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Impact of Closeout Conventions and of First to Default Risk on Bilateral CVA (Impact des conventions de valorisation au moment du défaut sur la CVA bilatérale)

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Overview

1. Bilateral Valuation Adjustment: CVA and DVA
   - Unilateral vs bilateral CVA, arbitrage-free valuation
   - DVA Hedging?
2. Contagion analysis: impact of closeout conventions
   - Risk Free or Substitution Closeout?
3. Can we neglect first to default risk in pricing?
4. Restructuring Counterparty Risk
   - Basel III and CVA capital requirements
   - Fully collateralised deals - prohibitive?
   - Restructuring CVA: securitization via margin lending
5. Conclusions and References

Counterparty credit risk - why?

- Financial crisis 2008-2009 $\rightarrow$ regulatory banking reform $\rightarrow$ strengthen Basel rules: Basel (1988), Basel II (2004), Basel 2.5 (2011), Basel III (2013-2019)
- implemented in the EU via Capital Requirements Directive (CRD)
- aims (among others) to strengthen the capital requirements for counterparty credit risk (and in CRD III for market risk) resulting in higher Pillar I requirements for both (Pillar 1: "sets out the minimum capital requirements firms will be required to meet for credit, market and operational risk" according to FSA- *http://www.fsa.gov.uk/about/what/international/basel*)

Counterparty credit risk - why?

Regulatory news

- The European Market Infrastructure Regulation (EMIR) in Europe was voted through the European Parliament on 29th March (part of the 2009 pledge of the G20 leaders to get all standardized OTC derivatives clear through Central Counterparties (CCP))
- new standardized Credit Support Annex (CSA) to be released by ISDA in the next few months

Regulatory framework is dynamic and technical details of the rules to be implemented are of increasing importance!

Counterparty credit risk - what?

[Basel II, Annex IV, 2/A]

"The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default."

Definitions (Basel II)

"Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, the counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction." [Basel II, Annex IV, 2/A]

Conterparty risk:

- asymmetric $\rightarrow$ unilateral CVA
- symmetric $\rightarrow$ bilateral CVA

The case of asymmetric counterparty risk

"0" = the investor; "2"= the counterparty;
("1"= the underlying name/risk factor of the contract).

τ_2: default time of "2"; T= final maturity, $\tau_0 = \infty$

$REC_2 = 1 - LGD_2$ = recovery rate for unsecured claims in case of default of "2"

$\Pi_0(t, T) =$ sum of all future cash flows seen by '0' between times t and T, discounted to t

$NPV(t, T) = E_t[\Pi(t, T)]$ is the NPV under the risk-neutral expectation

The case of asymmetric counterparty risk

General payoff seen from the point of view of “0”:

$$\begin{aligned}\Pi_0^D(t,T) &= 1_{\{\tau_2>T\}}\Pi_0(t,T)\\ +1_{\{t<\tau_2<T\}} &\left[\Pi_0(t,\tau_2) + D(t,\tau_2)(REC_2(NPV_0(\tau_2))^+ - (-NPV_0(\tau_2))^+)\right]\end{aligned}$$

Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. in case of early default of the counterparty, the payments due before default occurs are received (second term)
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_2 is received (third term),
4. whereas if it is negative it is paid in full by the investor (fourth term)

Unilateral Counterparty Valuation Adjustment (UCVA)

Valuation of unilateral counterparty risk (after simplifying the cash flows and taking risk-neutral expectation):

$$
\begin{aligned}
E_t \Pi_0^D(t,T) &= 1_{\{\tau_2 > t\}} E_t \Pi_0(t,T) \\
&\quad - E_t \left\{ LGD_2 1_{\{t < \tau_2 < T\}} D(t,\tau_2) [NPV_0(\tau_2)]^+ \right\}
\end{aligned}
$$

- first term is value in the absence of counterparty risk
- second term = $UCVA_0$ (as computed by “0”)

Credit risk: changing the nature of the contract

Default-free derivative + counterparty risk = hybrid (credit) derivative

- regular payoff + counterparty risk $\Rightarrow$ a level of optionality to the payoff

In particular, *model independent products* $\Rightarrow$ model dependent in the underlying market

Counterparty credit analysis incorporates an **opinion** of the:

- underlying market dynamics
- underlying market volatility

Counterparty's analysis

In the same unilateral credit default risk, counterparty "2" computes the payoff allowing for its own default:

$$\begin{aligned}\Pi_2^D(t,T) \quad &= 1_{\{\tau_2>T\}}\Pi_2(t,T) \\ +1_{\{t<\tau_2<T\}} \quad &[\Pi_2(t,\tau_2)+D(t,\tau_2)((NPV_2(\tau_2))^+ - REC_2(-NPV_2(\tau_2))^+)]\end{aligned}$$

Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty "2", the payments due before default occurs go through (second term)
3. and then if the residual net present value is positive to the defaulted "2", it is received in full from "0" (third term),
4. whereas if it is negative, only the recovery fraction REC_2 is paid to "0" by liquidators (fourth term).

Unilateral Debit Valuation Adjustment (UDVA)

The above formula simplifies, after taking expectation, to:

$$\begin{aligned} E_t\{\Pi_2^D(t,T)\} &= 1_{\{\tau_2>t\}} E_t \Pi_2(t,T) \\ &\quad + E_t\left\{ LGD_2 1_{\{t<\tau_2<T\}} D(t,\tau_2)[-NPV_2(\tau_2)]^+ \right\} \end{aligned}$$

The adjustment term applied to the risk-free price $E_t\Pi_2(t,T)$ is called Unilateral DEBIT Valuation Adjustment $UDVA_2(t)$.

Note:

- $UDVA_2 = UCVA_0 > 0$
- $UDVA_0 = UCVA_2 = 0$ (here, since $\tau_0 = \infty$)

Example: UCVA, UDVA

A deal between "0" and "2" is valued at 100 at $t = 0$ in favour of "0" without credit risk.

Assume that the expected loss of "0" in case of default of "2" is 2.

The value of the deal, including credit risk, becomes:

- for "0": $+100 - 2 = +100 - UCVA_0$ (counterparty "2" may default)
- for "2": $-100 + 2 = -100 + UDVA_2$ ("2" itself may default)

We used: $\Pi_0(0, T) = -\Pi_2(0, T)$,
$E_0(\Pi_0(0, T)) = 100$
$UDVA_0 = UCVA_2 = 0$ (since "0" cannot default)
and $UDVA_2 = UCVA_0 = 2$.

Include the default of both parties? (symmetry)

Assuming “0” is default-free is either unrealistic or just an approximation for the case when “2” has a much higher probability of default in comparison to “0”.

If we allow in the calculations for both parties to default, a counterparty “2” may see this as more conducive to closing a deal, since the adjustment would be cheaper to “2”.

The case of symmetric counterparty risk

What is the counterparty risk adjustment when both parties:

"0" = the investor; "2"= the counterparty;

("1"= the underlying name/risk factor of the contract).

can default (symmetric risk)?

(τ_0, τ_2: default times of "0" and "2"; T= final maturity,
$REC_2 = 1 - LGD_2$ = recovery rate for unsecured claims)
$\Pi_0(t, T) =$ sum of all future cash flows for '0' between times t and T, discounted to t

Partition of events ordering the default times:

$$
\begin{aligned}
A &= \{\tau_0 \le \tau_2 \le T\} & E &= \{T \le \tau_0 \le \tau_2\} \\
B &= \{\tau_0 \le T \le \tau_2\} & F &= \{T \le \tau_2 \le \tau_0\} \\
C &= \{\tau_2 \le \tau_0 \le T\} & 1_{(A \cup B)} &= 1_{(t < \tau^{1st} = \tau_0 < T)} \\
D &= \{\tau_2 \le T \le \tau_0\} & 1_{(C \cup D)} &= 1_{(t < \tau^{1st} = \tau_2 < T)}
\end{aligned}
$$

The case of symmetric counterparty risk

$$\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T)$$
$$+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\,(\mathrm{NPV}_0(\tau_2))^+ - (-\mathrm{NPV}_0(\tau_2))^+\right)\right]$$
$$+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left((\mathrm{NPV}_0(\tau_0))^+ - REC_0\,(-\mathrm{NPV}_0(\tau_0))^+\right)\right]$$

1. If no early default ⇒ payoff of a default-free claim (1st term).
2. In case of early default of the counterparty, the payments due before default occurs are received (2nd term),
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_2 is received (3rd term),
4. whereas if negative, it is paid in full by the investor (4th term).
5. In case of early default of the investor, the payments due before default occurs are received (5th term),
6. and then if the residual net present value is positive it is paid in full by the counterparty to the investor (6th term),
7. whereas if it is negative only the recovery value of the investor REC_0 is paid to the counterparty (7th term).

The case of symmetric counterparty risk

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{DVA}_0(t) - \mathrm{CVA}_0(t)$$

$$\mathrm{DVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_0 \cdot \mathbf{1}_{(t<\tau^{1st}=\tau_0<T)} \cdot D(t,\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{CVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2 \cdot \mathbf{1}_{(t<\tau^{1st}=\tau_2<T)} \cdot D(t,\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

- Obtained simplifying the previous formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_0 < \tau_2$. This is positive to the investor 0 and is called "Debit Valuation Adjustment" (DVA)
- 3rd term : Counterparty risk adj due to scenarios $\tau_2 < \tau_0$
- Bilateral Valuation Adjustment as seen from 0:

  $\mathrm{BVA}_0 = \mathrm{DVA}_0 - \mathrm{CVA}_0.$
- If computed from the opposite point of view of "2" having counterparty "0", $\mathrm{BVA}_2 = -\mathrm{BVA}_0$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MtM
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MtM

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS ⇒ books POSITIVE MtM
- credit quality of investor IMPROVES ⇒ books NEGATIVE MtM
- "Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements" FAS 157 (see also IAS 39)

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

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- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: “Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi’s CDS spreads”

The case of symmetric counterparty risk: DVA?

More recently, From the Wall Street Journal

`October 18, 2011, 3:59 PM ET. Goldman Sachs Hedges Its Way to Less Volatile Earnings.`

"Goldman's DVA gains in the third quarter totaled $450 million, about $300 million of which was recorded under its fixed-income, currency and commodities trading segment and another $150 million recorded under equities trading.

That amount is comparatively smaller than the $1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported $1.7 billion of DVA gains in its investment bank.

Analysts estimated that Morgan Stanley will record $1.5 billion of net DVA gains when it reports earnings on Wednesday [...]"

The case of symmetric counterparty risk: DVA?

How can DVA be hedged? One should sell protection on oneself, an impossible feat, unless one buys back bonds that one had issued earlier. This may be hard to implement, though.

Most times DVA is hedged by proxy. Instead of selling protection on oneself, one sells protection on a number of names that one thinks are highly correlated to oneself.

The case of symmetric counterparty risk: DVA?

Again from the WSJ article above:

"[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials.
A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldn't say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained a peer group. Most would consider peers to Goldman to be other large banks with big investment-banking divisions, including Morgan Stanley, J.P. Morgan Chase, Bank of America, Citigroup and others. The performance of these companies bonds would be highly correlated to Goldman's."

This can approximately hedge the spread risk of DVA, but not the jump to default risk (Merrill hedging DVA risk by selling protection on Lehman would not have been a good idea.)

Closeout Impact: Substitution (ISDA?) vs Risk Free

When we computed the bilateral adjustment formula from

$$\Pi^D_0(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T)$$
$$+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right]$$
$$+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left(\left(-\mathrm{NPV}_2(\tau_0)\right)^+ - REC_0\left(\mathrm{NPV}_2(\tau_0)\right)^+\right)\right]$$

(where we now substituted $\mathrm{NPV}_0 = -\mathrm{NPV}_2$ in the last two terms) we used the risk free NPV upon the first default, to close the deal. But what if upon default of the first entity, the deal needs to be valued by taking into account the credit quality of the surviving party? What if we make the substitutions

$$\mathrm{NPV}_0(\tau_2) \to \mathrm{NPV}_0(\tau_2) + \mathrm{UDVA}_0(\tau_2)$$

$$\mathrm{NPV}_2(\tau_0) \to \mathrm{NPV}_2(\tau_0) + \mathrm{UDVA}_2(\tau_0)?$$

Closeout Impact: Substitution (ISDA?) vs Risk Free

This seems to be supported by:

ISDA (2009) Close-out Amount Protocol.

"In determining a Close-out Amount, the Determining Party may consider any relevant information, including, without limitation, one or more of the following types of information: (i) quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that may take into account the creditworthiness of the Determining Party at the time the quotation is provided [...]"

This makes valuation more consistent: upon default we still price including the DVA, as we were doing before default.

Closeout Impact: Substitution (ISDA?) vs Risk Free

Comparative Analysis

Brigo and Morini (2010) use a Zero Coupon Bond as a contract and consider:

1. Defaults of ‘0’ and ‘2’ are independent
2. Defaults of ‘0’ and ‘2’ are co-monotonic

Suppose ‘0’ (the lender) holds the bond,
and ‘2’ (the borrower) will pay the notional 1 at maturity T.

The risk free price of the bond at time 0 to ’0’ is denoted by $P(0, T)$.

Closeout Impact: Substitution (ISDA?) vs Risk Free

If we assume deterministic interest rates, the above formulas reduce to

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T)]$$

$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_0 < \tau_2 < T) + REC_2\mathbb{Q}(\tau_2 \leq \min(\tau_0, T))]$$

$$= P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T) + LGD_2\mathbb{Q}(\tau_0 < \tau_2 < T)]$$

Undesirable contagion impact of risk-free closeout

The adjusted price of the bond DEPENDS ON THE CREDIT RISK OF THE LENDER ‘0’, unlike the Substitution Closeout.

Closeout: Substitution (ISDA?) vs Risk Free

Co-Monotonic Case

If we assume the default of ‘0’ and ‘2’ to be co-monotonic, and the spread of the lender ‘0” to be larger, we have that the lender ‘0” defaults first in ALL SCENARIOS (e.g. ‘2’ is a subsidiary of ‘0’, or a company whose well being is completely driven by ‘0’: ‘2’ is a tyre factory whose only client is car producer ‘0”). In this case

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T)]$$
$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_2 < T)] = P(0,T)$$

Risk free closeout is correct. Either ‘0’ does not default, and then ‘2’ does not default either, or if ‘0’ defaults, at that precise time ‘2’ is solvent, and ‘0’ recovers the whole payment. Credit risk of ‘2’ should not impact the deal, but it does for substitution closeout (contagion).

Closeout Impact: Substitution (ISDA?) VS Risk Free

The independence case: Contagion with Risk Free closeout

The Risk Free closeout shows that *upon default of the lender*, the mark to market to the lender itself jumps up, or equivalently **the mark to market to the borrower jumps down**. The effect can be quite dramatic.

The substitution closeout instead shows no such contagion, as the mark to market does not change upon default of the lender.

The co-monotonic case: Contagion with Substitution closeout

The Risk Free closeout behaves nicely in the co-monotonic case, and there is no change upon default of the lender.

Instead the Substitution closeout shows that *upon default of the lender* the mark to market to the lender jumps down, or equivalently **the mark to market to the borrower jumps up**.

Closeout Impact: Substitution (ISDA?) VS Risk Free

Contagion (negative) impact of an early default of the Lender:

Dependence→ Closeout↓	independence	co-monotonicity
Risk Free	Negatively affects Borrower	No contagion
Replacement	No contagion	Further Negatively affects Lender

For a numerical case study and more details see Brigo and Morini (2010, 2011).

A simplified BVA formula without τ^{1st}

Instead of the full bilateral formula, the industry at times uses the difference of two unilateral formulas. Replace (this is the risk free closeout case) the correct formula WITH first to default risk:

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{DVA}_0(t) - \mathrm{CVA}_0(t)$$

$$\mathrm{DVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(t < \tau^{1st} = \tau_0 < T) \cdot D(t,\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{CVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(t < \tau^{1st} = \tau_2 < T) \cdot D(t,\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

with the approximated formula WITHOUT first to default risk:

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{UDVA}_0(t) - \mathrm{UCVA}_0(t)$$

$$\mathrm{UDVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(t < \tau_0 < T) \cdot D(t,\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{UCVA}_0(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(t < \tau_2 < T) \cdot D(t,\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

A simplified BVA formula without τ^{1st}

- The simplified formula is attractive because it allows for the construction of a bilateral counterparty risk pricing system based only on a unilateral one.
- ignores that upon the first default at τ^{1st} closeout proceedings are started, thus involving a degree of double counting
- ignores the default dependence between the two parties via τ^{1st}
- A simplified bilateral formula is possible also in case of substitution closeout, but it turns out to be identical to the simplified formula of the risk free closeout case.

A simplified formula without τ^{1st} for bilateral VA

Brigo, Buescu and Morini (2011): analyze the impact of default dependence between investor '0' and counterparty '2' on the difference between the two formulas in the case of a Zero Coupon Bond and of an equity forward.

The difference between the full correct formula and the simplified formula can be easily computed as:

$$\begin{aligned} & E_0[1_{\{\tau_0<\tau_2<T\}} LGD_2 D(0,\tau_2)(E_{\tau_2}(\Pi(\tau_2,T)))^+] \\ - \quad & E_0[1_{\{\tau_2<\tau_0<T\}} LGD_0 D(0,\tau_0)(-E_{\tau_0}(\Pi(\tau_0,T)))^+]. \end{aligned} \tag{1}$$

A simplified formula without τ^{1st}: a Zero Coupon Bond

We assume deterministic interest rates. Consider $P(t, T)$ held by '0' (lender) who will receive the notional 1 from '2'(borrower) at final maturity T if there has been no default of '2'.

The difference between the correct bilateral formula and the simplified one is, under risk free closeout,

$$LGD_2 P(0, T)\mathbb{Q}(\tau_0 < \tau_2 < T).$$

The case with substitution closeout is instead trivial and the difference is null. For a bond, the simplified formula coincides with the full substitution closeout formula.

Therefore the difference above is the same difference between risk free closeout and substitution closeout formulas, and has been examined earlier, also in terms of contagion.

A simplified formula without τ^{1st}: an Equity forward

In this case the payoff at maturity time T is given by $S_T - K$,

where S_T is the price of the underlying equity at time T

and K the strike price of the forward contract (typically $K = S_0$, 'at the money', or $K = S_0/P(0,T)$, 'at the money forward').

We compute the difference D^{02} between the correct bilateral risk free closeout formula and the simplified one.

A simplified formula without τ^{1st}: The case of an Equity forward

$D^{02} := A_1 - A_2$, where

$$A_1 = E_0\left\{1_{\{\tau_0<\tau_2<T\}} LGD_2 D(0,\tau_2)(S_{\tau_2} - P(\tau_2,T)K)^+\right\}$$
$$A_2 = E_0\left\{1_{\{\tau_2<\tau_0<T\}} LGD_0 D(0,\tau_0)(P(\tau_0,T)K - S_{\tau_0})^+\right\}$$

The worst cases will be the ones where the terms A_1 and A_2 do not compensate. For example assume there is a high probability that $\tau_0 < \tau_2$ and that the forward contract is deep in the money. In such case A_1 will be large and A_2 will be small.

Similarly, a case where $\tau_2 < \tau_0$ is very likely and where the forward is deep out of the money will lead to a large A_2 and to a small A_1.

However, we show with a numerical example that even when the forward is at the money the difference can be relevant. For more details see Brigo, Buescu and Morini (2011).

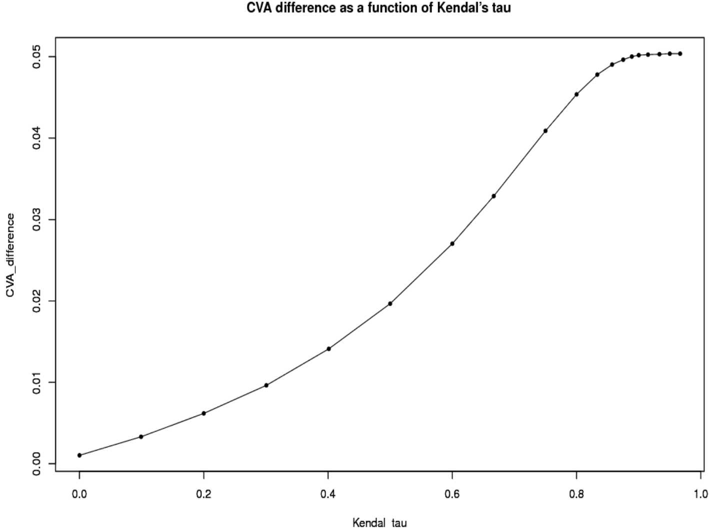


Figure : D^{02} plotted against Kendall's tau between τ_0 and τ_2, all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_0 = 0.1$, $\lambda_2 = 0.05$.

Basel III and CVA capital requirements

"Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults." Basel Committee on Banking Supervision, BIS (2011). Press release available at *http://www.bis.org/press/p110601.pdf*

Given the above situation, Basel III is imposing very severe capital requirements for CVA (*http://www.bis.org/publ/bcbs189.pdf*).

This may lead to forms of securitization of CVA such as margin lending on the whole exposure or on tranches of the exposure.

Basel III and CVA securitization

Such ”securitization of CVA” would be very difficult to model and to manage, requiring a global valuation perspective.

Few papers have appeared in the literature that are attempting to address CVA securitization, see for example Albanese et al (2011).

The problem with the traditional upfront charge for unilateral CVA is that it leaves CVA volatility with the investor and not with the risky counterparty.

In the unilateral case, the investor charges an upfront for CVA to the counterparty and then implements a hedging strategy. The investor is thus exposed to CVA mark to market volatility in the future.

Fully collateralised deals - prohibitive?

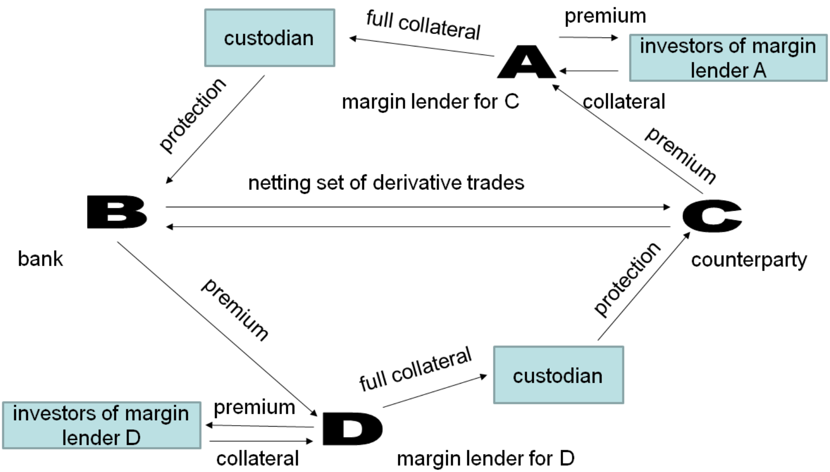
Alternatively the investor may request collateral from the counterparty, but not all counterparties are able to regularly post collateral, and this can be rather punitive for some corporate counterparties.

See recent example on Lufthansa from Risk magazine:

> *The airline's Cologne-based head of finance, Roland Kern, expects its earnings to become more volatile not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions".*

Margin lending is a possible solution this problem.

Margin Lending - Albanese, Brigo and Oertel (2011) I



Margin Lending - Albanese, Brigo and Oertel (2011) II

Traditionally, the CVA is typically charged by the structuring bank *B* (investor) either on an upfront basis or it is built into the structure as a fixed coupon stream.

Margin lending instead is predicated on the notion of floating rate CVA.

Assume we are in a bi-partite transaction between the default-free bank/investor B and the defaultable counterparty (say a corporate client) C. The bank may require a CVA payment at time 0 for protection on the exposure up to 6 months. Then in 6 months the bank will require a CVA payment for protection for further six months, prevailing at that time, on what will be the exposure up to one year, and on and on, up to the final maturity.

Such a CVA would be a Floating rate CVA.

Margin Lending - Albanese, Brigo and Oertel (2011) III

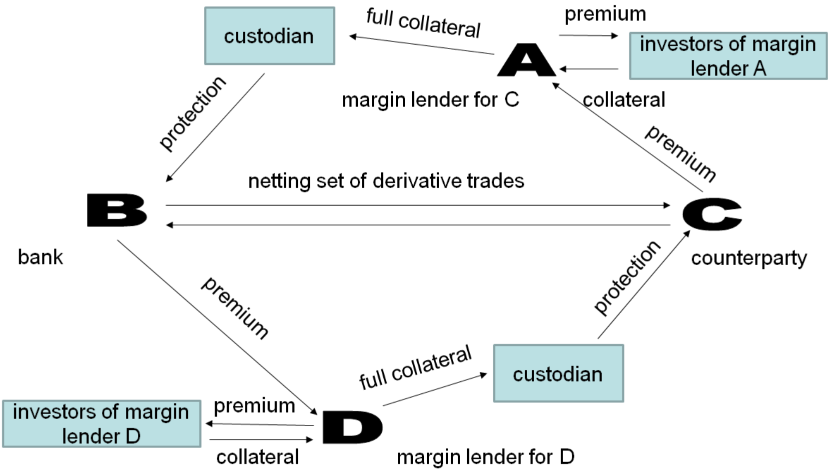
Margin lending is designed in such a way as to transfer:

- the conditional credit spread volatility risk and
- the mark-to-market volatility risk

or in other terms CVA volatility, from the bank to the counterparties.

We may explain this more in detail by following the arrows in the Figure.

Margin Lending - Albanese, Brigo and Oertel (2011) IV



Margin Lending - Albanese, Brigo and Oertel (2011) V

To avoid posting collateral, C enters into a margin lending transaction. C pays periodically (say semi-annually) a floating rate CVA to margin lender A ('premium' arrow connecting C to A), which A pays to investors (premium arrow connecting A to Investors). This latest payment can have a seniority structure similar to that of a cash CDO.

In exchange, for six months the investors provide A with daily collateral posting ('collateral' arrow connecting Investors to A) and A passes the collateral to a custodian ('collateral' arrow connecting A to the custodian).

This collateral need not be cash, but it can be in the form of hypothecs

If C defaults within the semi-annual period, the collateral is paid to B to provide protection ('protection' arrow connecting the custodian to B) and the loss in taken by the Investors who provided the collateral.

Margin Lending - Albanese, Brigo and Oertel (2011) VI

At the end of the six months period, the margin lender may decide whether to continue with the deal or to back off. With this mechanism C is bearing the CVA volatility risk, whereas B is not exposed to CVA volatility risk, which is the opposite of what happens with traditional upfront CVA charges.

Albanese, Brigo and Oertel (2011) argue that whenever an entity's credit worsens, it receives a subsidy from its counterparties in the form of a DVA positive mark to market which can be monetized by the entity's bond holders only upon their own default.

Margin Lending - Albanese, Brigo and Oertel (2011) VII

Whenever an entity's credit improves instead, it is effectively taxed as its DVA depreciates.

Wealth is thus transferred from the equity holders of successful companies to the bond holders of failing ones, the transfer being mediated by banks acting as financial intermediaries and implementing the traditional CVA/DVA mechanics.

Rewarding failing firms with a cash subsidy may be a practice of debatable merit as it skews competition. But rewarding failing firms with a DVA benefit is without question suboptimal from an economic standpoint: the DVA benefit they receive is paid in cash from their counterparties but, once received in this form, it cannot be invested and can only be monetized by bond holders upon default.

Margin Lending - Albanese, Brigo and Oertel (2011) VIII

Again, Albanese, Brigo and Oertel (2011) submit that margin lending structures may help reversing the macroeconomic effect by eliminating long term counterparty credit risk insurance and avoiding the wealth transfer that benefits the bond holders of defaulted entities.

There are a number of possible problems with this. First, proper valuation and hedging of this to the investor who are providing collateral to the lender is going to be tough. There is no satisfactory standard for even simple synthetic CDOs.

Admittedly this requires and effective global valuation framework, see for example the discussion in Albanese et al (2011).

Margin Lending - Albanese, Brigo and Oertel (2011) IX

The other problem is: what if all margin lenders pull off at some point due to a systemic crisis?

That would be a problem, indeed, but one may argue that the market is less likely to arrive in such a situation in the first place if the wrong incentives to defaulting firms are stopped and an opposite structure, such as the one in margin lending, is implemented.

There is also a penta-partite version including a clearing house. But there is much more work to do to assess this framework properly.

CVA Restructuring: Global Valuation? I

A fair valuation and risk management of CVA restructuring through margin lending requires a global model, in order to have consistency and sensible greeks.

But even when staying with traditional upfront CVA and DVA in large portfolios, as our examples above pointed out, different models are typically used in different asset classes. This can lead to models that are inconsistent with each other.

For example, an equity example uses a firm value model, whereas in the other asset classes we use reduced form models.

CVA Restructuring: Global Valuation? II

What if one has a portfolio with all asset classes together? And more generally, how does one ensure a consistent modeling framework that is needed to get meaningful prices and especially cross correlation sensitivities?

The problem is rather difficult and involves important computational resources and intelligent systems architecture.

Few papers have appeared in the literature that are attempting a global valuation framework, see for example Albanese et al (2010, 2011).

CVA Restructuring: Global Valuation? III

Delicate points include:

Modeling dependencies across defaults (we do not have even a good model for synthetic corporate CDO, base correlation still used there, see for example Brigo, Pallavicini and Torresetti (2010))

Modeling dependencies between defaults and each other asset class

Modeling dependencies between different asset classes

Properly including credit volatility with positive credit spreads

IT IS DIFFICULT!

Conclusions

- Bilateral CVA brings in symmetry, but also paradoxical statements
- Bilateral CVA requires a choice of closeout (risk free or substitution), and this is relevant.
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.
- Approximations ignoring first to default risk (sometimes used in the industry) do not work well.
- Basel III will make capital requirements rather severe
- CVA restructuring through margin lending and hypothecs is a possible alternative (requires a Consistent Global Valuation approach)
- This also holds for possible forms of CVA Securitization

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Easter Egg: Regulatory inconsistency on DVA

- Capital Requirements regulation:

NO DVA: Basel III, page 37, July 2011 release

"This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75."

- Accounting regulation:

YES DVA: FAS 157 (see also IAS 39)

"Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements"

Easter Egg 2: Basel III CVA risk capital charge

Basel III CCR changes:

- assume a larger correlation between SIFI's
- assume larger risk weights of CCR for banks using internal models (with EPE)
- additional Pillar I capital charge for the potential one year increase in CVA due to the widening counterparties' credit spreads (MtM CVA losses)

CVA Risk Capital charge calculation:

- advanced (internal VAR bond model treats CVA as a bond)
- standardized (simplified formula provided by regulator: paragraph 98 of *http://www.bis.org/publ/bcbs189.pdf*)

Easter Egg 3: Basel III on the choice of probability measure

"The first factor within the sum represents an approximation of the market implied marginal probability of a default occurring between times t_{i-1} and t_i. Market implied default probability (also known as risk neutral probability) represents the market price of buying protection against a default and is in general different from the real-world likelihood of a default." (in paragraph 98 of *http://www.bis.org/publ/bcbs189.pdf*)