(\lambda)}(T-t) - \frac{\lambda\pi}{4\gamma}(T-t)^2\Big) + 2\gamma\Big(\hat{q}_t^{(\lambda)} - \frac{\lambda\pi}{2\gamma}(T-t)\Big)$$

- Thus,

$$\overline{Y}_t = c'\big(\overline{D}_t - \overline{X}_t\big)$$

- Optimal trading strategy is still to make the forecast marginal cost equal to the forecast price.

Interpretation

Consequence

- Suppose that at time 0, the intraday price is equal to the day-ahead spot price and the producer is balanced.
- Here, balanced means :

$$Y_0 + \lambda\pi T = c'\left(\widehat{D}_0 - \widehat{X}_0\right)$$

- Thus, at time 0 shoud be taken the precautionnary position :

$$\overline{q}_0^{(\lambda)} = \frac{\lambda\pi T + \frac{\lambda\pi}{4\gamma}(r(\eta,\beta)+\nu)T^2}{(r(\eta,\beta)+\nu)T + 2\gamma}$$

Delay h in generation

- Consider the auxilary relaxed problem without jumps.
- It is always better to wait until $T - h$ to take the generation decision.
- Between $T - h$ and T, we face an optimal trading problem with no generation and with inventory $X_{T-h} + \xi$.
- Knowing the optimal trading rate as function of ξ, it is possible to compute the optimal generation level.
- Knowing the optimal generation level to be applied at time $T - h$ and the optimal trading rate between $T - h$ and T, we are brougth back to an instance of our problem between 0 and $T - h$ with a (slightly) more complex terminal cost function.

Consequence

After tedious computations, we found that :

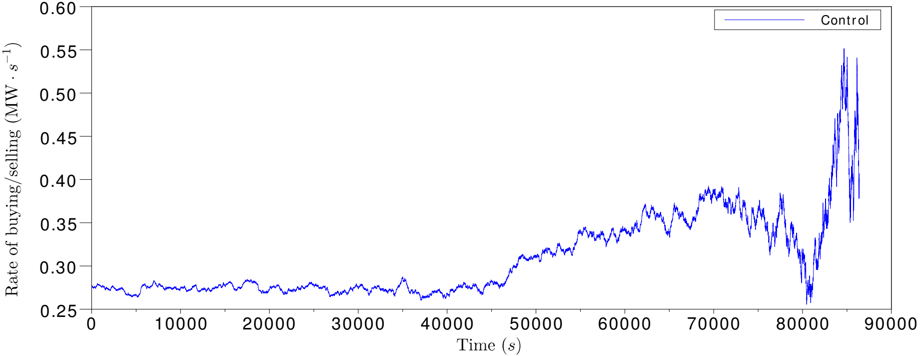
- Between 0 and $T - h$, the control with and without delay is the same.
- Only the value functions differ.

Numerical applications and simulations

Parameter values for nice simulations

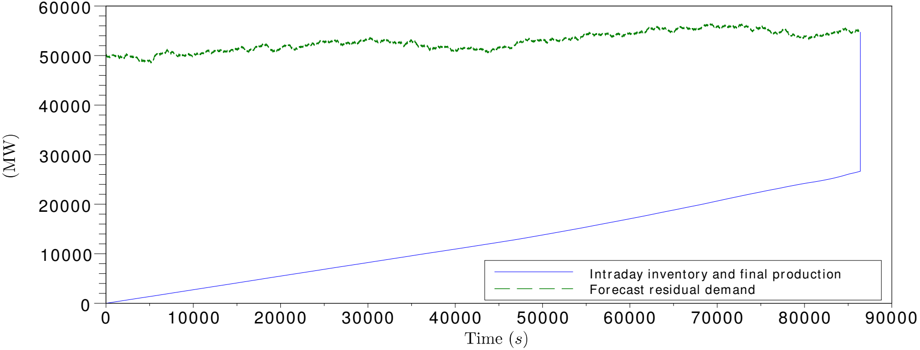
Time period	$T = 24$	h
Initial inventory level	$X_0 = 0$	MWh
Initial demand forecast	$D_0 = 50,000$	MWh
Initial intraday price	$Y_0 = 50$	€·(MWh)$^{-1}$
Demand forecast trend	$\mu = 0$	MWh·s^{-1}
Intraday price volatility	$\sigma_0 = 1/60$	€·(MWh)$^{-1} \cdot s^{-1/2}$
Demand forecast volatility	$\sigma_d = 1000/60$	MWh·$s^{-1/2}$
Correlation	$\rho = 0.8$	
Cost function parameter	$\beta = 0.002$	€·(MWh)$^{-2}$
Inbalance penality	$\eta = 200$	€·(MWh)$^{-2}$
Permanent impact	$\nu = 4.00 \cdot 10^{-5}$	€·(MWh)$^{-2}$
Instantaneous impact	$\gamma = 2.22$	€·s·(MWh)$^{-2}$
Probability of positive jumps	$p^+ = 1$	
Intensity of jump	$\lambda = 1.5/(3600 \cdot 24)$	s^{-1}
Size of the price jump	$\pi^+ = 10$	€·(MWh)$^{-1}$
Size of the demand forecast jump	$\delta^+ = 1500$	MWh

No jumps, no delay — Optimal trading rate



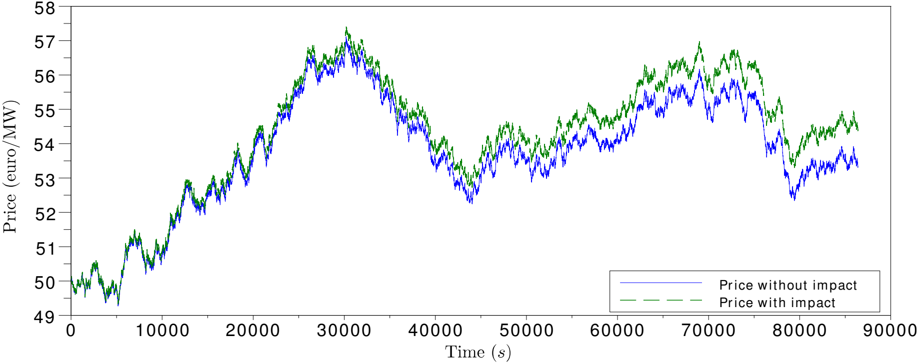
Final oscillation to adjust production and demand.

No jumps, no delay — Inventory



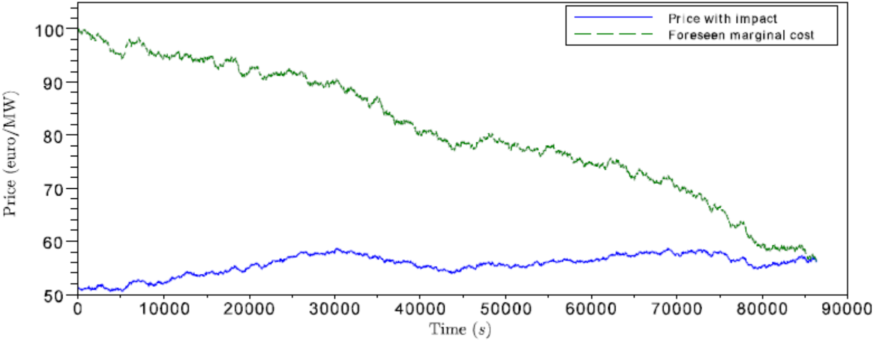
Linear growth of the inventory with final generation to adjust to demand.

No jumps, no delay — Prices



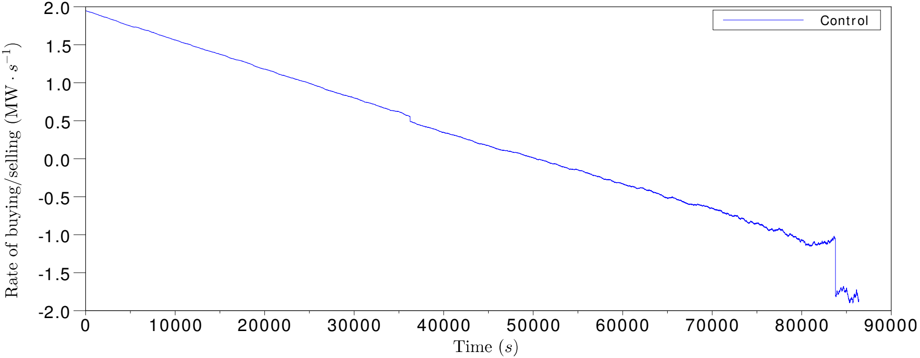
Small yet persistent impact on prices.

No jumps, no delay — Marginal cost and price



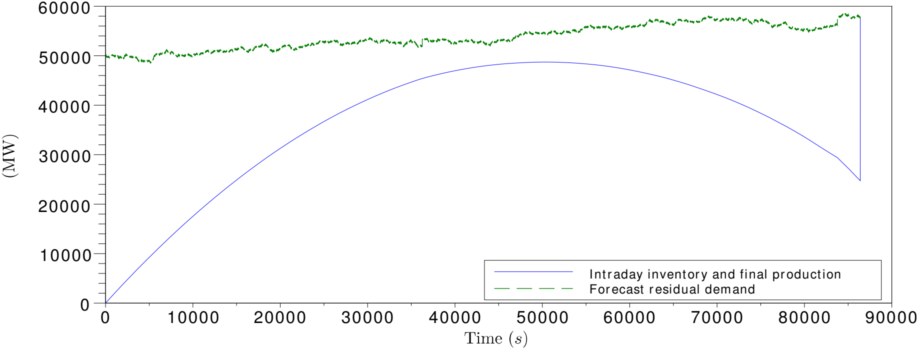
The marginal cost decreases until it reaches the increasing price.

Jumps $\pi > 0$, no delay — Optimal trading rate



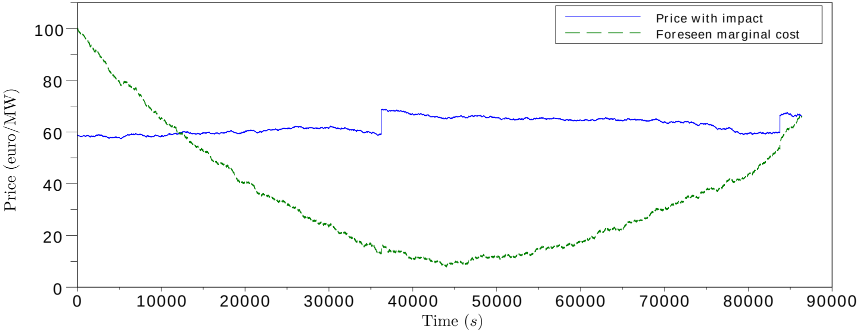
Decreasing trading rate. Starts with positive value (buy), at some point in time, becomes negative (sell).

Jumps $\pi > 0$, no delay — Inventory



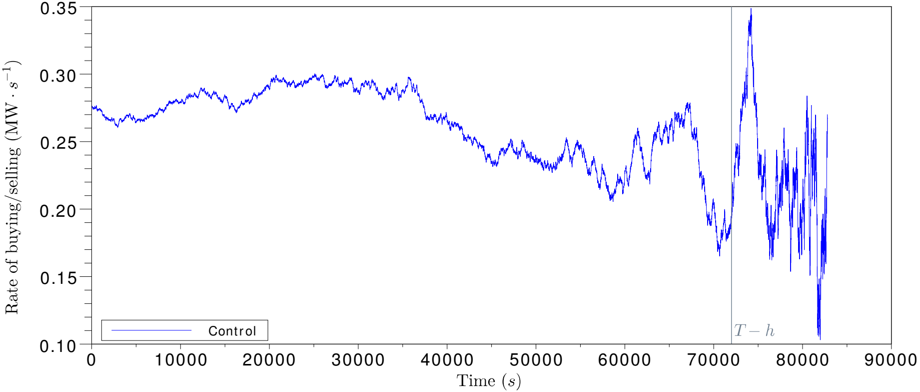
First, increasing inventory, then decreasing inventory, and final generation to adjust to demand.

Jumps, no delay — Marginal cost and price



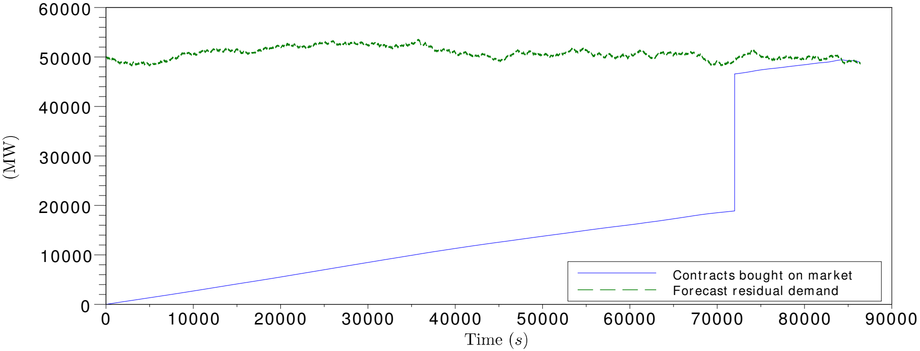
First decreasing forecast marginal cost (positive large inventory means less to be produced at maturity) then, increasing forecast marginal cost.

Delay $h = 4$ hours — Optimal trading rate



Trading rate after time $T - h$ is the only tool to fit the demand. Thus becomes more volatile.

Delay $h = 4$ hours — Inventory



At $T - h$, the rate at which the inventory increases suddenly changes.

Application — Precautionnary position

Question

What is the size of the anticipated precautionary position that should be taken in the case of jumps in the residual demand forecast ?

Answer

$$\overline{q}_0^{(\lambda)} = \frac{\lambda\pi T + \frac{\lambda\pi}{4\gamma}(r(\eta,\beta)+\nu)T^2}{(r(\eta,\beta)+\nu)T + 2\gamma}$$

with reasonnable parameter values :
$\lambda = 1.5\ 10^{-5}$ (1 jump per month), $\gamma = 2$ €$\cdot h \cdot$(MWh)$^{-2}$, $\nu = 0.1$ €$\cdot$(MWh)$^{-2}$,
$\pi = 500$ €/MWh, $\eta = 200$ €/MWh, $\beta = 2.0\ 10^{-3}$ €/MW^2, $T = 24$ hours.

$$\overline{q}_0^{(\lambda)} \approx 3.6 \text{ MWh}/h$$

For one jump per week, the precautionnary position is $\approx$ 14 MWh/h.

Conclusion

Conclusion

- Analysis of electricity intraday trading with a small and tractable stochastic control model.
- Extension of Almgren and Chriss (2000) optimal execution model with linear impact to stochastic target.
- Confort the operational strategy.
- Jumps in the residual demand process lead to non-zero yet small precautionnary initial position.

Perspective

- Intraday prices models.
- Statistical arbitrage.
- Risk management.

References

- Talk based on paper "An optimal trading problem in intraday electricity market" available on arXiv and to appear in *Mathematical and Financial Economics*.
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