

time using the function $D_L(z; \varphi)$. This is very much in the spirit of [Th] and underlines the importance of Jacobi forms.

Paragraph 4 contains an application of the additive distribution law (i). Replacing classical Gauss sums by certain resolvents constructed from $D_L(z; \varphi)$ and studying their prime ideal factorization, one obtains an elliptic analogue to Stickelberger's celebrated theorem. More precisely, let $l \geq 5$ denote a rational prime, F a number field such that there exists an elliptic curve E/F together with an F -rational l -torsion point P , and set $N = F(\zeta_l + \zeta_l^{-1})$. Then one can construct a quadratic Stickelberger element $\theta_2 \in \mathbb{Z}[\text{Gal}(N/F)]$, which annihilates a certain subquotient of the ideal class group of N .

In [11] Bayad has defined a remarkable p -adic analogue of $D_L(z; \varphi)$, replacing complex elliptic curves \mathbb{C}/L by Tate curves $\bar{K}^\times/q^{\mathbb{Z}}$. His results here are of independent interest and also provide an efficient method to simplify the computation of the prime ideal factorization of the elliptic Gauss sums in [9].

The main result of [Th], paragraph 6, is a multiplicative distribution law for the function $K(z; L, \Lambda)$. There are already several results of this sort for different normalizations of (powers of) $K(z; L, \Lambda)$ in the literature, see e.g. Coates, Kubert, deShalit, and most notably Robert in [50]. In my opinion it is Robert's merit to define the 'correct' normalization of $K(z; L, \Lambda)$, which results in a simple and beautiful distribution law. However, all previous results are formulated and proved for lattices L, L', M such that $L = L' \cap M$ and $([L' : L], 6) = 1$. Bayad and Ayala in [16] get rid of the restriction $([L' : L], 6) = 1$ and prove a very general distribution relation for $K(z, L, \Lambda)$ itself. It seems, that this result is the genesis of all previously known results. I could imagine that a careful analysis of this result would directly lead to Robert's normalization.

In paragraph 7 of [Th] Bayad considers a multiplicative distribution relation for Siegel functions (viewed as functions on divisors of \mathbb{C}) proved by Jarvis in [38]. Jarvis' proof is of more algebraic nature and only gives the distribution relation up to a root of unity. The proof in [16] (by Bayad and Ayala) is of analytic nature and one certainly gets the impression that this is the 'correct' way to prove this sort of result. As a benefit one gets an equality, i.e. the root of unity in Jarvis' formula is explicitly specified.

In paragraph 8 Bayad reports on his work on elliptic analogues of Zagier's multiple Dedekind sums, essentially replacing the function $\cot(x)$ in Zagier's definition by $D_L(z; \varphi)$, $L = \mathbb{Z}\tau + \mathbb{Z}$, $\text{Im}(\tau) > 0$. His main result is a reciprocity law in the style of Dedekind's reciprocity law for classical Dedekind sums. Letting $\text{Im}(\tau)$ tend to infinity one recovers and generalizes results of Zagier. It will be very interesting to see applications of these new results which generalize the known applications of Zagier's multiple Dedekind sums. There are certainly many open problems which could lead to interesting projects in the near future.

Paragraph 9 is concerned with yet unpublished work. In [15] an additive distribution law for the (additively normalized) Weierstrass ζ -function is proved. As an application one obtains a new and fast algorithm for determining the polynomial

$$H(X) = \prod_{\substack{C \subset A/L \\ C \neq \{0\}}} (X - \nu_C(t)).$$