Restructuring Counterparty Credit Risk

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–

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Counterparty Risk Frontiers: collateral damages
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This is work in progress. In particular, definitions, abbreviations and symbolic language in this work can be subject of change (ambiguity of terms in the literature).

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Framework

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- In this talk will adapt a "network view".
- Let $k \in \{0, 2\}$ be given and let $X = (X(t))_{0 \le t \le T}$ denote a stochastic process describing a (possibly vulnerable) cash flow between party 0 and party 2 (or a random sequence of prices). If, at time t, X_t is seen from the point of view of party k, we denote its value equivalently as $X_t(k)$ or $X_t(k; 2-k)$ or $X_k(t)$ - depending on its eligibility.

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- Moreover, we will make use of the important notation $Y_t(k \,|\, 2-k)$ to describe a cash flow Y from the point of view of party k at time t contingent on the default of party $2-k$.

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- Moreover, we will make use of the important notation $Y_t(k \,|\, 2-k)$ to describe a cash flow Y from the point of view of party k at time t contingent on the default of party $2-k$.
- Notice that the permutation $s : \{0, 2\} \longrightarrow \{0, 2\}, k \mapsto 2-k$ is bijective. It satisfies $s \circ s = s$. (Or put $s(k) := 3-k$ if s should permute the numbers 1 and 2 instead.)

The set of all bilateral CCR scenarios

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$$\Omega = N \,\dot{\cup}\, A_0^- \,\dot{\cup}\, A_2^- \,\dot{\cup}\, A_0 \,.$$

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$$\Omega = N \,\dot\cup\, A_0^- \,\dot\cup\, A_2^- \,\dot\cup\, A_0 \,.$$

In the following we assume that $\mathbb{Q}(A_0 = \emptyset) = 1$ (where $\mathbb{Q}$ denotes a "risk neutral measure" . . .).

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Definition

Let $X \equiv (X_t)_{t\in[0,T]}$ be an arbitrary stochastic process. X is called non-vulnerable if X and $\mathbb{1}_N X$ almost surely have the same sample paths, else X is called vulnerable.

Money Conservation Principle (MCP)

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Definition (Money Conservation Principle)

Let x be an arbitrary amount of money (which could be a negative number), measured in a single fixed currency unit $\mathbb{U}$ (e.g., $\mathbb{U}$: = €). Let $k \in \{0, 2\}$. TFAE:

- Party k receives x currency units $\mathbb{U}$ from party $2 - k$;
- Party $2 - k$ pays x currency units $\mathbb{U}$ to party k;
- Party $2 - k$ receives $-x$ currency units $\mathbb{U}$ from party k.

Thus, paying x currency units $\mathbb{U}$ is by definition equivalent to receiving $-x$ currency units $\mathbb{U}$ for all real money values x, implying that any non-vulnerable cash flow $X = (X_t)_{0 \leq t \leq T}$ between party k and its counterpart $2 - k$ satisfies $X_t(k) = -X_t(2 - k)$ for all $0 \leq t \leq T$. Hence, an asset for party k represents a liability for party $2 - k$.

Valuation of defaultable claims I

Defaultable claims can be valued by interpreting them as portfolios of claims between non-defaultable counterparties including the riskless claim and mutual default protection contracts. From the point of view of party k latter says:

Fix $k \in \{0, 2\}$. Party k sells to party $2 - k$ default protection on party $2 - k$ contingent to an amount specified by a close-out rule.

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Fix $k \in \{0, 2\}$. Party k sells to party $2-k$ default protection on party $2-k$ contingent to an amount specified by a close-out rule.

Let $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$ and let

- $M_t(k)$ be the mark-to-market value to party k in case both, party k and party $2-k$ are default-free;
- $CVA_t(k \,|\, 2-k)$ be the value of default protection that party k sells to party $2-k$ contingent on the default of party $2-k$.

Valuation of defaultable Claims II

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At t party k requires a payment of the "CCR risk premium" $CVA_t(k \,|\, 2-k) > 0$ from party $2-k$ to be compensated for the risk of a default of party $2-k$.

Valuation of defaultable Claims II

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At t party k requires a payment of the "CCR risk premium" $CVA_t(k \,|\, 2-k) > 0$ from party $2-k$ to be compensated for the risk of a default of party $2-k$. Conversely, party $2-k$ requires a payment of $CVA_t(2-k \,|\, k) > 0$ from party k to be compensated for the risk of a default of party k. Therefore, party $2-k$ reports at t the "bilaterally CCR-adjusted" value (defined as "fair value" in FAS 157):

$$V_t(2-k) := -CVA_t(2-k \,|\, k) + M_t(2-k) + CVA_t(k \,|\, 2-k)$$

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The inclusion of DVA began 2005. In September 2006 the accounting standard in relation to fair value measurements FAS 157 (*The Statements of Financial Accounting Standard, No 157*) asked banks to record a DVA entry (implying that the DVA of one party is the CVA of the other).

Valuation of defaultable Claims III

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FAS 157 namely says: "... *Because nonperformance risk includes the reporting entity's credit risk, the reporting entity should consider the effect of its credit risk (2-kredit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements*..."

Valuation of defaultable Claims III

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The European equivalent of FAS 157 is the fair value provision of IAS 39 which had been published by the International Accountancy Standards Board in 2005, showing similar wording with respect to the valuation of CCR.

So, we define

$$DVA_t(k) := DVA_t(k; 2-k) \quad := \quad CVA_t(2-k \,|\, k)\,. \tag{1}$$

Valuation of defaultable Claims IV

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Consequently,

$$V_t(2-k) \quad = \quad -CVA_t(2-k\,|\,k) + M_t(2-k) + CVA_t(k\,|\,2-k)$$

Valuation of defaultable Claims IV

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Consequently,

$$\begin{aligned} V_t(2-k) &= -CVA_t(2-k \mid k) + M_t(2-k) + CVA_t(k \mid 2-k) \\ &= M_t(2-k) + DVA_t(2-k; k) - CVA_t(2-k \mid k) \end{aligned}$$

Valuation of defaultable Claims IV

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Consequently,

$$\begin{aligned} V_t(2-k) &= -CVA_t(2-k\,|\,k) + M_t(2-k) + CVA_t(k\,|\,2-k) \\ &= M_t(2-k) + DVA_t(2-k;k) - CVA_t(2-k\,|\,k) \\ &= M_t(2-k) - BVA_t(2-k;k)\,, \end{aligned} \tag{2}$$

where

$$BVA_t(2-k;k) := CVA_t(2-k\,|\,k) - DVA_t(2-k;k) \overset{\checkmark}{=} -BVA_t(k;2-k)\,.$$

Valuation of defaultable Claims IV

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Consequently,

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\begin{aligned}
V_t(2-k) &= -CVA_t(2-k\,|\,k) + M_t(2-k) + CVA_t(k\,|\,2-k) \\
&= M_t(2-k) + DVA_t(2-k;k) - CVA_t(2-k\,|\,k) \\
&= M_t(2-k) - BVA_t(2-k;k)\,, \qquad (2)
\end{aligned}
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where

$$BVA_t(2-k;k) := CVA_t(2-k\,|\,k) - DVA_t(2-k;k) \overset{\checkmark}{=} -BVA_t(k;2-k)\,.$$

Similarly (due to the MCP and permutation):

$$V_t(k) \overset{(2)}{=} M_t(k) - BVA_t(k;2-k) \overset{(!)}{=} -V_t(2-k) \qquad (3)$$

for all $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$. Hence, both parties agree. $\Rightarrow$ More risky parties pay less risky parties in order to trade with them.

Valuation of defaultable Claims V

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Actually we have seen more: namely the following fact which completely ignores the construction/definition of DVA:

Observation

Assume that the MCP holds, and suppose that both parties include a possible future default of their respective counterpart. Then

$$V_t(k) := M_t(k) - \big(CVA_t(k\,|\,2-k) - CVA_t(2-k\,|\,k)\big) = -V_t(2-k)$$

for all $k \in \{0, 2\}$ *and* $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$*, implying that the MCP can be transferred to the vulnerable cash flow* V*.*

Implications of the traditional CVA/DVA mechanics I

Let us assume that party k is default-free (such as e. g. the French National Bank (hopefully...)). Then $CVA_t(2-k\,|\,k) = DVA_t(k; 2-k) = 0$ for all $0 \leq t < \tau_{2-k} \wedge T$. If however, party $2-k$ converges to its own default, $CVA_t(k\,|\,2-k) \uparrow \ldots$ if $t \longrightarrow \tau_{2-k}$. Consequently,

$$V_t(k) = M_t(k) - CVA_t(k\,|\,2-k) \downarrow \ldots \text{ if } t \longrightarrow \tau_{2-k}\,,$$

implying that the default-free party k would be strongly exposed to an increase of $CVA_t(k\,|\,2-k)$ - transferred from the risky party $2-k$ to the solvent party k.

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And what does the risky party report?

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And what does the risky party report?

$$V_t(2-k) = -V_t(k) = M_t(2-k) + DVA_t(2-k;k) \uparrow \ldots \text{ if } t \longrightarrow \tau_{2-k}\,!$$

Implications of the traditional CVA/DVA mechanics II

Saying it in other words again:

Whenever an entity's credit worsens, it receives a subsidy from its counterparties in the form of a DVA positive mark to market which can be monetised by the entity's bond holders only at their own default. Whenever an entity's credit improves instead, it is effectively taxed as its DVA depreciates.

Wealth is thus transferred from the equity holders of successful companies to the bond holders of failing ones, the transfer being mediated by banks acting as financial intermediaries and implementing the traditional CVA/DVA mechanics.

The role of partial information

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Fix $0 \leq t < u \leq T$. Let $\mathcal{F}_t$ denote the information of a specific investor at t, representing all observable market quantities but the default events or any factors that might be linked to credit ratings of the both parties, and let $\mathcal{G}_t$ represent the investor's enlarged information at time t, consisting of knowledge of the behaviour of market prices up to time t as well as (possible) default times until t. With respect to the information $\mathcal{F}_t$ defaults until t would arrive suddenly, as opposed to the case of the enlarged information $\mathcal{G}_t$.

In the following we assume that both filtrations coincide: $\mathbb{F} = \mathbb{G}$, and that both, τ_0 and τ_2 are $\mathbb{G}$-stopping times (cf. [2]).

Representation of the MtM M - FTAP I

Fix $k \in \{0, 2\}$ and let $0 \leq t < T$. Consider the random variable

$$\Pi_k^{(t,T]} := D(0,t)^{-1} \int_t^T D(0,u) d\Phi_k(u) \, ,$$

where Φ_k (viewed from party k) denotes a non-vulnerable cumulative dividend process of the portfolio over the time horizon $[0, T]$ and $D(0, \cdot)$ a continuous $\mathbb{F}$-adapted discount factor process (which both are assumed to be of finite variation).

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Representation of the MtM M - FTAP II

Seen from party k's point of view the CCR clean mark-to-market process $M_k = (M_k(t))_{0 \le t < T} \equiv (M_t(k))_{0 \le t < T}$ is then given by

$$M_k(t) = \mathbb{E}^{\mathbb{Q}}\left[\Pi_k^{(t,T]} \middle| \mathcal{F}_t\right] = \mathbb{E}^{\mathbb{Q}}\left[\int_t^T D(t,u)d\Phi_k(u) \middle| \mathcal{F}_t\right] = -M_{2-k}(t)\,,$$

where $\mathbb{Q}$ is a "risk-neutral measure" (due to the risk-neutral valuation formula). In the following we fix $\mathbb{Q}$ and omit its extra description in the notation of (conditional) expectation operators.

Another important piece of notation: For any stochastic process X we put $\widetilde{X}_t := D(0,t)X_t$ and obtain the discounted process $\widetilde{X}$ with numéraire $D(0,\cdot)$.

The main bilateral CCR building blocks

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$\Pi_k^{(t,u]} = -\Pi_{2-k}^{(t,u]}$

Random CCR clean cumulative cash flows from the claim in $(t, u]$, discounted to time t - seen from k's point of view

The main bilateral CCR building blocks

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$\Pi_k^{(t,u]} = -\Pi_{2-k}^{(t,u]}$	Random CCR clean cumulative cash flows from the claim in $(t,u]$, discounted to time t - seen from k's point of view
$M_k(t) = \mathbb{E}_{\mathbb{Q}}[\Pi_k^{(t,T]} \vert \mathcal{F}_t] = -M_{2-k}(t)$	Random NPV (or MtM) of $\Pi_k^{(t,u]}$ given as conditional expectation w.r.t. a risk neutral measure $\mathbb{Q}$, given the information $\mathcal{F}_t$ (cf. [2])

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$0 \leq R_k < 1$	k's (rdm.) recovery rate; i. e., the portion of the payoff from the MtM paid by party k to party $2-k$ in case of k's default

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$0 \leq R_k < 1$	k's (rdm.) recovery rate; i. e., the portion of the payoff from the MtM paid by party k to party $2-k$ in case of k's default
$0 < LGD_k := 1 - R_k \leq 1$	k's (random) Loss Given Default
$D(t,u) := D(0,u)/D(0,t)$	random discount factor at time t for time $u > t$

CCR free close-out I

So, how is $BVA_t(k; 2-k) = CVA_t(k \,|\, 2-k) - DVA_t(k; 2-k)$ actually determined? More precisely: what role do play ISDA's close-out rules here?

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Throughout this presentation, the letter ω describes a random "event" and Ω the set of all possible random "events". We consider functions of type $\mathbb{1}_A$, where $\mathbb{1}_A(\omega) := 1$ if $\omega \in A$ and $\mathbb{1}_A(\omega) := 0$ if $\omega \notin A$. The whole analysis of CCR is based on the functions $x^+ := \max\{x, 0\}$ and $x^- := x^+ - x = (-x)^+ = \max\{-x, 0\}$.

CCR free close-out I

So, how is $BVA_t(k;2-k) = CVA_t(k\,|\,2-k) - DVA_t(k;2-k)$ actually determined? More precisely: what role do play ISDA's close-out rules here?

Throughout this presentation, the letter ω describes a random "event" and Ω the set of all possible random "events". We consider functions of type $\mathbb{1}_A$, where $\mathbb{1}_A(\omega) := 1$ if $\omega \in A$ and $\mathbb{1}_A(\omega) := 0$ if $\omega \notin A$. The whole analysis of CCR is based on the functions $x^+ := \max\{x, 0\}$ and $x^- := x^+ - x = (-x)^+ = \max\{-x, 0\}$.

Fix $k \in \{0, 2\}$, and assume that party $2-k$ defaults first; i. e., $A_{2-k}^- \neq \emptyset$. Suppose that the close-out is settled at τ_{2-k} (no margin period of risk) and that no collateral is exchanged between party k and party $2-k$ until $\tau_{2-k} \wedge T$.

CCR free close-out II

Let $\omega \in A_{2-k}^{-}$. A CCR free close-out (in a given netting set) is reflected in the following table:

	$M_k(\tau_{2-k})(\omega) > 0$	$M_k(\tau_{2-k})(\omega) \leq 0$
Party k receives from party $2-k$	$R_{2-k}(\omega) \cdot M_k(\tau_{2-k})(\omega)$	0
Party k pays to party $2-k$	0	$-M_k(\tau_{2-k})(\omega)$

Hence, for all $k \in \{0, 2\}$ it follows that:

$$\begin{aligned} \mathbb{1}_{A_{2-k}^{-}} V_k(\tau_{2-k}) &= \mathbb{1}_{A_{2-k}^{-}} \left(R_{2-k}(M_k(\tau_{2-k}))^{+} - (-M_k(\tau_{2-k}))^{+}\right) \\ &= \mathbb{1}_{A_{2-k}^{-}} M_k(\tau_{2-k}) - \mathbb{1}_{A_{2-k}^{-}} LGD_{2-k}(M_k(\tau_{2-k}))^{+} . \end{aligned}$$

CCR free close-out II

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Let $\omega \in A^-_{2-k}$. A CCR free close-out (in a given netting set) is reflected in the following table:

	$M_k(\tau_{2-k})(\omega) > 0$	$M_k(\tau_{2-k})(\omega) \leq 0$
Party k receives from party $2-k$	$R_{2-k}(\omega) \cdot M_k(\tau_{2-k})(\omega)$	0
Party k pays to party $2-k$	0	$-M_k(\tau_{2-k})(\omega)$

Hence, for all $k \in \{0, 2\}$ it follows that:

$$\begin{aligned}
\mathbb{1}_{A^-_{2-k}} V_k(\tau_{2-k}) &= \mathbb{1}_{A^-_{2-k}} \left(R_{2-k}(M_k(\tau_{2-k}))^+ - (-M_k(\tau_{2-k}))^+ \right) \\
&= \mathbb{1}_{A^-_{2-k}} M_k(\tau_{2-k}) - \mathbb{1}_{A^-_{2-k}} LGD_{2-k}(M_k(\tau_{2-k}))^+ .
\end{aligned}$$

However, what about the inclusion of DVA?

ISDA's replacement close-out rule

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Let $\omega \in A_{2-k}^{-}$ and put $M_k^*(t) := M_k(t) + DVA_t(k; 2-k)$. According to *"ISDA's replacement close-out rule"* from 2009 (in a given netting set) we derive the following table:

	$M_k^*(\tau_{2-k})(\omega) > 0$	$M_k^*(\tau_{2-k})(\omega) \leq 0$
Party k receives from party $2-k$	$R_{2-k}(\omega) \cdot M_k^*(\tau_{2-k})(\omega)$	0
Party k pays to party $2-k$	0	$-M_k^*(\tau_{2-k})(\omega)$

Hence, for all $k \in \{0, 2\}$ it follows that:

$$\begin{aligned}\mathbb{1}_{A_{2-k}^{-}} V_k(\tau_{2-k}) &= \mathbb{1}_{A_{2-k}^{-}} \big(R_{2-k}(M_k^*(\tau_{2-k}))^+ - (-M_k^*(\tau_{2-k}))^+\big) \\ &= \mathbb{1}_{A_{2-k}^{-}} M_k^*(\tau_{2-k}) - \mathbb{1}_{A_{2-k}^{-}} LGD_{2-k}(M_k^*(\tau_{2-k}))^+ .\end{aligned}$$

An axiomatic approach to CVA I

Assume that *CCR free close-out* is applied to *both* parties 0 and 2. Put $\Delta_k := V_k - M_k$. Then

$$\mathbb{1}_{A^-_{2-k}} \Delta_k(\tau_{2-k}) = -\mathbb{1}_{A^-_{2-k}} LGD_{2-k}(M_k(\tau_{2-k}))^+ \, . \tag{4}$$

Let us further assume that

$$-BVA_{\tau_{2-k}}(k; 2-k) = \Delta_{\tau_{2-k}}(k) \equiv \Delta_k(\tau_{2-k}) \text{ on } A^-_{2-k} \tag{5}$$

and that $\widetilde{BVA}_\bullet(k; 2-k)$ is a càdlàg UI-$\mathbb{F}$-martingale.

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and that $\widetilde{BVA}_\bullet(k; 2-k)$ is a càdlàg UI-$\mathbb{F}$-martingale.

Let $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$. Since τ_{2-k} is $\mathbb{F}$-measurable, equation (3) and an application of the Optional Sampling Theorem implies

$$\begin{aligned} \mathbb{1}_{A^-_{2-k}} \widetilde{\Delta}_t(k) &\overset{(3)}{=} -\mathbb{1}_{A^-_{2-k}} \widetilde{BVA}_t(k; 2-k) \overset{(5)}{=} \mathbb{E}_t\left[\mathbb{1}_{A^-_{2-k}} \widetilde{\Delta}_{\tau_{2-k}}(k)\right] \\ &\overset{(4)}{=} -\mathbb{E}_t\left[\mathbb{1}_{A^-_{2-k}} LGD_{2-k}(\widetilde{M}_k(\tau_{2-k}))^+\right] . \end{aligned} \tag{6}$$

An axiomatic approach to CVA II

Observe that

$$\widetilde{\Delta}_t(k) \overset{(3)}{=} -\widetilde{BVA}_t(k; 2-k) = \widetilde{BVA}_t(2-k; k) \overset{(3)}{=} -\widetilde{\Delta}_t(2-k)\,.$$

Consequently (since equation (7) holds for all $k \in \{0, 2\}$), we obtain

$$\begin{aligned}
\widetilde{\Delta}_t(k) &= \mathbb{1}_{A_k^-}\widetilde{\Delta}_t(k) + \mathbb{1}_{A_{2-k}^-}\widetilde{\Delta}_t(k) + \mathbb{1}_N\widetilde{\Delta}_t(k) \\
&= -\mathbb{1}_{A_k^-}\widetilde{\Delta}_t(2-k) + \mathbb{1}_{A_{2-k}^-}\widetilde{\Delta}_t(k) + \mathbb{1}_N\widetilde{\Delta}_t(k) \\
&= \mathbb{E}_t\left[\mathbb{1}_{A_k^-} LGD_k(\widetilde{M}_{2-k}(\tau_k))^+\right] - \mathbb{E}_t\left[\mathbb{1}_{A_{2-k}^-} LGD_{2-k}(\widetilde{M}_k(\tau_{2-k}))^+\right] \\
&+ \mathbb{1}_N\widetilde{\Delta}_t(k)\,.
\end{aligned}$$

Next, let us assume that $\widetilde{\Delta}_t(k) = 0$ on N (a very “reasonable” assumption since neither party k nor party $2-k$ will default before T). Really?

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Assumption (B-Zero)

$$\mathbb{1}_N \widetilde{BVA}_t(k; 2-k) = 0$$

for all $k \in \{0, 2\}$ *and for all* $0 < t < \tau_0 \wedge \tau_2 \wedge T$.

An axiomatic approach to CVA III

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Assumption (B-Zero)

$$\mathbb{1}_N \widetilde{BVA}_t(k; 2-k) = 0$$

for all $k \in \{0, 2\}$ *and for all* $0 < t < \tau_0 \wedge \tau_2 \wedge T$.

(*i. e.,*

$$\mathbb{1}_N(\omega) \widetilde{BVA}_t(k; 2-k)(\omega) = 0$$

for all $(\omega, t) \in \Omega \times [0, T]$ *satisfying* $t < \tau_0(\omega) \wedge \tau_2(\omega)$).

Vulnerable cash flows I

Now, keeping the latter assumption in mind, let us revisit Brigo-Capponi's construction of the following vulnerable cash flow (which actually is an "existence result" - cf. [4]):

$$\widehat{\Pi}_k^{(t,T]} := \mathbb{1}_N \cdot \Pi_k^{(t,T]} + \mathbb{1}_{A_{2-k}^-} \cdot \Pi_k^{(2-k)} + \mathbb{1}_{A_k^-} \cdot \Pi_k^{(k)}\,, \qquad (7)$$

where the 2×2 random matrix $(\Pi_k^{(l)})_{l,k \in \{0,2\}}$ is given by

$$\Pi_k^{(l)} := \Pi_k^{(t,\tau_l]} + (-1)^{\frac{k+l}{2}} D(0,t)^{-1} \Big(LGD_l \cdot (\widetilde{M}_l(\tau_l))^- + \widetilde{M}_l(\tau_l) \Big)$$

for all $l \in \{k, 2-k\}$.

Vulnerable cash flows II

Studying carefully the proof of Brigo and Capponi one can see that in fact

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_k^{(t,T]}|\mathcal{F}_t\big] &\overset{(!)}{=} M_t(k) \\ &- D(0,t)^{-1}\mathbb{E}_{\mathbb{Q}}\Big[\mathbb{1}_{A^-_{2-k}} LGD_{2-k}(\widetilde{M}_k(\tau_{2-k}))^+\big|\mathcal{F}_t\Big] \\ &+ D(0,t)^{-1}\mathbb{E}_{\mathbb{Q}}\Big[\mathbb{1}_{A^-_k} LGD_k(\widetilde{M}_{2-k}(\tau_k))^+\big|\mathcal{F}_t\Big]\,.\end{aligned}$$

Hence, if we put $X_t(k) := \mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_k^{(t,T]}|\mathcal{F}_t\big]$ both, Brigo-Capponi's vulnerable cash flow representation above and our assumption lead to

$$\widetilde{X}_t(k) - \widetilde{M}_t(k) \overset{(!)}{=} (1 - \mathbb{1}_N)\widetilde{\Delta}_t(k) = \widetilde{\Delta}_t(k)\,.$$

Consequently, $\mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_k^{(t,T]}|\mathcal{F}_t\big] = X_t(k) \overset{(!)}{=} V_t(k)$.

FTDCVA, FTDDVA and FTDBVA I

BaFin

Put

$$
\begin{aligned}
\text{FTDCVA}_t(k \,|\, 2-k) &:= \mathbb{E}_t\left[\mathbb{1}_{A^-_{2-k}} LGD_{2-k} D(t,\tau_{2-k})(M_k(\tau_{2-k}))^+\right], \\
\text{FTDDVA}_t(k; 2-k) &:= \text{FTDCVA}_t(2-k \,|\, k), \\
\text{FTDBVA}_k(t;T) &:= \text{FTDCVA}_t(k \,|\, 2-k) - \text{FTDDVA}_t(k; 2-k)\,.
\end{aligned}
$$

FTDCVA, FTDDVA and FTDBVA II

Definition

Let $k = 0$ or $k = 2$ and $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$ ($\mathbb{Q}$ – a.s.).

(i) The positive $\mathcal{F}_t$-measurable random variable $\text{FTDCVA}_t(k \mid 2-k)$ is called First-to-Default Credit Valuation Adjustment at t.

(ii) The positive $\mathcal{F}_t$-measurable random variable $\text{FTDDVA}_t(k; 2-k) := \text{FTDCVA}_t(2-k \mid k)$ is called First-to-Default Debit Valuation Adjustment at t.

(iii) The real $\mathcal{F}_t$-measurable random variable $\text{FTDBVA}_t(k; 2-k) := \text{FTDCVA}_t(k \mid 2-k) - \text{FTDDVA}_t(k; 2-k)$ is called First-to-Default Bilateral Valuation Adjustment at t.

Brigo and Capponi revisited

Theorem (Brigo-Capponi (2008))

Assume No-Arbitrage and that each CCR clean contingent claim between party 0 *and party* 2 *of a given portfolio is attainable. Let* $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$ $(\mathbb{Q} -$ *a.s.*$)$*. Assume that the MCP holds. Let* $M_t(k)$ *denote the mark-to-market value of the portfolio to party* k *in case both,* 0 *and* 2 *are default-free. If both parties apply the CCR free close-out rule it follows that*

Brigo and Capponi revisited

Theorem (Brigo-Capponi (2008))

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$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]} | \mathcal{F}_t\right] = M_t(k) - \mathit{FTDBVA}_t(k; 2-k)$$

for all $k \in \{0, 2\}$.

Bilateral CCR risk premium vs FTDBVA

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Theorem

Let $k \in \{0, 2\}$. Assume No-Arbitrage and attainability of each CCR clean contingent claim between party 0 and party 2 of a given portfolio. Assume that the MCP holds. Let $0 \leq t < \tau_0 \wedge \tau_2 \wedge T$ and $V_t(k) := M_t(k) - B_t(k; 2-k)$, where

- *$M_t(k)$ denotes the mark-to-market value of the portfolio to party k in case both, 0 and 2 are default-free,*
- *$B_t(k; 2-k) = -B_t(2-k; k)$,*
- *$V_{\tau_{2-k}}(k) = M_{\tau_{2-k}}(k) - B_{\tau_{2-k}}(k; 2-k)$ on A^-_{2-k},*
- *$\widetilde{B}_\bullet(k; 2-k)$ is a càdlàg and uniformly integrable $\mathbb{F}$-martingale which satisfies condition (B-Zero).*

If both parties 0 and 2 apply the CCR free close-out rule, then $B_t(k; 2-k) = \text{FTDBVA}_t(k; 2-k)$ and $V_t(k) = \mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]} | \mathcal{F}_t\right]$.

UCVA$_t(k \,|\, 2-k)$ as a special case of FTDCVA$_t(k \,|\, 2-k)$

Special Case (A single default only $\rightsquigarrow$ Basel III)

Fix $k \in \{0, 2\}$. Assume that in addition $\tau_k = +\infty$ (i. e., no default of party k). Then $A^-_{2-k} = \{\tau_{2-k} \le T\}$ and $A^-_k = \emptyset$. Consequently, FTDDVA$_t(k; 2-k) = 0$,

$$\mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_k^{(t,T]} | \mathcal{F}_t\big] = M_t(k) - \textit{FTDCVA}_t(k \,|\, 2-k)\,,$$

$\text{UCVA}_t(k \,|\, 2-k)$ as a special case of $\text{FTDCVA}_t(k \,|\, 2-k)$

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$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]} | \mathcal{F}_t\right] = M_t(k) - \text{FTDCVA}_t(k \,|\, 2-k) \,,$$

and

$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_{2-k}^{(t,T]} | \mathcal{F}_t\right] = M_t(2-k) + \text{FTDDVA}_t(2-k; k) .$$

$\text{UCVA}_t(k \mid 2-k)$ as a special case of $\text{FTDCVA}_t(k \mid 2-k)$

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Fix $k \in \{0, 2\}$. Assume that in addition $\tau_k = +\infty$ (i. e., no default of party k). Then $A^-_{2-k} = \{\tau_{2-k} \leq T\}$ and $A^-_k = \emptyset$. Consequently, FTDDVA$_t(k; 2-k) = 0$,

$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]} | \mathcal{F}_t\right] = M_t(k) - \mathit{FTDCVA}_t(k \mid 2-k)\,,$$

and

$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_{2-k}^{(t,T]} | \mathcal{F}_t\right] = M_t(2-k) + \mathit{FTDDVA}_t(2-k; k).$$

Hence, if party k were the investor, and if $\tau_k = +\infty$ the Unilateral CVA UCVA$_t(k, 2-k) :=$ FTDCVA$_t(k \mid 2-k)$ would have to be paid by party $2-k$ to the default free party k at t to cover a potential default of party $2-k$ after t.

Structure of $\text{FTDCVA}_t(k \mid 2-k)$

Although we write "$\text{FTDCVA}_t(k \mid 2-k)$" it always should be kept in mind that we actually are working with a very complex object, namely:

$$\boxed{\text{FTDCVA}_k(t, T, LGD_{2-k}, \tau_k, \tau_{2-k}, D(t, \tau_{2-k}), M_k(\tau_{2-k}))} \quad !$$

UCVA and Basel III - Part I

Firstly we list a very restrictive case of a possible calculation of UCVA, encoded in the much too simple "CVA $=$ PD $*$ LGD $*$ EE" formula which however seems to be used often in financial institutes.

Proposition (Rough Approximation – Part I)

Let $k \in \{0, 2\}$. Assume that

(i) *party k will not default until T: $\tau_k := +\infty$;*

(ii) *LGD_{2-k} is constant and non-random;*

(iii) *$(\widetilde{M}_k(\tau_{2-k}))^+$ and τ_{2-k} are independent under $\mathbb{Q}$ (i. e., WWR or RWR is ignored completely).*

Then

$$UCVA_0(k \,|\, 2-k) = \mathbb{Q}(\tau_{2-k} \le T) \cdot LGD_{2-k} \cdot \mathbb{E}_{\mathbb{Q}}\left[(\widetilde{M}_k(\tau_{2-k}))^+\right].$$

UCVA and Basel III - Part II

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Suppose there exists a further random variable $\mathfrak{M}$ (a "market risk factor") so that $M_k(\tau_{2-k})$ is a function of $\mathfrak{M}$ as well, $M_k(\tau_{2-k}, \mathfrak{M})$ say.

Proposition (Rough Approximation – Part II)

Assume that

(i) *Party k will not default until T: $\tau_k := +\infty$;*

(ii) *LGD_{2-k} is constant and non-random;*

(iii) *For all t $D(0, t)$ does not depend on $\mathfrak{M}$;*

(iv) *$\mathfrak{M}$ and τ_{2-k} are independent under $\mathbb{Q}$.*

Then

$$UCVA_k(0 \mid T) = LGD_{2-k} \int_0^T D(0, t) \mathbb{E}_{\mathbb{Q}}\left[(M_k(t, \mathfrak{M}))^+\right] dF^{\mathbb{Q}}_{\tau_{2-k}}(t),$$

where $F^{\mathbb{Q}}_{\tau_{2-k}}(t) := \mathbb{Q}(\tau_{2-k} \leq t)$ for all $t \in \mathbb{R}$ (unconditional df).

Proof.

Put $\Phi(t,m) := \mathbb{1}_{[0,T]}(t)\cdot\psi(t,m)$, where $(t,m)^\top \in \mathbb{R}^+ \times \mathbb{R}$ and $\psi(t,m) := D(0,t)\cdot(M_k(t,m))^+$. Let $F^{\mathbb{Q}}_{(\tau_{2-k},\mathfrak{M})}$ denote the *bivariate* df of the random vector $(\tau_{2-k},\mathfrak{M})^\top$ w.r.t. $\mathbb{Q}$. Then

$$
\begin{aligned}
\text{UCVA}_0(k\,|\,2-k) &\overset{(i),(ii)}{=} LGD_{2-k}\mathbb{E}_{\mathbb{Q}}[\Phi(\tau_{2-k},\mathfrak{M})] \\
&= LGD_{2-k}\int_{\mathbb{R}^+\times\mathbb{R}} \Phi(t,m)dF^{\mathbb{Q}}_{(\tau_{2-k},\mathfrak{M})}(t,m) \\
&\overset{(iv),\text{Fubini}}{=} LGD_{2-k}\int_{[0,T]}\Big(\int_{\mathbb{R}}\psi(t,m)dF^{\mathbb{Q}}_{\mathfrak{M}}(m)\Big)dF^{\mathbb{Q}}_{\tau_{2-k}}(t) \\
&\overset{(iii)}{=} LGD_{2-k}\int_0^T D(0,t)\mathbb{E}_{\mathbb{Q}}[(M_k(t,\mathfrak{M}))^+]dF^{\mathbb{Q}}_{\tau_{2-k}}(t).
\end{aligned}
$$

□

Wrong-Way Risk and Right-Way Risk I

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$\mathrm{EE}_k^{(\mathfrak{M})}(t) := \mathbb{E}_{\mathbb{Q}}[(M_k(t,\mathfrak{M}))^+]$ is known as party k's Expected Exposure at t. In general it can be identified by MC simulation only.

Wrong-Way Risk and Right-Way Risk I

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$\text{EE}_k^{(\mathfrak{M})}(t) := \mathbb{E}_{\mathbb{Q}}[(M_k(t,\mathfrak{M}))^+]$ is known as party k's Expected Exposure at t. In general it can be identified by MC simulation only.

The situation where $\mathbb{Q}(\tau_{2-k} \leq t)$ is positively dependent on $\text{EE}_k^{(\mathfrak{M})}(t)$, is referred to as Wrong-Way Risk (WWR). In the case of WWR, there is a tendency for party $2-k$ to default when party k's exposure to party $2-k$ is relatively high. The situation where $\mathbb{Q}(\tau_{2-k} \leq t)$ is negatively dependent on $\text{EE}_k^{(\mathfrak{M})}(t)$ is referred to as Right-Way Risk (RWR). In the case of RWR, there is a tendency for party $2-k$ to default when party k's exposure to party $2-k$ is relatively low (cf. [5], [6]).

Wrong-Way Risk and Right-Way Risk II

A simple way to include WWR is to use the "alpha" multiplier α of Basel II to increase $\mathrm{EE}_k^{(\mathfrak{M})}(t)$ - under the tacid assumption of independence between $\mathrm{EE}_k^{(\mathfrak{M})}(t)$ and $\mathbb{Q}(\tau_{2-k} \leq t)$. The effect of α is to increase UCVA. Basel II sets $\alpha := 1.4$ or allows banks to use their own models, with $\alpha \geq 1.2$. This means that, at minimum, the UCVA has to be 20% higher than that one given in the case of the independence assumption. If a bank does not have its own model for WWR it has to be 40% higher. Estimates of α reported by banks range from 1.07 to 1.10 (cf. [6]).

Is the ISDA formula (Para 98) of Basel III true?

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Technical Remark

Regarding the calculation of $UCVA_0(k \,|\, 2-k)$ in Basel III (para 98), observe that the integral in the above Proposition in fact is a Lebesgue-Stieltjes integral. Hence, if $t \mapsto EE_k^{(\mathfrak{M})}(t)$ were not continuous (in time) and if it oscillated too strongly, that integral would not necessarily be a Riemann-Stieltjes integral, implying that we seemingly cannot simply approximate it numerically through a Riemann-Stieltjes sum of the type

$$UCVA_0(k \,|\, 2-k) \approx \sum_{i=1}^{n} D(0, t_i^*) \cdot EE_k^{(\mathfrak{M})}(t_i^*) \cdot \left(F_{\tau_{2-k}}^{\mathbb{Q}}(t_i) - F_{\tau_{2-k}}^{\mathbb{Q}}(t_{i-1})\right)$$

$$= \sum_{i=1}^{n} D(0, t_i^*) \cdot EE_k^{(\mathfrak{M})}(t_i^*) \cdot \mathbb{Q}(t_{i-1} < \tau_{2-k} \leq t_i), \quad (8)$$

Basel III UCVA slightly modified

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where $0 = t_0 < \ldots < t_n = T$ and $2t_i^* := t_{i-1} + t_i$. However:

Corollary

Assume that

(i) *The assumptions* (*i*), (*ii*) *and* (*iv*) *of the previous Proposition are satisfied;*

(ii) *For all* $i = 1, \ldots, n$, *for all* $t \in [t_{i-1}, t_i]$, $\mathbb{Q}(\tau_{2-k} > t) = \exp\big(-\lambda_{2-k}^{(i)} t\big)$, *where* $\lambda_{2-k}^{(i)} > 0$ *is a constant;*

(iii) *For all* $i = 1, \ldots, n$, *for all* $t \in [t_{i-1}, t_i]$, $r(t) \equiv r_i$ *is constant;*

(iv) LGD_{2-k} *is calibrated from a CDS curve with constant CDS spread* $s_{2-k}^{(i)}$ *on each* $[t_{i-1}, t_i]$.

Then $s_{2-k}^{(i)} = \lambda_{2-k}^{(i)} \cdot LGD_{2-k}$ (*"Credit Triangle"*), *and*

$$UCVA_0(k \,|\, 2-k) = \sum_{i=1}^{n} s_{2-k}^{(i)} \int_{t_{i-1}}^{t_i} e^{-r_i t} EE_k^{(\mathfrak{M})}(t) \exp\left(-\frac{s_{2-k}^{(i)} t}{LGD_{2-k}}\right) dt.$$

CVA risk in Basel III (Para 99)

Assuming both, the approximation (8) of Basel III and the "spread representation"

$$\mathbb{Q}(t^*_{i-1} < \tau_{2-k} \le t^*_i) = e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big) - e\big(s^{(i)}_{2-k}, t^*_i\big),$$

where $e(s,t) := \exp(-s \cdot t / LGD_{2-k})$, a Taylor series approximation of 2^{nd} order leads to the so called "CVA risk" of Basel III, i. e., to a delta/gamma approximation for $\text{UCVA}_0(k \,|\, 2-k)$, viewed as a function $f(s_{2-k})$ of the n-dimensional spread vector $s_{2-k} \equiv (s^{(1)}_{2-k}, \ldots s^{(n)}_{2-k})^\top$ only:

$$f(s_{2-k} + h) - f(s_{2-k}) \overset{(\|h\| \text{ small })}{\approx}$$

$$\sum_{i=1}^{n} D(0,t^*_i)\mathrm{EE}^{(\mathfrak{M})}_k(t^*_i)\, h_i\Big(t^*_i e\big(s^{(i)}_{2-k}, t^*_i\big) - t^*_{i-1} e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big)\Big) \; +$$

$$\frac{1}{2\,LGD_{2-k}} \sum_{i=1}^{n} D(0,t^*_i)\mathrm{EE}^{(\mathfrak{M})}_k(t^*_i)\, h_i^2\Big(t^{*2}_{i-1} e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big) - t^{*2}_i e\big(s^{(i)}_{2-k}, t^*_i\big)\Big)$$

CVA risk in Basel III: Flaws I

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An analysis of "CVA volatility risk" and its capitalisation should particularly treat the following serious flaws:

(i) CVA risk (and hedges) extend far beyond the risk of credit spread changes. It includes all risk factors that drive the underlying counterparty exposures as well as dependent interactions between counterparty exposures and the credit spreads of the counterparties (and their underyings). By solely focusing on credit spreads, the Basel III UCVA VaR and stressed VaR measures in its advanced approach for determining a CVA risk charge do not reflect the real risks that drive the P&L and earnings of institutes. Moreover, banks typically hedge these non-credit-spread risk factors. The Basel III capital calculation does not include these hedges.

CVA risk in Basel III: Flaws II

(ii) The non-negligible and non-trivial problem of a more realistic inclusion of WWR should be analysed deeply. In particular, the “alpha” multiplier $1.2 \leq \alpha$ should be revisited, and any unrealistic independence assumption should be strongly avoided.

(iii) Credit and market risks in UCVA are not different from the same risks, embedded in many other trading positions such as corporate bonds, CDSs, or equity derivatives. CVA risk can be seen as just another source of market risk. Consequently, it should be managed within the trading book. Basel III requires that the CVA risk charge is calculated on a stand alone basis, separated from the trading book. This seems to be an artificial segregation. A suitable approach would be to include UCVA and all of its hedges into the trading book capital calculation.

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Restructuring of CVA/DVA cash flows

Apart from FTDCVA the following approaches are subject of current research:

- Portable CVA (not a topic of this talk);
- Tripartite structures with one-sided collateralisation and margin lending;
- Quadripartite structures with two-sided collateralisation and margin lending;
- CCP structures with margin lending.

Margin lending I

Traditionally, the CVA is typically charged by the investing institute party k either on an upfront basis or it is built into the structure as a fixed coupon stream.

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The principle of "margin lending" instead builds on a "floating rate CVA". Its application would imply that the investing institute party k no longer is endangered by CVA volatility risk (i. e., by the credit spread volatility risk and the mark-to-market volatility risk of party k's risky counterparty). Latter would then be shifted from party k to the risky counterparty. Default risk instead would be forwarded in form of a "CVA volatility risk securitisation" to the investors who finance the margin lenders.

Margin lending II

For example let us firstly assume a bilateral (“bipartite”) transaction between a default-free investor party k and a defaultable counterparty $2-k$ (such as e. g. a corporate client). party k may require a CVA payment at time 0 for protection on the exposure up to 6 months. Then at the end of these 6 months party k will require another CVA payment regarding a protection for further 6 months, and so on - up to the final maturity of the trade. We would call such a CVA a “floating rate CVA”.

Margin lending II

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Now let us assume that the investing institute party k enters into derivative transactions with a counterp, and both evade the mutual counterparty credit risk by entering into "collateral revolvers" with liquidity providers A and D. To understand this mechanism let us take a look at the following picture.

Quadripartite structure with margin lending

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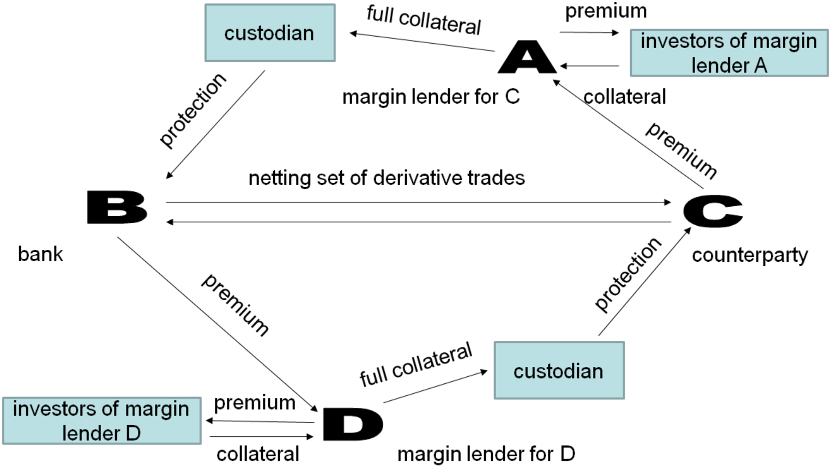

Margin lending III

To avoid posting collateral, party $2-k$ enters into a margin lending transaction. party $2-k$ pays periodically (say all 6 months) a floating rate CVA to a margin lender A ("premium" arrow connecting party $2-k$ to A) which A pays to investors ("premium" arrow connecting A to "investors of margin lender A").

Margin lending III

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In exchange, for 6 months the investors of A provide A with daily collateral posting ("collateral" arrow connecting "investors" to A) and A passes the collateral to a custodian ("collateral" arrow connecting A to the custodian). This collateral need not be cash, but it can be in the form of hypothecs.

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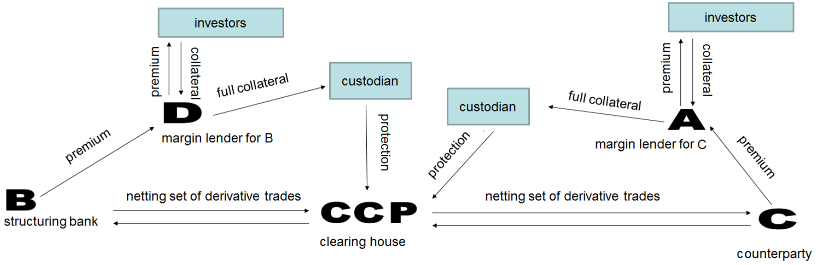
If party $2-k$ defaults within the 6 months-period, the collateral is paid to party k to provide protection (“protection” arrow connecting the custodian to party k) and the loss is taken by the investors of A who provided the collateral.

CCP structure with margin lending



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Are there any questions, comments or remarks?