

Counterparty Risk Frontiers: Collateral damages
Paris, 4 May 2012. LES RENCONTRES DES CHAIRES FBF
Next Generation CVA: From Funding Liquidity to
Margin Lending

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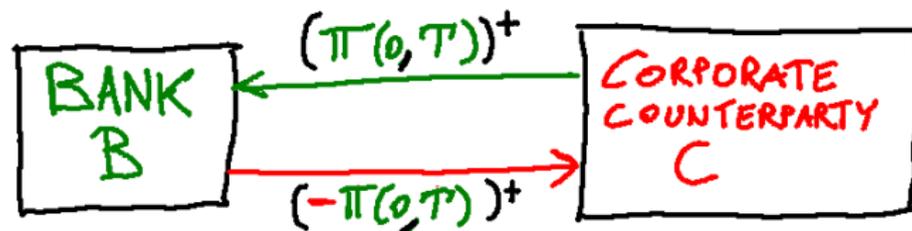
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Agenda I

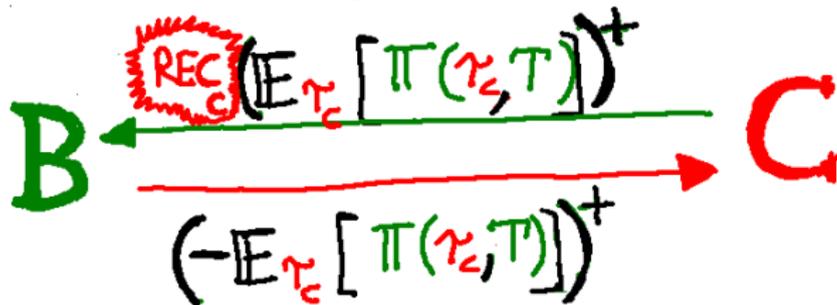
- 1 Counterparty Credit Risk pricing: Payout mathematics
 - CVA, DVA and 1st to Default
 - Risk Free Closeout or Replication Closeout? 1st to default?
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Context



π :
PORTFOLIO
CASH FLOWS
TO B

τ_C
DEFAULT OF C
CLOSEOUT:



The case of symmetric counterparty risk

Suppose now that we allow for both parties to default. Counterparty risk adjustment allowing for default of “B”?

“B” : the investor; “C”: the counterparty;

(“1”: the underlying name/risk factor of the contract).

τ_B, τ_C : default times of “B” and “C”. T : final maturity

We consider the following events, forming a partition

Four events ordering the default times

$$\mathcal{A} = \{\tau_B \leq \tau_C \leq T\} \quad \mathcal{E} = \{T \leq \tau_B \leq \tau_C\}$$

$$\mathcal{B} = \{\tau_B \leq T \leq \tau_C\} \quad \mathcal{F} = \{T \leq \tau_C \leq \tau_B\}$$

$$\mathcal{C} = \{\tau_C \leq \tau_B \leq T\}$$

$$\mathcal{D} = \{\tau_C \leq T \leq \tau_B\}$$

Define $\text{NPV}_{\{B,C\}}(t) := \mathbb{E}_t[\Pi_{\{B,C\}}(t, T)]$, and recall $\Pi_B = -\Pi_C$.

The case of symmetric counterparty risk

$$\begin{aligned} \Pi_B^D(t, T) = & \mathbf{1}_{E \cup F} \Pi_B(t, T) \\ & + \mathbf{1}_{C \cup D} \left[\Pi_B(t, \tau_C) + D(t, \tau_C) \left(\text{REC}_C (\text{NPV}_B(\tau_C))^+ - (-\text{NPV}_B(\tau_C))^+ \right) \right] \\ & + \mathbf{1}_{A \cup B} \left[\Pi_B(t, \tau_B) + D(t, \tau_B) \left((\text{NPV}_B(\tau_B))^+ - \text{REC}_B (-\text{NPV}_B(\tau_B))^+ \right) \right] \end{aligned}$$

- 1 If no early default \Rightarrow payoff of a default-free claim (1st term).
- 2 In case of early default of the counterparty, the payments due before default occurs are received (second term),
- 3 and then if the residual net present value is positive only the recovery value of the counterparty REC_C is received (third term),
- 4 whereas if negative, it is paid in full by the investor (4th term).
- 5 In case of early default of the investor, the payments due before default occurs are received (fifth term),
- 6 and then if the residual net present value is positive it is paid in full by the counterparty to the investor (sixth term),
- 7 whereas if it is negative only the recovery value of the investor REC_B is paid to the counterparty (seventh term).

The case of symmetric counterparty risk

$$\mathbb{E}_t \left\{ \Pi_B^D(t, T) \right\} = \mathbb{E}_t \left\{ \Pi_B(t, T) \right\} + \text{DVA}_B(t) - \text{CVA}_B(t)$$

$$\text{DVA}_B(t) = \mathbb{E}_t \left\{ \text{LGD}_B \cdot \mathbf{1}(t < \tau^{1st} = \tau_B < T) \cdot D(t, \tau_B) \cdot [-\text{NPV}_B(\tau_B)]^+ \right\}$$

$$\text{CVA}_B(t) = \mathbb{E}_t \left\{ \text{LGD}_C \cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T) \cdot D(t, \tau_C) \cdot [\text{NPV}_B(\tau_C)]^+ \right\}$$

$$\mathbf{1}(A \cup B) = \mathbf{1}(t < \tau^{1st} = \tau_B < T), \quad \mathbf{1}(C \cup D) = \mathbf{1}(t < \tau^{1st} = \tau_C < T)$$

- Obtained simplifying the previous formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_B < \tau_C$. This is positive to the investor 0 and is called "Debit Valuation Adjustment" (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_C < \tau_B$
- Bilateral Valuation Adjustment as seen from 0:

$$\text{BVA}_B = \text{DVA}_B - \text{CVA}_B.$$
- If computed from the opposite point of view of "C" having counterparty "B", $\text{BVA}_C = -\text{BVA}_B$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS \Rightarrow books POSITIVE MARK TO MKT
- credit quality of investor IMPROVES \Rightarrow books NEGATIVE MARK TO MKT
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5\$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

The case of symmetric counterparty risk: DVA?

October 18, 2011, 3:59 PM ET, WSJ. Goldman Sachs Hedges Its Way to Less Volatile Earnings.

Goldman's DVA gains in the third quarter totaled \$450 million [...] That amount is comparatively smaller than the \$1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported \$1.7 billion of DVA gains in its investment bank. Analysts estimated that Morgan Stanley will record \$1.5 billion of net DVA gains when it reports earnings on Wednesday [...]

Is DVA real? **DVA Hedging**. Buying back bonds? Proxying?

DVA hedge? One should sell protection on oneself, impossible, unless one buys back bonds that he had issued earlier. Very Difficult. Most times: proxying. Instead of selling protection on oneself, one sells protection on a number of names that one thinks are highly correlated to oneself.

The case of symmetric counterparty risk: DVA?

Again from the WSJ article above:

[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials. A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldn't say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained 'a peer group.'

This can approximately hedge the spread risk of DVA, but not the jump to default risk. Merrill hedging DVA risk by selling protection on Lehman would not have been a good idea. Worsens systemic risk.

DVA or no DVA? Accounting VS Capital Requirements

NO DVA: Basel III, page 37, July 2011 release

This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75.

YES DVA: FAS 157

Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

DVA or no DVA? Accounting VS Capital Requirements

Stefan Walter says:

“The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognise it, says Stefan Walter, *secretary-general of the Basel Committee*: The main reason for not recognising DVA as an offset is that it would be inconsistent with the overarching supervisory prudence principle under which we do not give credit for increases in regulatory capital arising from a deterioration in the firms own credit quality.”

Closeout: Replication (ISDA?) VS Risk Free

When we computed the bilateral adjustment formula from

$$\begin{aligned} \Pi_B^D(t, T) &= \mathbf{1}_{E \cup F} \Pi_B(t, T) \\ &+ \mathbf{1}_{C \cup D} \left[\Pi_B(t, \tau_C) + D(t, \tau_C) \left(\text{REC}_C (\text{NPV}_B(\tau_C))^+ - (-\text{NPV}_B(\tau_C))^+ \right) \right] \\ &+ \mathbf{1}_{A \cup B} \left[\Pi_B(t, \tau_B) + D(t, \tau_B) \left((-\text{NPV}_C(\tau_B))^+ - \text{REC}_B (\text{NPV}_C(\tau_B))^+ \right) \right] \end{aligned}$$

(where we now substituted $\text{NPV}_B = -\text{NPV}_C$ in the last two terms) we used the risk free NPV upon the first default, to close the deal. But what if upon default of the first entity, the deal needs to be valued by taking into account the credit quality of the surviving party? What if we make the substitutions

$$\text{NPV}_B(\tau_C) \rightarrow \text{NPV}_B(\tau_C) + \text{UDVA}_B(\tau_C)$$

$$\text{NPV}_C(\tau_B) \rightarrow \text{NPV}_C(\tau_B) + \text{UDVA}_C(\tau_B)?$$

Closeout: Replication (ISDA?) VS Risk Free

ISDA (2009) Close-out Amount Protocol.

"In determining a Close-out Amount, the Determining Party may consider any relevant information, including, [...] quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that **may take into account the creditworthiness of the Determining Party** at the time the quotation is provided"

This makes valuation more continuous: upon default we still price including the DVA, as we were doing before default.

B. and Morini (2010)

We analyze the Risk Free closeout formula in Comparison with the Replication Closeout formula for a Zero coupon bond when:

1. Default of 'B' and 'C' are independent
2. Default of 'B' and 'C' are co-monotonic

Closeout: Replication (ISDA?) VS Risk Free

Suppose 'B' (the lender) holds the bond, and 'C' (the borrower) will pay the notional 1 at maturity T .

The risk free price of the bond at time 0 to 'B' is denoted by $P(0, T)$.

If we assume deterministic interest rates, the above formulas reduce to

$$P^{D,Repl}(0, T) = P(0, T)[\mathbb{Q}(\tau_C > T) + REC_C \mathbb{Q}(\tau_C \leq T)]$$

$$P^{D,Free}(0, T) = P(0, T)[\mathbb{Q}(\tau_C > T) + \mathbb{Q}(\tau_B < \tau_C < T) + REC_C \mathbb{Q}(\tau_C \leq \min(\tau_B, T))]$$

$$= P(0, T)[\mathbb{Q}(\tau_C > T) + REC_C \mathbb{Q}(\tau_C \leq T) + LGD_C \mathbb{Q}(\tau_B < \tau_C < T)]$$

Risk Free Closeout and Credit Risk of the Lender

The adjusted price of the bond **DEPENDS ON THE CREDIT RISK OF THE LENDER 'B'** IF WE USE THE RISK FREE CLOSEOUT. This is counterintuitive and undesirable.

Closeout: Replication (ISDA?) VS Risk Free

Co-Monotonic Case

If we assume the default of B and C to be co-monotonic, and the spread of the lender 'B' to be larger, we have that the lender 'B' defaults first in ALL SCENARIOS (e.g. 'C' is a subsidiary of 'B', or a company whose well being is completely driven by 'B': 'C' is a tyre factory whose only client is car producer 'B'). In this case

$$P^{D,Repl}(0, T) = P(0, T)[\mathbb{Q}(\tau_C > T) + REC_C \mathbb{Q}(\tau_C \leq T)]$$

$$P^{D,Free}(0, T) = P(0, T)[\mathbb{Q}(\tau_C > T) + \mathbb{Q}(\tau_C < T)] = P(0, T)$$

Risk free closeout is correct. Either 'B' does not default, and then 'C' does not default either, or if 'B' defaults, at that precise time C is solvent, and B recovers the whole payment. Credit risk of 'C' should not impact the deal.

Closeout: Replication (ISDA?) VS Risk Free

The independence case: Contagion with Risk Free closeout

The Risk Free closeout shows that *upon default of the lender*, the mark to market to the lender itself jumps up, or equivalently **the mark to market to the borrower jumps down**. The effect can be quite dramatic.

The Replication closeout instead shows no such contagion, as the mark to market does not change upon default of the lender.

The co-monotonic case: Contagion with Replication closeout

The Risk Free closeout behaves nicely in the co-monotonic case, and there is no change upon default of the lender.

Instead the Replication closeout shows that *upon default of the lender* the mark to market to the lender jumps down, or equivalently **the mark to market to the borrower jumps up**.

Closeout: Replication (ISDA?) VS Risk Free

Impact of an early default of the Lender

Dependence → Closeout ↓	independence	co-monotonicity
Risk Free	Negatively affects Borrower	No contagion
Replication	No contagion	Further Negatively affects Lender

For a numerical case study and more details see Brigo and Morini (2010, 2011).

A simplified formula without τ^{1st} for bilateral VA

Instead of the full bilateral formula, the industry at times uses the difference of two unilateral formulas. Replace (this is the risk free closeout case) the correct formula with first to default risk

$$\mathbb{E}_t \left\{ \Pi_B^D(t, T) \right\} = \mathbb{E}_t \left\{ \Pi_B(t, T) \right\} + DVA_B(t) - CVA_B(t)$$

$$DVA_B(t) = \mathbb{E}_t \left\{ LGD_B \cdot \mathbf{1}(t < \tau^{1st} = \tau_B < T) \cdot D(t, \tau_B) \cdot [-NPV_B(\tau_B)]^+ \right\}$$

$$CVA_B(t) = \mathbb{E}_t \left\{ LGD_C \cdot \mathbf{1}(t < \tau^{1st} = \tau_C < T) \cdot D(t, \tau_C) \cdot [NPV_B(\tau_C)]^+ \right\}$$

with the approximated formula without first to default risk,

$$\mathbf{1}(t < \tau^{1st} = \tau_{B,C} < T) \rightarrow \mathbf{1}(t < \tau_{B,C} < T)$$

$$\mathbb{E}_t \left\{ \Pi_B^D(t, T) \right\} = \mathbb{E}_t \left\{ \Pi_B(t, T) \right\} + UDVA_B(t) \text{ (or UCVA}_C(t)) - UCVA_B(t)$$

For an equity forward, plot $\mathcal{D} = (DVA_B - CVA_B) - (UDVA_B - UCVA_B)$ as a function of Kendall's tau between the two default times

CVA difference as a function of Kendall's tau

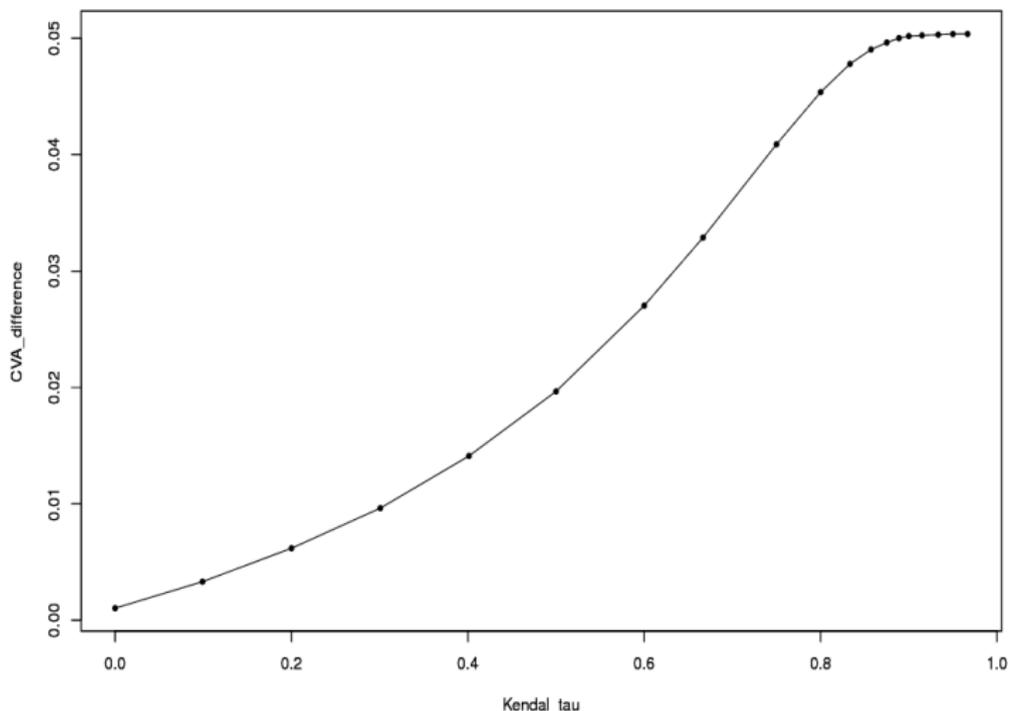


Figure: \mathcal{D} plotted against Kendall's tau between τ_B and τ_C , all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_B = 0.1$, $\lambda_C = 0.05$.

PAYOFF RISK

The exact payout corresponding with the Credit and Debit valuation adjustment is not clear.

- DVA or not?
- Which Closeout?
- First to default risk or not?
- How are collateral and funding accounted for exactly?

At a recent industry panel (WBS) on CVA it was stated that 5 banks might compute CVA in 15 different ways.

Counterparty default model: CIR++ stochastic intensity

If we cannot assume independence, we need a default model.

Counterparty instantaneous credit spread: $\lambda(t) = y(t) + \psi(t; \beta)$

- 1 $y(t)$ is a CIR process with possible jumps

$$dy_t = \kappa(\mu - y_t)dt + \nu\sqrt{y_t}dW_t^y + dJ_t, \quad \tau_C = \Lambda^{-1}(\xi), \quad \Lambda(T) = \int_0^T \lambda(s)ds$$

- 2 $\psi(t; \beta)$ is the shift that matches a given CDS curve
- 3 ξ is standard exponential independent of all brownian driven stochastic processes
- 4 In CDS calibration we assume deterministic interest rates.
- 5 Calibration : Closed form Fitting of model survival probabilities to counterparty CDS quotes
- 6 B and El Bachir (2010) (Mathematical Finance) show that this model with jumps has closed form solutions for CDS options.

4 cases: Rates, Credit, Commodities and Equity

Impact of dynamics, volatilities, correlations, wrong way risk

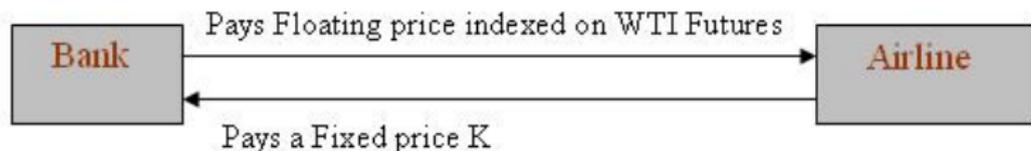
- **Interest Rate Swaps and Derivatives Portfolios** (B. Masetti (2005), B. Pallavicini 2007, 2008, B. Capponi P. Papatheodorou 2011, B. C. P. P. 2012 with collateral and gap risk)
- **Commodities swaps (Oil)** (B. and Bakkar 2009)
- **Credit: CDS on a reference credit** (B. and Chourdakis 2009, B. C. Pallavicini 2012 Mathematical Finance)
- **Equity: Equity Return Swaps** (B. and Tarenghi 2004, B. T. Morini 2011)
- Equity uses AT1P firm value model of Brigo and Tarenghi (2004) (barrier options with time-inhomogeneous GBM). Davis and Pistorious (2010) resort to Bessel Bridges.

Further asset classes are studied in the literature. For example see Biffis et al (2011) for CVA on **longevity swaps**.

Commodities: Futures, Forwards and Swaps

- **Forward:** OTC contract to buy a commodity to be delivered at a maturity date T at a price specified today. The cash/commodity exchange happens at time T .
- **Future:** Listed Contract to buy a commodity to be delivered at a maturity date T . Each day between today and T margins are called and there are payments to adjust the position.
- **Commodity Swap: Oil Example:**

FIXED-FLOATING (for hedge purposes)



Commodities: Modeling Approach

Schwartz-Smith Model

$$\begin{aligned} \ln(S_t) &= x_t + l_t + \varphi(t) \\ dx_t &= -kx_t dt + \sigma_x dW_x \\ dl_t &= \mu dt + \sigma_l dW_l \\ dW_x dW_l &= \rho_{x,l} dt \end{aligned}$$

Correlation with credit

$$\begin{aligned} dW_x dW_y &= \rho_{x,y} dt, \\ dW_l dW_y &= \rho_{l,y} dt \end{aligned}$$

Variables

S_t : Spot oil price;
 x_t, l_t : short and long term components of S_t ;
 This can be re-cast in a classic convenience yield model

Calibration

φ : defined to exactly fit the oil forward curve.
 Dynamic parameters k, μ, σ, ρ are calibrated to At the money implied volatilities on Futures options.

Commodities

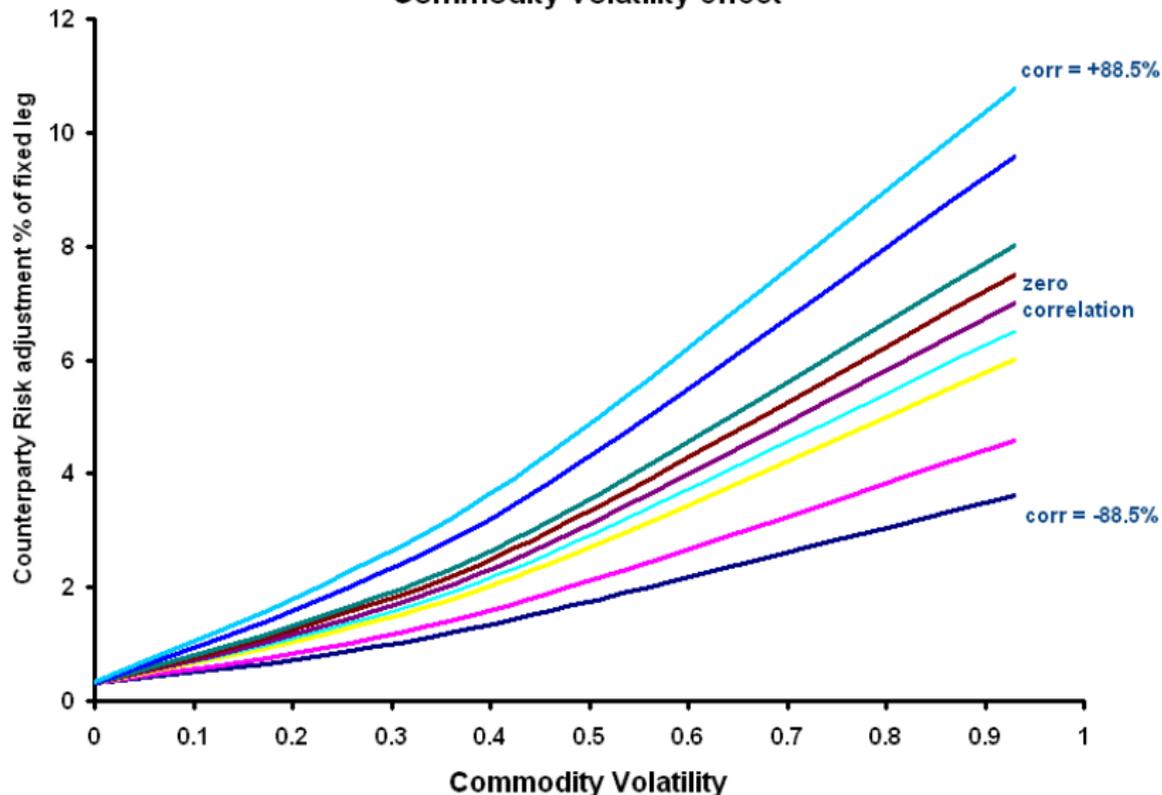
Total correlation Commodities - Counterparty default

$$\bar{\rho} = \frac{d\langle \lambda, S \rangle_t}{\sqrt{d\langle \lambda, \lambda \rangle_t d\langle S, S \rangle_t}}$$

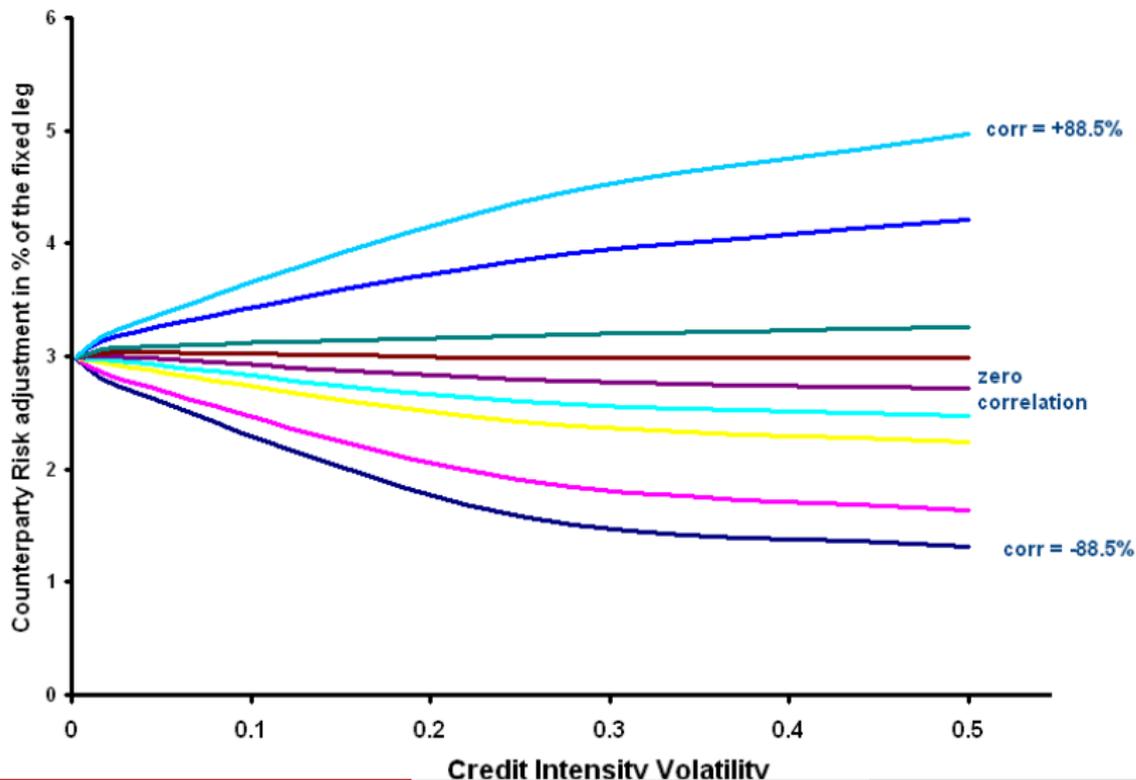
$$=: \text{corr}(d\lambda_t, dS_t) = \frac{\sigma_x \rho_{x,y} + \sigma_L \rho_{L,y}}{\sqrt{\sigma_x^2 + \sigma_L^2 + 2\rho_{x,L} \sigma_x \sigma_L}}$$

We assumed no jumps in the intensity

Commodities: Commodity Volatility Effect

Counterparty Risk adjustment for 7Y Payer WTI Swap
Commodity volatility effect

Commodities: Credit Volatility Effect

Counterparty Risk adjustment for 7Y Payer WTI Swap
Credit volatility effect

Wrong Way Risk?

Basel 2, under the "Internal Model Method", models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

What did we get in our cases? Two examples:

$$(4.973 - 2.719)/2.719 = 82\% \gg 40\%$$

$$(1.878 - 1.79)/1.79 \approx 5\% \ll 20\%$$

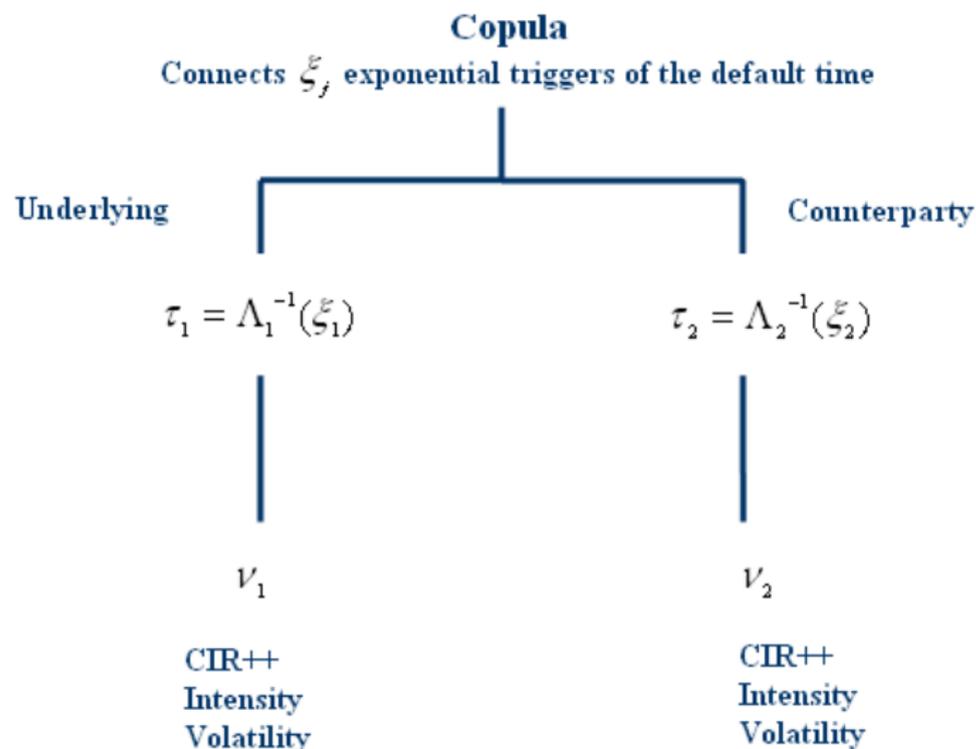
Credit (CDS)

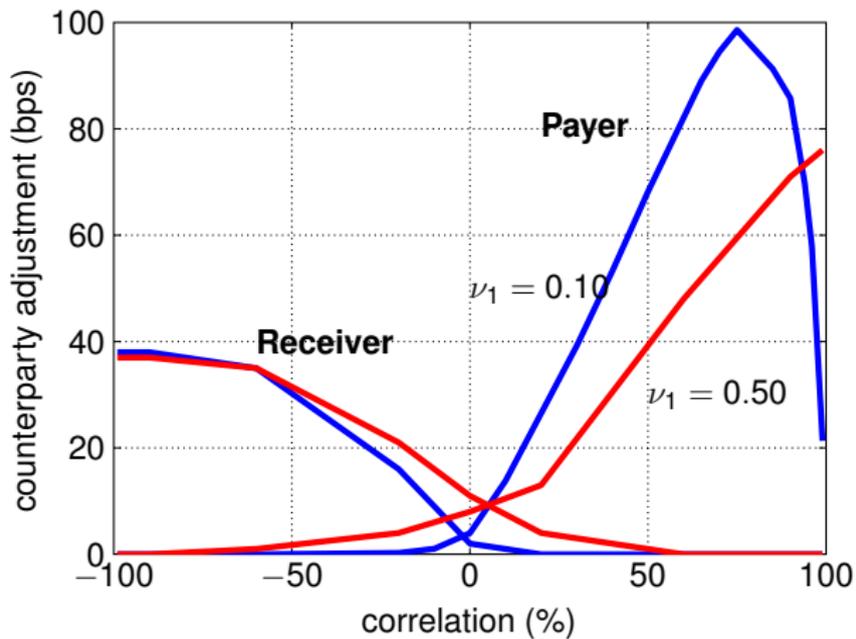
- Model equations: ("1" = CDS underlying, "C" = counterparty)

$$dy_j(t) = k_j(\mu_j - y_j(t))dt + \nu_j \sqrt{y_j(t)} dZ_j(t), \quad \lambda_j = y_j + \psi_j, \quad j = 1, C$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s) ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_1 and ξ_C are connected through a gaussian copula with correlation parameter ρ (or with Kendall's tau = $2 \arcsin(\rho)/\pi$).
- In our approach, we take into account default correlation between default times τ_1 and τ_2 **and** credit spreads volatility $\nu_j, j = 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
Taking into account ρ and $\nu \implies$ better representation of market information and behavior, especially for wrong way risk.

Credit (CDS) : Overview





Credit Spread Volatility as a Smoothing Parameter

The dropping blue correlation pattern is due to a feature inherent in the copula notion (any copula).

Take for example the case with constant deterministic (zero volatility) intensities for simplicity. Push dependence to co-monotonicity ($\rho = 1$ in the Gaussian case), so that

$$\tau_1 = \frac{\xi}{\lambda_1} \quad \tau_C = \frac{\xi}{\lambda_C} \quad (*)$$

Usually $\lambda_1 > \lambda_C$ because one does not buy default protection for name 1 from an entity C that is riskier than 1.

Then $\tau_1 < \tau_C$ in all scenarios.

Then whenever τ_C hits, the CDS has already defaulted and there is no loss faced by B. This is why CVA drops to zero when $\rho \rightarrow 1$.

Credit Spread Volatility as a Smoothing Parameter

$$\tau_1 = \frac{\xi}{\lambda_1} \quad \tau_C = \frac{\xi}{\lambda_C} \quad (*)$$

However, if we increase Credit Volatility ν to values that are realistic (Brigo 2005 on CDS options) the uncertainty in (*) comes back in the "denominator" and the pattern goes back to be increasing.

The fundamental role of Credit Volatility

Credit Vol is a fundamental risk factor and should be taken into account. Current models for multiname credit derivatives (CDO, Default Baskets) ignore credit volatility assuming it is zero. This can lead to very funny results when the correlation becomes very high (unrealistic representation of systemic risk)

Collateral Management and Gap Risk I

Collateral (CSA) is considered to be the solution to counterparty risk.

Periodically, the position is re-valued ("marked to market") and a quantity related to the change in value is posted on the collateral account from the party who is penalized by the change in value.

This way, the collateral account, at the periodic dates, contains an amount that is close to the actual value of the portfolio and if one counterparty were to default, the amount would be used by the surviving party as a guarantee (and viceversa).

Gap Risk is the residual risk that is left due to the fact that the realignment is only periodical. If the market were to move a lot between two realigning ("margining") dates, a significant loss would still be faced.

Folklore: Collateral completely kills CVA and gap risk is negligible.

Collateral Management and Gap Risk I

Folklore: Collateral completely kills CVA and gap risk is negligible.

We are going to show that there are cases where this is not the case at all (B. Capponi and Pallavicini 2012, Mathematical Finance)

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo *et al.* (2011) and citations therein.
- Example: Collateralized bilateral CVA for a netted portfolio of IRS with 10y maturity and 1y coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See B, Capponi, Pallavicini and Papatheodorou (2011)

Collateral Management and Gap Risk II

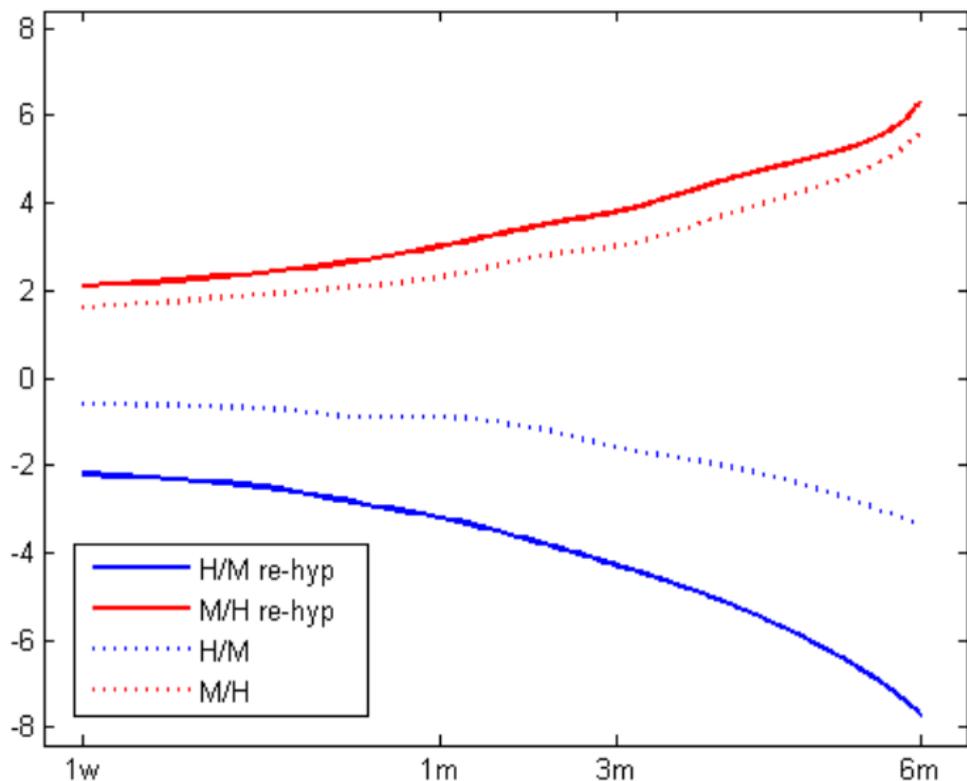


Figure explanation

Bilateral valuation adjustment, margining and rehypothecation

The figure shows the BVA(DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency δ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well.

Continuous lines represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou ‘Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypothecation and Netting’ available at <http://arxiv.org/abs/1101.3926>

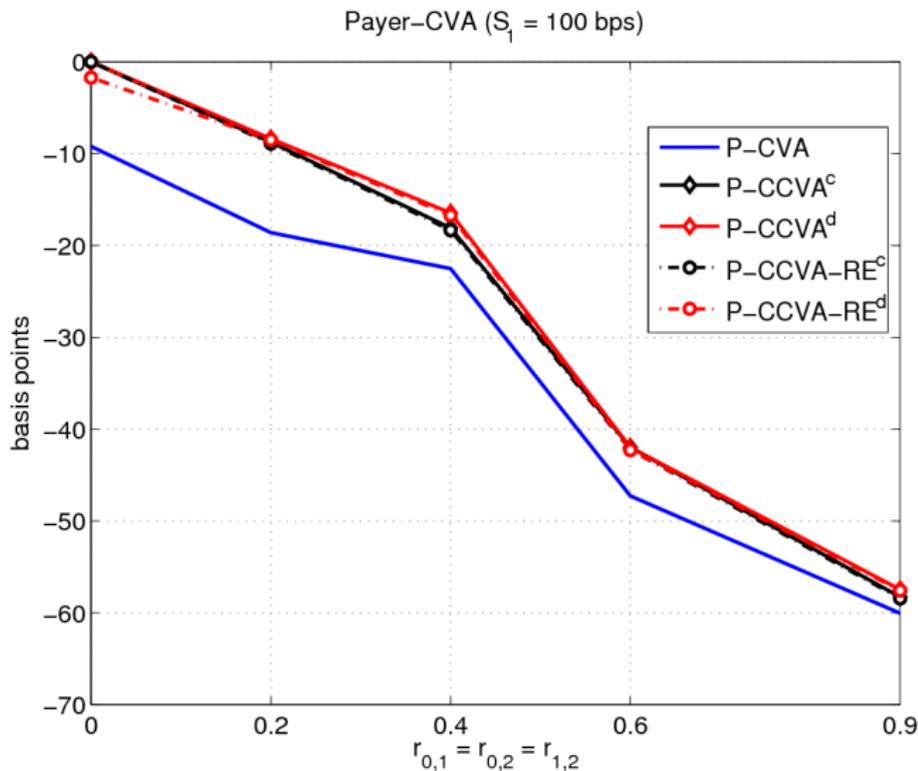
Figure explanation

From the fig, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.

Re-hypothecation enhances the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

Let us now look at a case with more contagion: a CDS.

Collate



Collateral Management and Gap Risk II

The figure refers to a payer CDS contract as underlying. See the full paper by Brigo, Capponi and Pallavicini (2011) for more cases.

If the investor holds a payer CDS, he is buying protection from the counterparty, i.e. he is a protection buyer.

We assume that the spread in the fixed leg of the CDS is 100 while the initial equilibrium spread is about 250.

Given that the payer CDS will be positive in most scenarios, when the investor defaults it is quite unlikely that the net present value be in favor of the counterparty.

We then expect the CVA term to be relevant, given that the related option will be mostly in the money. This is confirmed by our outputs.

Collateral Management and Gap Risk III

We see in the figure a relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 bps when under high correlation.

We also see that, for zero correlation, collateralization succeeds in completely removing CVA, which goes from 10 to 0 basis points.

However, collateralization seems to become less effective as default dependence grows, in that collateralized and uncollateralized CVA become closer and closer, and for high correlations we still get 60 basis points of CVA, even under collateralization.

The reason for this is the instantaneous default contagion that, under positive dependency, pushes up the intensity of the survived entities, as soon as there is a default of the counterparty.

Collateral Management and Gap Risk IV

Indeed, the term structure of the on-default survival probabilities (see paper) lies significantly below the one of the pre-default survival probabilities conditioned on $\mathcal{G}_{\tau-}$, especially for large default correlation.

The result is that the default leg of the CDS will increase in value due to contagion, and instantaneously the Payer CDS will be worth more. This will instantly increase the loss to the investor, and most of the CVA value will come from this jump.

Given the instantaneous nature of the jump, the value at default will be quite different from the value at the last date of collateral posting, before the jump, and this explains the limited effectiveness of collateral under significantly positive default dependence.

Collateralization: Aggregating Cash Flows – I

But what is the precise calculation we did to compute CVA in presence of collateralization?

- Under collateralization, we can aggregate all cash flows, along with cash flows coming from the default of the Bank and the ones due in case of non-default, inclusive of the cash-flows of the collateral account.
- In the following equations we use the risk-free discount factor $D(t, T)$, which is implicitly used also in the definitions of the risk-free discounted payoff $\Pi(t, T)$, and in the accumulation curve used for the collateral account C_t .

Collateralization: Aggregating Cash Flows – II

- We obtain by summing all the contributions

$$\begin{aligned}
 \bar{\Pi}(t, T; C) = & \\
 & \mathbf{1}_{\{\tau > T\}} \Pi(t, T) \\
 & + \mathbf{1}_{\{\tau < T\}} (\Pi(t, \tau) + D(t, \tau) C_\tau) \\
 & + \mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{B, \tau} < 0\}} \mathbf{1}_{\{C_\tau > 0\}} (\varepsilon_{B, \tau} - C_\tau) \\
 & + \mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{B, \tau} < 0\}} \mathbf{1}_{\{C_\tau < 0\}} ((\varepsilon_{B, \tau} - C_\tau)^- + \text{REC}'_C (\varepsilon_{B, \tau} - C_\tau)^+) \\
 & + \mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{B, \tau} > 0\}} \mathbf{1}_{\{C_\tau > 0\}} ((\varepsilon_{B, \tau} - C_\tau)^- + \text{REC}_C (\varepsilon_{B, \tau} - C_\tau)^+) \\
 & + \mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{B, \tau} > 0\}} \mathbf{1}_{\{C_\tau < 0\}} (\text{REC}_C \varepsilon_{B, \tau} - \text{REC}'_C C_\tau) \\
 & + \mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{C, \tau} > 0\}} \mathbf{1}_{\{C_\tau < 0\}} (\varepsilon_{C, \tau} - C_\tau) \\
 & + \mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{C, \tau} > 0\}} \mathbf{1}_{\{C_\tau > 0\}} ((\varepsilon_{C, \tau} - C_\tau)^+ + \text{REC}'_B (\varepsilon_{C, \tau} - C_\tau)^-) \\
 & + \mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{C, \tau} < 0\}} \mathbf{1}_{\{C_\tau < 0\}} ((\varepsilon_{C, \tau} - C_\tau)^+ + \text{REC}_B (\varepsilon_{C, \tau} - C_\tau)^-) \\
 & + \mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \mathbf{1}_{\{\varepsilon_{C, \tau} < 0\}} \mathbf{1}_{\{C_\tau > 0\}} (\text{REC}_B \varepsilon_{C, \tau} - \text{REC}'_B C_\tau)
 \end{aligned}$$

Collateralization: Aggregating Cash Flows – III

- Hence, by a straightforward calculation we get

$$\begin{aligned}
 \bar{\Pi}(t, T; C) = & \Pi(t, T) \\
 & - 1_{\{\tau < T\}} D(t, \tau) (\Pi(\tau, T) - 1_{\{\tau = \tau_C\}} \varepsilon_{B, \tau} - 1_{\{\tau = \tau_B\}} \varepsilon_{C, \tau}) \\
 & - 1_{\{\tau = \tau_C < T\}} D(t, \tau) (1 - \text{REC}_C) (\varepsilon_{B, \tau}^+ - C_\tau^+)^+ \\
 & - 1_{\{\tau = \tau_C < T\}} D(t, \tau) (1 - \text{REC}'_C) (\varepsilon_{B, \tau}^- - C_\tau^-)^+ \\
 & - 1_{\{\tau = \tau_B < T\}} D(t, \tau) (1 - \text{REC}_B) (\varepsilon_{C, \tau}^- - C_\tau^-)^- \\
 & - 1_{\{\tau = \tau_B < T\}} D(t, \tau) (1 - \text{REC}'_B) (\varepsilon_{C, \tau}^+ - C_\tau^+)^-
 \end{aligned}$$

- Notice that the collateral account enters only as a term reducing the exposure of each party upon default of the other one, keeping into account which is the party who posted the collateral.

Collateralized Bilateral CVA

- Now, by taking risk-neutral expectation of both sides of the above equation, and by plugging in the definition of mid-market exposure, we obtain the general expression for collateralized bilateral CVA.

$$\begin{aligned}
 \text{CVA}(t, T; C) = & -\mathbb{E}_t \left[\mathbf{1}_{\{\tau < T\}} D(t, \tau) (\varepsilon_\tau - \mathbf{1}_{\{\tau = \tau_C\}} \varepsilon_{B, \tau} - \mathbf{1}_{\{\tau = \tau_B\}} \varepsilon_{C, \tau}) \right] \\
 & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \text{LGD}_C (\varepsilon_{B, \tau}^+ - C_\tau^+)^+ \right] \\
 & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau = \tau_C < T\}} D(t, \tau) \text{LGD}'_C (\varepsilon_{B, \tau}^- - C_\tau^-)^+ \right] \\
 & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \text{LGD}_B (\varepsilon_{C, \tau}^- - C_\tau^-)^- \right] \\
 & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau = \tau_B < T\}} D(t, \tau) \text{LGD}'_B (\varepsilon_{C, \tau}^+ - C_\tau^+)^- \right]
 \end{aligned}$$

- Now, we need a recipe to calculate on-default exposures ε_{B, τ_C} and ε_{C, τ_B} , that, in the practice, are approximated from today exposure corrected for haircuts or add-ons.

Formulae for Collateralized Bilateral CVA – I

- We consider all the exposures being evaluated at mid-market, namely we consider:

$$\varepsilon_{B,t} \doteq \varepsilon_{C,t} \doteq \varepsilon_t$$

- Thus, in such case we obtain for collateralized bilateral CVA

$$\begin{aligned} \text{CVA}(t, T; \mathbf{C}) = & -\mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_C < T\}} D(t, \tau) \text{LGD}_C (\varepsilon_\tau^+ - C_\tau^+)^+ \right] \\ & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_C < T\}} D(t, \tau) \text{LGD}'_C (\varepsilon_\tau^- - C_\tau^-)^+ \right] \\ & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_B < T\}} D(t, \tau) \text{LGD}_B (\varepsilon_\tau^- - C_\tau^-)^- \right] \\ & - \mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_B < T\}} D(t, \tau) \text{LGD}'_B (\varepsilon_\tau^+ - C_\tau^+)^- \right] \end{aligned}$$

- After this section we show a possible way to relax such approximation.

Formulae for Collateralized Bilateral CVA – II

- If collateral re-hypothecation is not allowed ($LGD'_C \doteq LGD'_B \doteq 0$) the above formula simplifies to

$$CVA(t, T; C) = -\mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_C < T\}} D(t, \tau) LGD_C (\varepsilon_\tau^+ - C_\tau^+)^+ \right] \\ - \mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_B < T\}} D(t, \tau) LGD_B (\varepsilon_\tau^- - C_\tau^-)^- \right] \quad (1)$$

- On the other hand, if re-hypothecation is allowed and the surviving party always faces the worst case ($LGD'_C \doteq LGD_C$ and $LGD'_B \doteq LGD_B$), we get

$$CVA(t, T; C) = -\mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_C < T\}} D(t, \tau) LGD_C (\varepsilon_\tau - C_\tau)^+ \right] \\ - \mathbb{E}_t \left[\mathbf{1}_{\{\tau=\tau_B < T\}} D(t, \tau) LGD_B (\varepsilon_\tau - C_\tau)^- \right] \quad (2)$$

Inclusion of Funding Cost I

We now turn to the inclusion of funding costs. Q & A from

D. Brigo (2011). Counterparty Risk FAQ: Credit VaR, PFE, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, WWR, Basel, Funding, CCDS and Margin Lending. Available at arXiv.org, ssrn.com, defaultrisk.com, damianobrigo.it

- Q:** There is a further topic I keep hearing around. It's the inclusion of Cost of Funding into the valuation framework. What is that?
- A:** When you manage a trading position, you need to obtain cash in order to do a number of operations:
- hedging the position,
 - posting collateral,
 - paying coupons or notionals
 - set reserves in place

Inclusion of Funding Cost II

and so on. You may obtain cash from your Treasury department or in the market. You may also receive cash as a consequence of being in the position: a coupon, a notional reimbursement, a positive mark to market move, getting some collateral, a closeout payment.

All such flows need to be remunerated:

- if you are borrowing, this will have a cost,
- and if you are lending, this will provide you with some revenues.

Including the cost of funding into your valuation framework means to properly account for such features.

Q: What is available in the literature?

A:

- Crepey (2011) is the most comprehensive treatment I have seen so far. The only limitation is that it does not allow for underlying credit instruments in the portfolio, and has possible issues with FX.

Inclusion of Funding Cost III

- A related framework that is more general, is in Pallavicini, Perini and B. (2011). Earlier works are partial.
- Piterbarg (2010) considers an initial analysis of the problem of replication of derivative transactions under collateralization but in a very simplistic standard Black Scholes framework without default risk, considering only two basic special cases.
- The fundamental funding implications in presence of default risk have been considered instead in Morini and Prampolini, focusing only on particularly simple products, such as zero coupon bonds or loans, in order to highlight some essential features of funding costs.
- Fujii and Takahashi (2010) analyzes implications of currency risk for collateral modeling.
- Burgard resorts to a PDE approach, being therefore quite unrealistic in high dimensions.

Inclusion of Funding Cost IV

- Crepey (2011) and Pallavicini, Perini and B. (2011) are the most comprehensive frameworks so far.

The trick is doing this consistently with all other aspects, especially counterparty risk. A number of practitioners advocate a “Funding Valuation Adjustment”, or FVA, that would be additive so that the total price of the portfolio would be

$$\text{RISK FREE PRICE} + \text{DVA} - \text{CVA} + \text{FVA}$$

However, it is not that simple.

Proper inclusion of funding costs leads to a recursive pricing problem that may be formulated as a backwards stochastic differential equation (BSDE, as in Crepey, or to a discrete time backward induction equation, as in Pallavicini Perini B. . The simple additive structure above is not there.

Inclusion of Funding Cost V

- But why is the problem inherently recursive?
- The value of the cash and collateral processes may depend on the price of the derivative, which, in turn, depends on such processes, transforming the BCCFVA pricing equation into a recursive equation.
 - Thus, funding and investing costs cannot be considered as a simple additive term to a price obtained by disregarding them.
- Importantly, **identifying DVA with Funding is wrong in general.**

Restructuring CVA: Contingent CDS (CCDS)

Definition

When the reference credit defaults at τ , the protection seller pays protection on a notional that is not fixed but given by the NPV of a reference Portfolio Π at that time if positive. This amount is: $(\mathbb{E}_{\tau_C} \Pi(\tau_C, T))^+$, minus a recovery R_{EC} fraction of it.

CCDS default leg payoff = asymmetric counterparty risk adjustm

The payoff of the default leg of a Contingent CDS is exactly

$$(1 - R_{EC}) \mathbf{1}_{\{(t < \tau_C < T)\}} D(t, \tau_C) (\mathbb{E}_{\tau_C} \Pi(\tau_C, T))^+$$

Precise Valuation? Liquidity?

Counterparty risk of the Protection Seller? Standardization?

General Remarks on CCDS

"[...]Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope. [...] Counterparty risk has become a particular concern in the markets for interest rate, currency, and commodity swaps - because these trades are not always backed by collateral.[...] Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions (Financial Times, April 10, 2008)."

Being the two payoffs equivalent, UCVA valuation will hold as well for the default leg of a CCDS.

Interest on CCDS has come back in 2011 now that CVA capital charges risk to become punitive.

Basel III and CVA I

When the valuation of a risk is more dangerous than the risk itself

“Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults.”

Basel Committee on Banking Supervision, BIS (2011). Press release available at <http://www.bis.org/press/p110601.pdf>

Given the above situation, Basel III is imposing very severe capital requirements for CVA.

Basel III and CVA II

This may lead to forms of securitization of CVA such as margin lending on the whole exposure or on tranches of the exposure.

Such "securitization of CVA" would be very difficult to model and to manage, requiring a global valuation perspective.

CVA Volatility the wrong way

The problem with the traditional upfront charge for unilateral CVA is that it leaves CVA volatility with the investor and not with the risky counterparty that generated it.

In the unilateral case, the investor charges an upfront for CVA to the counterparty and then implements a hedging strategy. The investor is thus exposed to CVA mark to market volatility in the future.

Basel III and CVA III

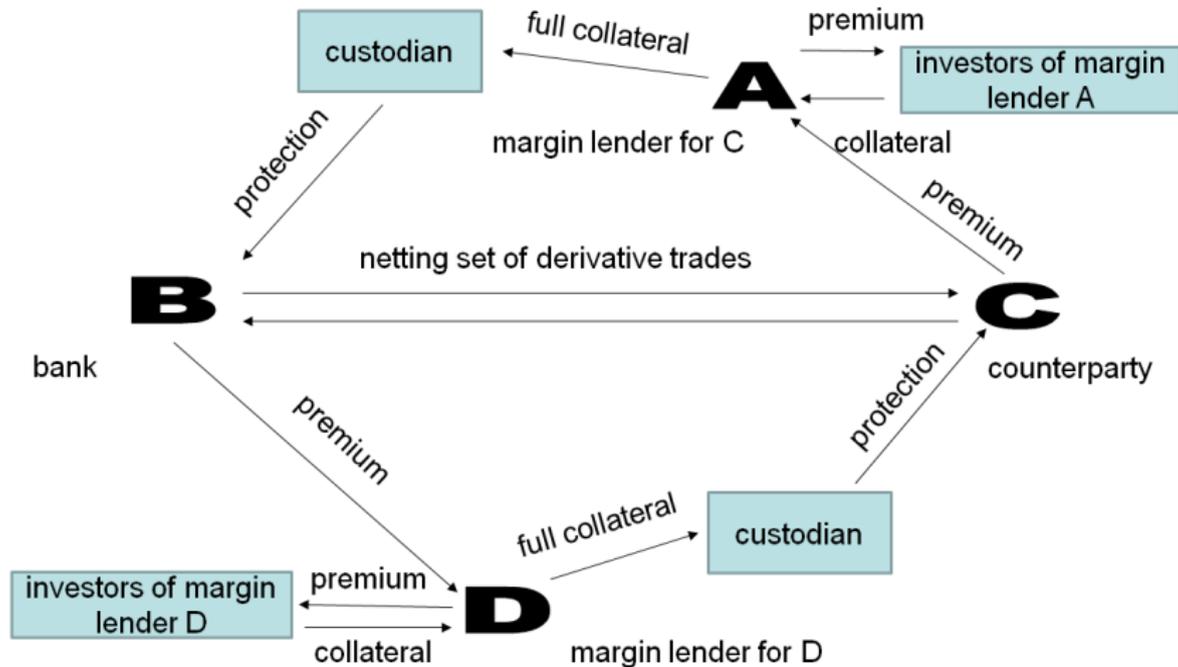
Alternatively the investor may request collateral from the counterparty, but not all counterparties are able to regularly post collateral, and this can be rather punitive for some corporate counterparties.

See recent example on Lufthansa from Risk magazine:

The airline's Cologne-based head of finance, Roland Kern, expects its earnings to become more volatile ' not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions".

Margin lending is a possible solution this problem.

Margin Lending I



Margin Lending II

To avoid posting collateral, C pays semi-annually a floating rate CVA to margin lender A ('premium' arrow connecting C to A), which A pays to investors (premium arrow connecting A to Investors). This latest payment can have a seniority structure similar to that of a cash CDO.

In exchange, for six months the investors provide A with daily collateral posting ('collateral' arrow connecting Investors to A) and A passes the collateral to a custodian ('collateral' arrow connecting A to the custodian).

This collateral need not be cash, but it can be in the form of hypothecs

Margin Lending III

Upon Default:

If C defaults within the 6m period, the collateral is paid to B ('protection' arrow connecting the custodian to B) and the loss is taken by the Investors.

At the end of the 6m period, the margin lender may decide whether to continue with the deal or to back off. C is bearing the CVA volatility risk, whereas B is not exposed to CVA volatility risk. *This is the opposite of what happens with traditional upfront CVA charges.*

Margin Lending IV

Possible problems

- Proper valuation and hedging of this to the investor who are providing collateral to the lender is going to be very difficult. There is no satisfactory standard for even simple synthetic CDOs.
- Requires an effective global valuation framework (but even standard CVA does).
- Very difficult to achieve given how Banks front offices are organized
- What if all margin lenders pull off at some point due to a systemic crisis?

On the last point, one may argue that the market is less likely to arrive in such a situation in the first place if the wrong incentives to defaulting firms are stopped and an opposite structure, such as the one in margin lending, is implemented.

CVA Restructuring: Global Valuation? I

A fair valuation and risk management of CVA requires a global model, in order to have consistency and sensible greeks.

This can be difficult. For example, our equity study used a firm value model AT1P, whereas in the other asset classes we used intensity models.

What if one has a portfolio with all asset classes together? And more generally, how does one ensure a consistent modeling framework that is needed to get meaningful prices and especially cross correlation sensitivities?

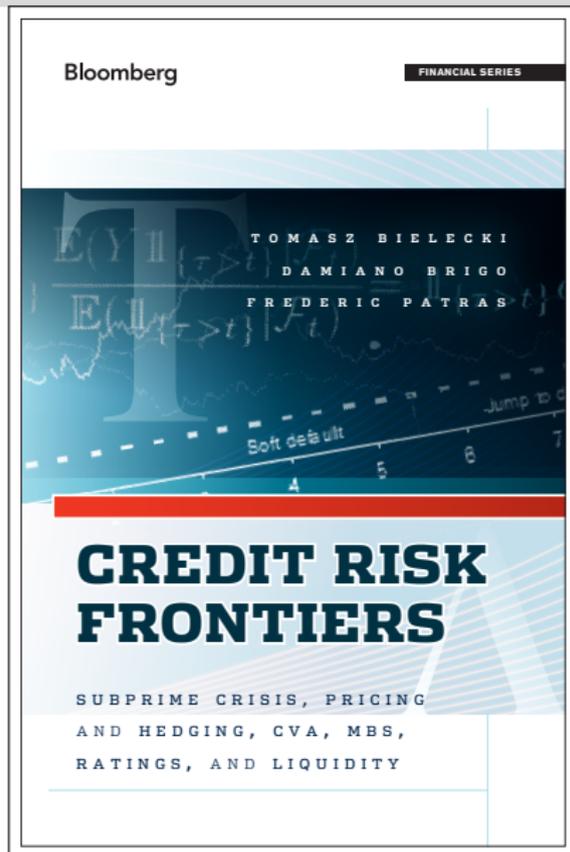
The problem is rather difficult and involves important computational resources and intelligent systems architecture.

CVA Restructuring: Global Valuation? II

Delicate points on Global Valuation include

- Modeling dependencies across defaults (we do not have even a good model for synthetic corporate CDO, base correlation still used there, see again B. Pallavicini and Torresetti (2010))
- Modeling dependencies between defaults and each other asset class
- Modeling dependencies between different asset classes
- Properly including large credit volatility and jumps with positive credit spreads

Wiley / Bloomberg Press,
2011
Credit Risk Frontiers:
subprime crisis, pricing and
hedging, cva, mbs, ratings,
and liquidity



Conclusions I

More than a simple additive adjustment or multiplier

- Counterparty Risk adds one level of optionality.
- Analysis including underlying asset/ counterparty default correlation requires a credit model.
- Highly specialized hybrid modeling framework.
- Accurate scenarios for wrong way risk.
- Outputs vary and can be very different from Basel II multipliers
- Outputs are strongly model dependent and involve model risk and model choices

Conclusions II

Payout modeling uncertainty and subtleties

- Bilateral CVA brings in symmetry but also paradoxical statements
- Bilateral CVA requires a choice of closeout (risk free or substitution), and this is relevant.
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.
- Approximations ignoring first to default risk (sometimes used in the industry) do not work well.

Conclusions III

Collateral and Gap Risk

- Inclusion of Collateral and netting rules is possible in a fully consistent way
- Collateral is not always effective against CVA
- Gap risk in collateralization remains relevant in presence of strong contagion

Cost of Funding

- Inclusion of Funding costs leads to a recursive problem
- No simple additive adjustment
- DVA is not a funding component in general

Conclusions IV

CVA Capital requirements and Restructuring

- Basel III will make capital requirements rather severe
- Contingent CDS as hedging instruments have limited effectiveness
- CVA restructuring through margin lending and hypothecs is a possible alternative
- Proper valuation and management of CVA and especially CVA restructuring requires a Consistent Global Valuation approach
- This also holds for all possible forms of CVA Securitization

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