Stochastic models for electricity markets
Lecture 05 - Electricity Retail Market Competition

Frontiers in Stochastic Modelling for Finance
Winter School - Università degli Studi di Padova

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Agenda

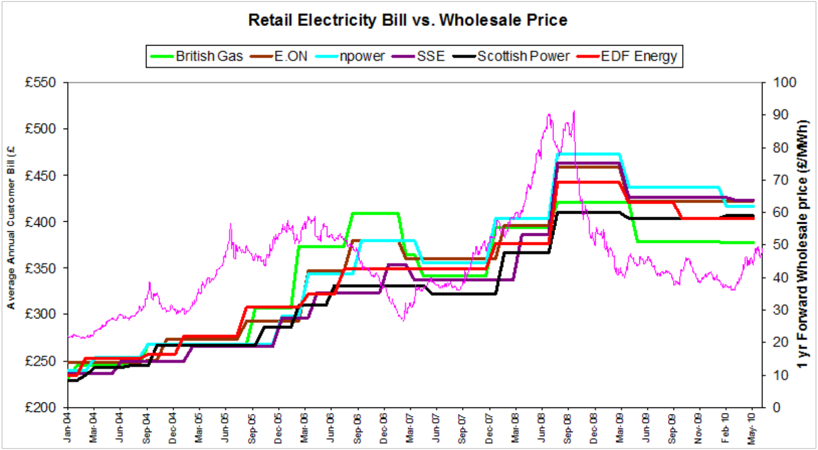
Competition on Electricity Retail Market

A bit of context

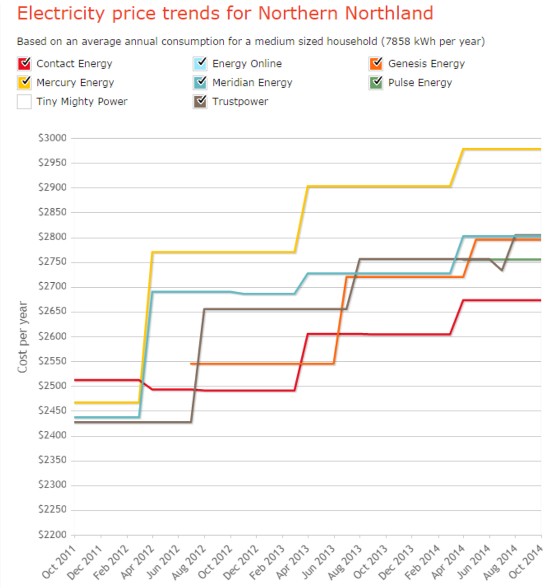
French retail electricity market

- The end of regulated tariff green and yellow on January 1st, 2016 in France.
- Competition on mass market is expected to increase.
- The churn is expected to increase.
- The pricing mechanisms are different than for the industrial customers.
- Mass market customers are offered fixed prices.

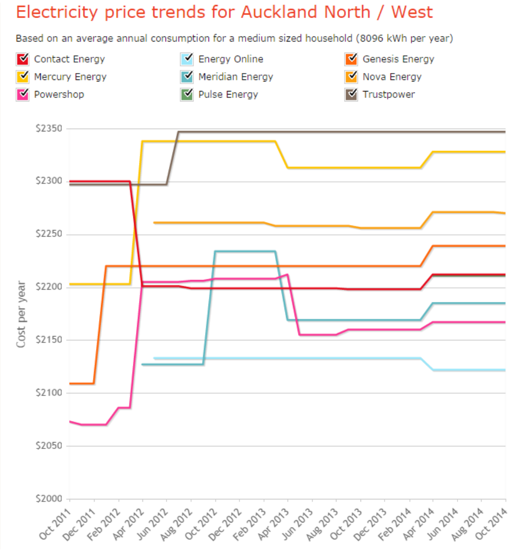
An idea of the future : competition in the UK electricity market



Retail competition in New-Zealand electricity market



Retail competition in New-Zealand



Electricity retail competition in the literature

- Mainly studied for the regulation point of view
- Joskow & Tirole (2006) : studies the effect of the lack of smart meters in households and load profiling.
- Finon & Boroumand (2011) : studies the relation between market structure (integration) and retail price policy.
- Suspicion that pricing behaviour in the UK results in tacite collusion.

Extract from Ofgem 2014 global investigation on retail market competition

The headline findings of the assessment were

(...) Possible tacit co-ordination : The assessment has not found evidence of explicit collusion between suppliers. However, there is evidence of possible tacit coordination reflected in the timing and size of price announcements and new evidence that prices rise faster when costs rise than they reduce when costs fall. Although tacit coordination is not a breach of competition law, it reduces competition and worsens outcomes for consumers. Published on Ofgem website on June 26th, 2014.

Problem

Question

- When is it optimal for a retailer to change her price (tariff) for the mass market ? and by how much ?

Remarks

- Profit may decrease because of an increase of the sourcing cost or because of a decrease of market share.
- Not possible to revise continuously the price to retail customers (not real-time pricing).
- Situation fits in impulse control games.
- Here, we want non-zero-sum impulse control game.

Previous works

- Friedman (1973) : zero-sum game optimal stopping problem
- Bensoussan & Friedman (1977) : non-zero-sum game optimal stopping problem
- Zhang (2011) : one player is plays continuously while the other plays with impulses.
- Cosso (2013) : zero-sum game impulse control game

Non-zero sum stochastic impulse games

Problem settings

- Let S be an open subset of $\mathbb{R}^d$.
- Underlying process when none of the players intervenes

$$dY_s^{t,y} = b(Y_s^{t,y})ds + \sigma(Y_s^{t,y})dW_s, \qquad s \in [t, \infty[, \tag{1}$$

  with initial condition $Y_t^{t,y} = y$.
- When player $i \in \{1, 2\}$ decides to intervene with impulse $\delta_i \in Z_i$, the process is shifted from state y to state $\Gamma^i(y, \delta_i)$.
- The action of the players is modelled by controls : an *impulse control for player* $i \in \{1, 2\}$ is a sequence

$$u_i = \{(\tau_{i,k}, \delta_{i,k})\}_{1 \le k \le M_i}, \tag{2}$$

  where M_i, $\{\tau_{i,k}\}_k$ are non-decreasing stopping times (the intervention times) and $\{\delta_{i,k}\}_k$ are Z_i-valued $\mathcal{F}_{\tau_{i,k}}$-measurable random variables (the corresponding impulses)

Objective functions

Each player aims at maximizing his payoff J^i, defined as follows :

$$J^i(x; u_1, u_2) := \mathbb{E}_x\Bigg[\int_0^{\tau_S} e^{-\rho_i s} f_i(X_s)ds + \sum_{1\le k\le M_i\,:\,\tau_{i,k}<\tau_S} e^{-\rho_i \tau_{i,k}} \phi_i\Big(X_{(\tau_{i,k})^-}, \delta_{i,k}\Big)$$

$$+ \sum_{1\le k\le M_j\,:\,\tau_{j,k}<\tau_S} e^{-\rho_i \tau_{j,k}} \psi_i\Big(X_{(\tau_{j,k})^-}, \delta_{j,k}\Big) + e^{-\rho_i \tau_S} h_i\big(X_{(\tau_S)^-}\big)\mathbb{1}_{\{\tau_S<+\infty\}}\Bigg], \quad (3)$$

for each $i, j \in \{1, 2\}$, $j \neq i$, $x \in S$ and u_1, u_2 impulse controls

Strategies

A strategy for player $i \in \{1,2\}$ is a couple $\varphi_i = (A_i, \xi_i)$, where A_i is a fixed subset of $\mathbb{R}^d$ and ξ_i is a continuous function from $\mathbb{R}^d$ to Z_i.

Once the strategies $\varphi_i = (A_i, \xi_i)$ have been chosen, a couple of impulse controls, which we denote $u_i(x; \varphi_1, \varphi_2)$, is uniquely defined by the following procedure :

- player i intervenes if and only if the process exits from A_i, in which case the impulse is given by $\xi_i(y)$, where y is the state ;
- a contemporary intervention is not possible : if both the players want to act, player 1 has the priority and player 2 does not intervene.

Remark

It is possible to precisely formalize the processes defined by this strategy.

Nash equilibrium

Nash equilibrium

Let $x \in S$. We say that a couple $(\varphi_1^*, \varphi_2^*) \in \mathcal{A}_x$, the set of admissible strategies with intial state x, is a Nash equilibrium if

$$\begin{aligned} J^1(x; \varphi_1^*, \varphi_2^*) &\geq J^1(x; \varphi_1, \varphi_2^*), \quad \forall \varphi_1 \,\text{s.t.}\, (\varphi_1, \varphi_2^*) \in \mathcal{A}_x, \\ J^2(x; \varphi_1^*, \varphi_2^*) &\geq J^2(x; \varphi_1^*, \varphi_2), \quad \forall \varphi_2 \,\text{s.t.}\, (\varphi_1^*, \varphi_2) \in \mathcal{A}_x. \end{aligned}$$

Quasi-variational inequality - Intervention operators

For two functions V_1, V_2, define :

$$\delta_i(x) = \arg\max_{\delta \in Z_i} \left\{ V_i(\Gamma^i(x,\delta)) + \phi_i(x,\delta) \right\}, \qquad x \in \bar{S},$$

and the four intervention operators

$$\begin{aligned} \mathcal{M}_i V_i(x) &= V_i\big(\Gamma^i(x,\delta_i(x))\big) + \phi_i\big(x,\delta_i(x)\big), \qquad x \in \bar{S}, \\ \mathcal{H}_i V_i(x) &= V_i\big(\Gamma^j(x,\delta_j(x))\big) + \psi_i\big(x,\delta_j(x)\big), \qquad x \in \bar{S}, \end{aligned}$$

for $i, j \in \{1,2\}$ and $i \neq j$.
Define

$$\mathcal{A}V_i = b \cdot \nabla V_i + \frac{1}{2}\mathrm{tr}\left(\sigma\sigma^t D^2 V_i\right), \qquad x \in \bar{S}.$$

Quasi-variational inequality - Conditions

Consider the conditions for V_1, V_2 :

$$\begin{aligned}
&V_i \text{ bounded if } \rho_i > 0,\ V_i(\infty) = 0 \text{ if } \rho_i = 0, && \text{if } S \text{ is unbounded}, && \text{(4a)}\\
&V_i = h_i, && \text{in } \partial S, && \text{(4b)}\\
&\mathcal{M}_j V_j - V_j \leq 0, && \text{in } S, && \text{(4c)}\\
&\mathcal{H}_i V_i - V_i = 0, && \text{in } \{\mathcal{M}_j V_j - V_j = 0\}, && \text{(4d)}\\
&\max\big\{ -\rho_i V_i + \mathcal{A} V_i + f_i, \mathcal{M}_i V_i - V_i \big\} = 0, && \text{in } \{\mathcal{M}_j V_j - V_j < 0\}. && \text{(4e)}
\end{aligned}$$

Remarks

- If player j intervenes (i.e. $\mathcal{M}_j V_j - V_j = 0$), at the equilibrium, we expect that player i does not lose anything (4d)
- On the contrary, if player j does not intervene (i.e. $\mathcal{M}_j V_j - V_j < 0$), then V_i behaves according to the PDE of a standard one-player impulse problem, $(\max\big\{ -\rho_i V_i + \mathcal{A} V_i + f_i, \mathcal{M}_i V_i - V_i \big\} = 0)$

Quasi-variational inequality - Verification theorem

Verification theorem

Let V_1, V_2 be functions satisfying the following regularity conditions for $i \in \{1, 2\}$:

- ∂D_i is a Lipschitz surface, where $D_i = \{\mathcal{M}_i V_i - V_i < 0\}$;
- $V_i \in C^2(S \setminus \partial D_i) \cap C^1(S) \cap C(\bar{S})$;
- V_i has locally bounded derivatives near ∂D_i.

Moreover, assume that V_1, V_2 satisfy (4). Let $x \in S$ and let

$$A_i^* = \{\mathcal{M}_i V_i - V_i < 0\}, \qquad \xi_i^* = \delta_i, \qquad \varphi_i^* = (A_i^*, \xi_i^*).$$

Assume that $(\varphi_1^*, \varphi_2^*) \in \mathcal{A}_x$. Then,

$$(\varphi_1^*, \varphi_2^*) \text{ is a Nash equilibrium and } J^i(x; \varphi_1^*, \varphi_2^*) = V_i(x) \text{ for } i \in \{1, 2\}.$$

A model of competition on electricity retail market

Model - hypothesis

- Each retailer wants to maximise her profit.
- They have one control variable : their retail market prices p_1 and p_2.
- They may change prices whenever they want.
- They may have different commercial costs
  - structure costs : quadratic w.r.t. market share α_i
  - switching costs : affine (fixed part + proportional w.r.t. market share)
- No possibility to change the cost structure.
- They have the same sourcing cost x_t (wholesale market price) which is random.
- Clients may change their retailer whenever they want.
- The dynamics of the market shares α_1 and α_2 is driven by the differences between the prices of the two retailers p_1 and p_2.
- The reaction of the market shares to a change of retailer prices is supposed to be instantaneous.

Model - Retailer objective

Retailer 1 has to determine the sequence of stopping time τ_n^1 and the changes in her price δ_n^1 such that :

$$\sup_{\tau_n^1,\delta_n^1} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(\left(p_t^1 - x_t\right)\alpha_t^1 - \frac{b_1}{2}(\alpha_t^1)^2\right)dt - \sum_n e^{-\rho\tau_n^1}\left(\lambda_1 \alpha_{\tau_n^1}^1 + c_1\right)\right] \quad (5a)$$

$$\alpha_t^1 = \Phi(p_t^1 - p_t^2) \quad (5b)$$

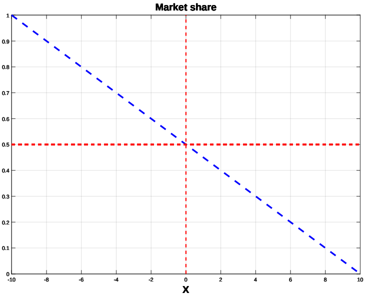
$$dx_t = \sigma dW_t + \mu dt \quad (5c)$$

$$p_t^1 = p_0^1 + \sum_n \delta_n^1 \mathbb{1}_{\{t \geq \tau_n^1\}} \quad (5d)$$

Remark

- Crude simplification of revenue function.
- With this structure cost, a retailer has no incentive to get all the market.

Model - Market share function



$$\Phi(y) = \begin{cases} 1 & y \le -\Delta \\ -\frac{1}{2\Delta}(y - \Delta) & y \in (-\Delta, \Delta) \\ 0 & y \ge \Delta. \end{cases}$$

Remarks

- Nothing prevents ex ante the prices to go to infinity.

Warm-up : the case of one retailer

- Same hypothesis as in the case with two retailers
- But, now the market share depends on the spread between the wholesale market price x_t and the retailer's price p_t.

The retailer has to determine the sequence of stopping time τ_n and the changes in her price δ_n such that :

$$\sup_{\tau_n, \delta_n} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left((p_t - x_t)\,\alpha_t - \frac{b}{2}(\alpha_t)^2\right) dt - \sum_n e^{-\rho\tau_n}\left(\lambda\alpha_{\tau_n} + c\right)\right] \tag{6a}$$

$$\alpha_t = \Phi(x_t - p_t) \tag{6b}$$

$$dx_t = \sigma dW_t + \mu dt \tag{6c}$$

$$p_t = p_0 + \sum_n \delta_n \mathbb{1}_{\{t \geq \tau_n\}} \tag{6d}$$

Remark

- The state of the problem is given by the state process $X_t := x_t - p_t$.

Optimal strategy in the one retailer case

- Standard impulse control problem [Oksendal & Sulem, 2009]
- Similar models suggest the optimal solution [Cadenillas et al. 2010] :
  - There exist two values $\underline{x} < \overline{x}$ both in $(0, \Delta)$ such that it is optimal to wait until the state process X reaches one of them.
  - Each time the state process X hits one $\underline{x}$ or $\overline{x}$, the intervention brings back the state process to a value x^*.
  - The optimal price change is given by $\delta = x^* - \underline{x}$ or $\delta = \overline{x} - x^*$ depending on which border is hit.
- Possible to compute numericaly $\underline{x}, x^*, \overline{x}$ given the parameters $b, c, \lambda, \Delta, \rho$

Back to the 2 retailer case

- Possible to reduce the dimension of the problem, by introducing the spread processes $X^1 := p^1 - x$ and $X^2 := p^2 - x$.
- The objective function for player 1 becomes :

$$v_1(x_1, x_2) = \sup_{\tau_n^1, \delta_n^1} \mathbb{E}_{x_1, x_2} \left[\int_0^\infty e^{-\rho t} \left(X_t^1 \Phi(X_t^1 - X_t^2) - \frac{b_1}{2} \Phi^2(X_t^1 - X_t^2) \right) dt \right.$$
$$\left. - \sum_n e^{-\rho \tau_n^1} \left(\lambda_1 \Phi(X_{\tau_{n-}^1}^1 - X_{\tau_{n-}^1}^2) + c_1 \right) \right]$$

where

$$dX_t^i = dp_t^i - dx_t = dp_t^i - \sigma dW_t - \mu dt, \quad i = 1, 2.$$

Continuation region

- We expect that the cost structure will force the spread $x^1 - x^2$ to stay at equilibrium in the interval $(-\Delta, \Delta)$.
- Player 1 will not intervene as long as the state variable X^1 stays in some region $D_1 := (\underline{x}_1, \overline{x}_1)$ with $\underline{x}_1 < \overline{x}_1$ possibly depending on the position of the other coordinate x^2.
- Each time x^1 hits the boundary, then player 1 intervenes to push it back to some value x_1^* in the interior of the continuation region D_1.
- By symetry, we expect that player 2 will behave in the same way.

Guessing the form of the value functions

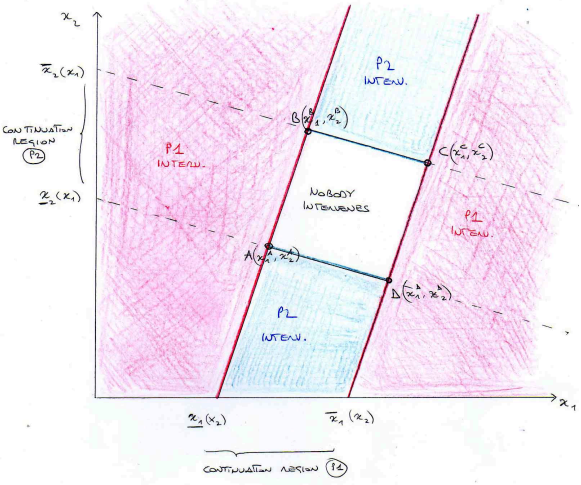


Figure: Matteo's hand-made drawing of the possible shape of the continuation region

Guessing the form of the value functions

$$R = \big\{(x_1, x_2) : x_1 \in [-\Delta, \underline{x}_1(x_2)] \cup [\bar{x}_1(x_2), \Delta]\big\},$$

where P1 intervenes (red area in the picture),

$$B = \big\{(x_1, x_2) : x_1 \in]\underline{x}_1(x_2), \bar{x}_1(x_2)[\ \text{ and } \ x_2 \in [-\Delta, \underline{x}_2(x_1)] \cup [\bar{x}_2(x_1), \Delta]\big\},$$

where P2 intervenes (blue area in the picture),

$$W = \big\{(x_1, x_2) : x_1 \in]\underline{x}_1(x_2), \bar{x}_1(x_2)[\ \text{ and } \ x_2 \in]\underline{x}_2(x_1), \bar{x}_2(x_1)[\big\},$$

where nobody intervenes (white area in the picture).

Guessing the form of the value functions

$$V_1 = \begin{cases} \mathcal{H}_1 V_1, & \text{in } B \text{ (blue area, P2 interv)}, \\ \varphi_1, & \text{in } W \text{ (white area, no interv)}, \\ \mathcal{M}_1 V_1, & \text{in } R \text{ (red area, P1 interv)}, \end{cases}$$

$$V_2 = \begin{cases} \mathcal{M}_2 V_2, & \text{in } B \text{ (blue area, P2 interv)}, \\ \varphi_2, & \text{in } W \text{ (white area, no interv)}, \\ \mathcal{H}_2 V_2, & \text{in } R \text{ (red area, P1 interv)}. \end{cases}$$

where φ_i is a sol. to $\mathcal{A}V_i - \rho V_i + f_i = 0$

Applying sufficient conditions

Optimality of $\underline{x}_1$

$$\left(\frac{\partial \varphi_1}{\partial x_1}\right)(x_1^*(x_2), x_2) = 0, \qquad x_2 \in [-\Delta, \Delta]. \tag{7}$$

Continuity on the curve $x_1 = \underline{x}_1(x_2)$. The function V_1 has two different expressions in the central vertical strip.

$$\varphi_1(x_1^*(x_2), x_2) - c_1 - \lambda_1 \Phi(\underline{x}_1(x_2) - x_2) = \varphi_1(\underline{x}_1(x_2), x_2), x_2 \in [x_2^A, x_2^B],$$
$$\varphi_1(x_1^*(x_2), x_2) - c_1 - \lambda_1 \Phi(\underline{x}_1(x_2) - x_2) = \varphi_1\Big(\underline{x}_1(x_2), x_2^*(\underline{x}_1(x_2))\Big), x_2 \in [-\Delta, \Delta] \setminus [x_2^A, x_2^B].$$

Continuity on the curve $x_1 = \bar{x}_1(x_2)$ (similar to above)

$$\varphi_1(x_1^*(x_2), x_2) - c_1 - \lambda_1 \Phi(\bar{x}_1(x_2) - x_2) = \varphi_1(\bar{x}_1(x_2), x_2), x_2 \in [x_2^D, x_2^C],$$
$$\varphi_1(x_1^*(x_2), x_2) - c_1 - \lambda_1 \Phi(\bar{x}_1(x_2) - x_2) = \varphi_1\Big(\bar{x}_1(x_2), x_2^*(\bar{x}_1(x_2))\Big), x_2 \in [-\Delta, \Delta] \setminus [x_2^D, x_2^C].$$

Applying sufficient conditions

Continuity on the segment AD, which belongs to the curve $x_2 = \underline{x}_2(x_1)$

$$\varphi_1\big(x_1, x_2^*(x_1)\big) = \varphi_1\big(x_1, \underline{x}_2(x_1)\big), \qquad x_1 \in \big]x_1^A, x_1^D\big[.$$

Continuity on the segment BC, which belongs to the curve $x_2 = \bar{x}_2(x_1)$

$$\varphi_1\big(x_1, x_2^*(x_1)\big) = \varphi_1\big(x_1, \bar{x}_2(x_1)\big), \qquad x_1 \in \big]x_1^B, x_1^C\big[.$$

Applying sufficient conditions

Differentiability on the segment AB, which belongs to the curve $x_1 = \underline{x}_1(x_2)$ (one condition for each derivative)

$$\left(\frac{\partial \varphi_1}{\partial x_1}\right)(\underline{x}_1(x_2), x_2) = \frac{\lambda_1}{2\Delta}, x_2 \in \left[x_2^A, x_2^B\right],$$
$$\left(\frac{\partial \varphi_1}{\partial x_2}\right)(\underline{x}_1(x_2), x_2) = \left(\frac{\partial \varphi_1}{\partial x_1}\right)(x_1^*(x_2), x_2) \cdot (x_1^*)'(x_2) + \left(\frac{\partial \varphi_1}{\partial x_2}\right)(x_1^*(x_2), x_2) - \frac{\lambda_1}{2\Delta},$$
$$x_2 \in [-\Delta, \Delta] \setminus \left[x_2^A, x_2^B\right].$$

Differentiability on the segment DC, which belongs to the curve $x_1 = \bar{x}_1(x_2)$ (one condition for each derivative)

$$\left(\frac{\partial \varphi_1}{\partial x_1}\right)(\bar{x}_1(x_2), x_2) = \frac{\lambda_1}{2\Delta}, \qquad x_2 \in \left[x_2^D, x_2^C\right],$$
$$\left(\frac{\partial \varphi_1}{\partial x_2}\right)(\bar{x}_1(x_2), x_2) = \left(\frac{\partial \varphi_1}{\partial x_1}\right)(x_1^*(x_2), x_2) \cdot (x_1^*)'(x_2) + \left(\frac{\partial \varphi_1}{\partial x_2}\right)(x_1^*(x_2), x_2) - \frac{\lambda_1}{2\Delta},$$
$$x_2 \in [-\Delta, \Delta] \setminus \left[x_2^D, x_2^C\right].$$

Final system

$$(i)\left(\frac{\partial\varphi_1}{\partial x_1}\right)\left(x_1^*(x_2), x_2\right) = 0, x_2 \in [-\Delta, \Delta],$$

$$(ii)\varphi_1\left(x_1^*(x_2), x_2\right) = \varphi_1\left(\underline{x}_1(x_2), x_2\right) + c_1 + \lambda_1\Phi\left(\underline{x}_1(x_2) - x_2\right), x_2 \in \left[x_2^A, x_2^B\right],$$

$$(iii)\varphi_1\left(x_1^*(x_2), x_2\right) = \varphi_1\left(\underline{x}_1(x_2), x_2^*\left(\underline{x}_1(x_2)\right)\right) + c_1 + \lambda_1\Phi\left(\underline{x}_1(x_2) - x_2\right), x_2 \in [-\Delta, \Delta] \setminus \left[x_2^A, x_2^B\right],$$

$$(iv)\varphi_1\left(x_1^*(x_2), x_2\right) = \varphi_1\left(\bar{x}_1(x_2), x_2\right) + c_1 + \lambda_1\Phi\left(\bar{x}_1(x_2) - x_2\right), x_2 \in \left[x_2^D, x_2^C\right],$$

$$(v)\varphi_1\left(x_1^*(x_2), x_2\right) = \varphi_1\left(\bar{x}_1(x_2), x_2^*\left(\bar{x}_1(x_2)\right)\right) + c_1 + \lambda_1\Phi\left(\bar{x}_1(x_2) - x_2\right), x_2 \in [-\Delta, \Delta] \setminus \left[x_2^D, x_2^C\right],$$

$$(vi)\varphi_1\left(x_1, x_2^*(x_1)\right) = \varphi_1\left(x_1, \underline{x}_2(x_1)\right), x_1 \in \left]x_1^A, x_1^D\right[,$$

$$(vii)\varphi_1\left(x_1, x_2^*(x_1)\right) = \varphi_1\left(x_1, \bar{x}_2(x_1)\right), x_1 \in \left]x_1^B, x_1^C\right[,$$

$$(viii)\left(\frac{\partial\varphi_1}{\partial x_1}\right)\left(\underline{x}_1(x_2), x_2\right) = \frac{\lambda_1}{2\Delta}, x_2 \in \left[x_2^A, x_2^B\right],$$

$$(ix)\left(\frac{\partial\varphi_1}{\partial x_2}\right)\left(\underline{x}_1(x_2), x_2\right) = \left(\frac{\partial\varphi_1}{\partial x_1}\right)\left(x_1^*(x_2), x_2\right) \cdot \left(x_1^*\right)'(x_2) + \left(\frac{\partial\varphi_1}{\partial x_2}\right)\left(x_1^*(x_2), x_2\right) - \frac{\lambda_1}{2\Delta}, x_2 \in [-\Delta, \Delta] \setminus \left[x_2^A, x_2^B\right]$$

$$(x)\left(\frac{\partial\varphi_1}{\partial x_1}\right)\left(\bar{x}_1(x_2), x_2\right) = \frac{\lambda_1}{2\Delta}, x_2 \in \left[x_2^D, x_2^C\right],$$

$$(xi)\left(\frac{\partial\varphi_1}{\partial x_2}\right)\left(\bar{x}_1(x_2), x_2\right) = \left(\frac{\partial\varphi_1}{\partial x_1}\right)\left(x_1^*(x_2), x_2\right) \cdot \left(x_1^*\right)'(x_2) + \left(\frac{\partial\varphi_1}{\partial x_2}\right)\left(x_1^*(x_2), x_2\right) - \frac{\lambda_1}{2\Delta}, x_2 \in [-\Delta, \Delta] \setminus \left[x_2^D, x_2^C\right]$$

Remarks

- Difficult to solve in general (even if $x_1^*, \underline{x}_1, \overline{x}_1$ were affine function of x_2).

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- For some situations, it is possible to go further.
- Case where the second player has infinite switching cost and the first one only a fixed finite swtiching cost (no proportional cost).
- In this situation, second player never moves and first player is set back to a one-player situation with a different market share function.

A special case

- For a one-player case with $\lambda = 0$ and $\mu = 0$, the optimal strategy is symetric around
$$x_v = \frac{\Delta\left(\Delta + b_1\right)}{2\Delta + b_1}$$

A special case

- For a one-player case with $\lambda = 0$ and $\mu = 0$, the optimal strategy is symetric around
$$x_v = \frac{\Delta\left(\Delta + b_1\right)}{2\Delta + b_1}$$
- The intervention frontiers $\underline{x}$ and $\overline{x}$ are given by $\underline{x} = x_v - y$ and $\overline{x} = x_v + y$ with y solution of the nonlinear system in (A, y) :

$$A\theta e^{\theta y} - A\theta e^{-\theta y} - 2k_2 y = 0$$
$$Ae^{\theta y} + Ae^{-\theta y} - k_2 y^2 - 2A + c_1 = 0$$

with $\theta = \frac{\sqrt{2\rho}}{\sigma}$ and $k_2 = \frac{\frac{2\Delta+b}{2\Delta^2}}{\rho}$

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- Thus, the first player has to solve this problem but with a market share function given by $\Phi(p_t^1 - p_0^2)$.

A special case

- In this case, the optimal strategy for player 1 given by

$$x_v^1 = \frac{(\Delta + b_1)}{2\Delta + b_1} \times (x_2 + \Delta)$$

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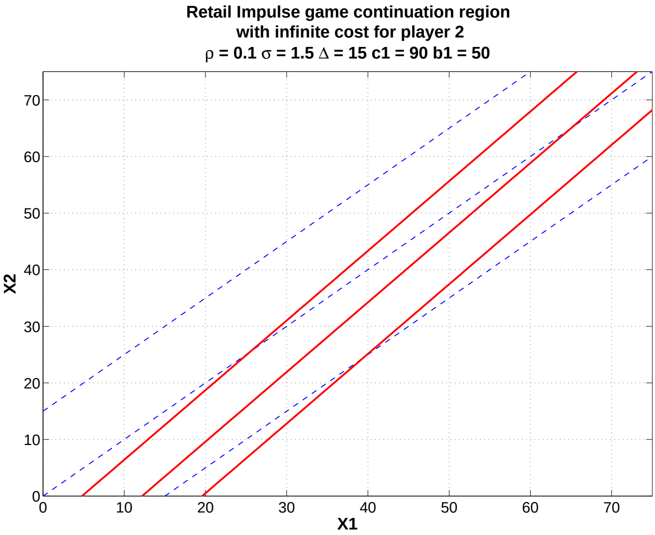
Numerical illustration



Conclusion

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- Possible to model and analyse the interaction between retailers on electricity markets as an impulse control game.
- Challenge for more than two players.

Perspective

- Alternative models for more than 2 players (1-Leader vs N-followers)
- Approximation using infinite number of players

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