School of HEC at University of Lausanne

Institute of Banking and Finance

Master of Science in Banking and Finance

Master Thesis

Capital Structure Arbitrage Strategies: Models, Practice and Empirical Evidence

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November 2003, Lausanne, Switzerland

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Acknowledgement

Special thanks are due to our supervisor Professor Salih Neftci, CUNY New York and HEC - University of Lausanne, who opened our eyes in finance during his very special and outstanding Financial Engineering course. Secondly, we would like to thank to our co-supervisors Dr. Norbert Dörr, Deutsche Bank AG London, who provided us with internal documents and helpful comments and Dr. Wilhelm Trinder, KPMG Frankfurt/Main, who supported us during our approach to find an interesting topic and reviewed the versions of the thesis. We would like to thank all our colleagues at the Master of Science in Banking and Finance (MBF) program at the Institute of Banking and Finance (IBF), School of HEC at University of Lausanne and the members of the Financial Asset Management and Engineering (FAME) program for there participation on our sometimes tiny problems. Especially we would like thank our secretaries Mena Jacquier and Sandrine Zaugg for their always warm welcome and friendly administrative support and help; the Director of IBF and of the MBF, Professor Michael Rockinger, who was also our professor in the Options & Futures course; the former MBF Director Professor Dider Cossin, now Professor at IMD, who taught and motivated us on how to apply credit derivatives during his very outstanding Applied Corporate Finance course. Amongst the many people who gave us the opportunity to discuss our ideas and provided valuable feedback or supported us with some helpful material we would like to thank immensely Dr. Alexander Passow, GOTTEX and FAME, Lausanne for his advise and help during all phases of the project, in particular for his insights in capital structure arbitrage strategies and for reading some parts of the thesis. Furthermore we would like to thank the members of AXA Investment Managers in Paris and London, Antoine Josserand, Olivier Lamotte, Jerome Vierling and in particular Guillaume Boulanger for providing us the data we needed for our project. Without this data we would never have been able to show the results we have in the empirical evidence part of the thesis. Furthermore, we would like to thank Dr. Lutz Schloegel, Lehman Brothers London, Dr. Günter Umlauf, KPMG London, Dr. Stefan Kremp, KPMG Frankfurt/Main, Ira Jersey, CSFB New York, Kirill Ilinski, JPMorgan New York, Joe Zou, Goldman Sachs New York and Ali Hirsa, Morgan Stanley New York. We also have to thank very gratefully our earlier mentors Professor Dr. Felix Ali Mehmeti, Valenciennes - France and Professor Dr.

Mirta Sataka Bugarin and Professor Dr. Francisco Cribari-Neto, Brasília - Brazil who taught us how to do scientific research. Last but not least, personal support from our partners and relatives was fantastic during the MBF program and during the last four month while doing this project.

Abstract

The objective of the thesis is the theoretical and practical background of capital structure arbitrage strategies and the empirical evidence of key relationships applying these strategies. Capital structure arbitrage involves taking long and short positions in different financial instruments of a company's capital structure, particularly between a company's debt and equity products. In general, capital structure arbitrage strategies can be viewed as an example of the interaction between market risk and credit risk, which often leads to an analysis of the relationship between the credit spreads and its proxy credit default swaps (CDS), the implied equity volatility skew, and the level of leverage. As an example the long-term relationship between France Telecom's CDS rates and volatility skew is analysed by means of cointegration tests. The results indicate that volatility skew and CDS rates are cointegrated over a three-year period. When a leverage indicator is used in a sub-sample the results are even more signficant than before.

Introduction

The objective of the thesis is the theoretical and practical background of capital structure arbitrage and hedging strategies, and the empirical evidence of key relationships applying these strategies. Capital structure arbitrage and hedging involves taking long and short positions in different instruments and asset classes of a company's capital structure, in particular between a company's debt and equity products. In general, capital structure arbitrage strategies can be viewed as an example for a the interaction between market risk and credit risk, which often leads to an analysis of the relationship between credit spreads and the implied equity volatility surface - so-called the volatility skew - or equity prices. Because of the lions share of the credit default swaps within the credit derivatives market and the general tremendous growth of this market during the last years, typical capital structure arbitrage strategies such as credit default swap (CDS) versus cash equities or equity options lead to the relationship between credit default swap rates and the volatility skew or equity prices. With the knowledge of these relationships and detailed information of the leverage cycles of firms, the implementation of a capital structure arbitrage strategy can be set as a convergence trade.

The thesis will be a first step in the relatively new area of capital structure arbitrage strategies with empirical analysis on the relationship between CDS rates, equity and equity options. The empirical result for a telecommunication sector firm, France Telecom, shows that there exists a long-run relationship between CDS rates and the corresponding equity volatility skew. This relationship, detected by cointegration tests, indicates that short-run dynamic deviations of these two variables (CDS rates and volatility skew) will revert to an equilibrium in a long-term; the variables are thus found to be cointegrated and the relationship between them is mean-reverting. When a proxy for the leverage is used the long-run relationship is even more significant, meaning that the relationship mean-reverts faster than before.

Therefore the thesis is organized in two parts, a descriptive part and an empirical analysis part. The completion of financial market information by using credit derivatives and, more specifically credit default swaps, as well as the consequences to the financial market from capital structure arbitrage strategies are discussed in the descriptive part of the thesis. The relation of the empirical results to the recent trading applications of capital structure arbitrage strategies are analysed and discussed in the empirical analysis part.

For a better understanding of the relatively new area of capital structure arbitrage strategies the descriptive part includes all relevant key facts on the bond pricing models based on equity price behavior, such as structural and reduced-form approaches. For example, to hedge the credit risk of convertible bonds with credit default swaps is also a favorite strategy with capital structure characteristics but is more known as classical convertible arbitrage. Merely the fact that in the near past the credit default swap market overcame the asset swap market in terms of liquidity brings this strategy also more in the light of capital structure arbitrage. Therfore an overview of convertibles is discussed as well in the descriptive part. Furthermore, before discussing in detail the market, the structures, valuation models and the implementation issues of credit default swaps, some basics about the important area of credit derivatives are presented in the descriptive part of the thesis. In the end of the descriptive part the main strategies and the market for capital structure arbitrage strategies is discussed.

The empirical analysis part includes the results of the cointegration analysis of the relationships described before. For these analysis the vector autoregression (VAR) and vector error-correction (VEC) methodologies and Granger causality as well other significant tests were used to show that the implied volatility skew is causing the CDS rates significantly in the long run especially in a leverage cycle of a firm. The descriptive statistics, discussion of the results and the conclusion is made in this part.

Capital structure arbitrage strategies currently are the fastest growing sector in the hedge fund market and in proprietary trading departments in large banks. For example, the number of hedge funds exploiting capital structure arbitrage strategies is expected to grow from 30 to approximately 200 funds until end of this year, according to Currie and Morris (2002). Additionally, Germany will open their financial market for hedge funds by a change in legislation for beginning 2004. This will also have an impact for the leading investment banks inside and outside Europe, which are highly active in such strategies, and therefore an increase in debt instruments versus equity instruments trading is expected.

One of the main reason is given by the fact that after the -world wide- equity bubble in 2000, investment-grades as well as high-yield default rates increased dramatically with respect to previous years. On the other side, the past months have been the best period for corporate bonds in at least 20 years, according to Zuckerman (2003). Trading debt versus equity of distressed companies has been highly attractive subsequently the bubble, because in times of widening credit spreads the correlation between movements in credit spread and the equity price increases, see Currie and Morris (2002). Another reason why investors are currently getting involved in capital structure arbitrage is related to the development of the credit default market.

By far the main users of credit derivatives are large banks, followed by securities firms and insurance companies. While banks and securities firms act both as sellers and buyers of protection, insurance companies, which have reportedly increased their market participation substantially in recent years, are primarily protection sellers, presumably using their expertise in evaluating and diversifying risk. Hedge funds are also relatively active participants, arbitraging perceived mis-pricing between the cash and the derivatives markets. Other participants include pension funds and mutual funds, but their participation in the market is very small. In terms of volume, according to the credit derivatives survey, Risk (2003), in the course of the year 2002 the credit derivatives market in terms of total notional amount of outstanding contracts has grown by more than 50% to $ 2,306 trillion and is expected to double until the end of 2004 up to $ 4.8 trillion, see also British Banker Association (BBA) forecasts.

The CDS capture nearly the half of the market share and become the dominant financial instrument in the credit derivatives market. CBOs and CDOs are also more and more used for balance sheet re-organisation. The risk replication of CDOs via portfolios of CDSs is one of the key transaction and one of the major reason why the CDS market remains more liquid than the markets for other credit derivative instruments. Furthermore, besides all these reasons and the fact that the CDS is the key instrument for completing market information, the CDS market is at the moment more liquid than the corresponding cash market for bonds and asset swaps, so that the CDS rate is the key information for the creditworthiness of companies and reflects the credit quality better than credit spreads in financial market.

Moreover, the results of Hull et al. (2003) and Zou (2003) provide new findings and alternatives for the relation between credit spreads and implied equity volatility surfaces, which we will also focus on throughout the thesis. Additionally, resent research interests as focused the sector such as the forthcoming works from Berd (2003) and Ilinski (2003) developing models to extract CDS rates from implied equity volatility surfaces. These models are based mainly on Merton approaches, but they use reduced form models or mixed versions as well, sometimes additional statistical models are also involved. None of these models is completely public at the moment but in the near future a lot of forthcoming papers will be produced by the industry and will be very interesting for academics as well.

Because of the overall development during the last years within the important area of capital structure arbitrage strategies the objective of the thesis is to do a first step in this new area. For the analysis of the relationship between CDS rates and the implied volatility skew we made use of powerful econometric techniques such as cointegration analysis, Granger causality test and VAR and VEC models.

Historically, Merton's model (1974) claims that the value of a company can be divided in an equity and a debt part. This model treats equity as an option on the firm's assets considering equity as a call option, the total liabilities as the strike price, and the value of the firm's assets as underlying. Merton takes the market value of a firm derived from the Black & Scholes formula. If the value of a company increases, the value of its equity increases. On the other hand, the probability of default declines, which results in a lower risk premium demanded from debtors. As a result, the value of debt increases as well. Both asset classes are therefore mutually correlated. A strategy to arbitrage could be to find undervalued bonds or stocks. Of the applications for arbitrage between equity and debt the Merton model has inspired the strategy of going short on stocks of high levered firms and long on their bonds, or taking the opportunity to profit from misalignment between CDS spreads and falling stocks. Alternatively a strategy could consist of comparing implied and historical volatilities between stock and bond markets. A corresponding sale of high implied volatility of one asset class could be hedged with the purchase of the lower volatility of the other asset class (trading across asset classes). As an alternative to bonds one could consider credit default swaps (CDS) which have become more liquid than the cash market particularly during the last 2 years after the equity bubble.

Recently, Lardy (2002) has shown a direct link between equity prices, implied volatilities and probabilities of default with a closed-formed formula and an uncertain default barrier. See also Finkelstein (2001). The commercial application of this issue is implemented in a web based tool called CreditGrades (www.creditgrades.com), see Finger et al (2002). Moodys KMV is an earlier commercial application, which is using the concept of distance to default. Arbitrage opportunities can be worked out by discrepancies between theoretical credit spreads and the traded spreads in the market. Furthermore, a similar arbitrage could be performed between different debt classes, such as senior and junior debt. A quite fashioned strategy in the past was convertible bond arbitrage. The typical strategy is to go long the convertible bond and short the equity. Convertible bonds rank as a more senior claim compare to ordinary equity in a company's balance sheet. In many cases the coupon, while less than the bond rate for that company, provides a yield higher than the equity dividend yield. The terms of convertibility from the bond into shares are usually fixed from the outset. If the share price rises then the convertible starts to behave like the equity. If the share price falls the convertible behaves like a corporate bond.

The plan of chapter 1 is the following: In Section 1.1 the thesis presents bond pricing models on equity price behavior based on structured models and reduced-form models. For the structured models initiated by Black and Scholes (1973) and Merton (1974) in this section we present the extensions during the last decades and a discussion of empirical evidence including newer works such as Collin-Dufrense (2000) and Goldstein (2001) and Gemmill (2002) for the analysis of credit spreads. In a recent development, Hull et al. (2003) provide a ranking ordering of creditworthiness via the credit spread implied by the Merton model. They use an analytical Merton framework to extract implied credit spreads from two implied volatilities as a measure for the implied volatility skew. To develop this framework they use a compound option pricing model introduced by Geske (1979). In section 1.1.1 a short description of this framework is presented including a corrected formula with detailed proves in appendix A. Also in section 1.1.1 some results from these framework are discussed and a extension for an analytical formula is shown in section 3.1.1, such that the implied put volatility is a function of the implied credit spread and the leverage of a company is shown with a proof in appendix A.5. In the following the main ideas of some commercial applications based on Merton model are presented in this section. Capital structure models use mainly Merton type models - so-called structured models - such as those provided by Moody's KMV and CreditGrades to work out discrepancies between a theoretical credit spread and the actual spread in the market. The basis of these models is mainly the assumption that buying a share in a company is equivalent to buying an option on the company's assets. The distance to default is the difference between the company's asset value and the value of its liabilities. The key thing is that the driver of the credit spread or the default probability of the debt is going to be the equity price of the firm. The impact in changes of these parameters is, however different for each security. In the end of section 1.1 a short discussion of reduced form models including empirical evidence is presented. In reduced-form models introduced by Jarrow and Turnbull (1995) the time to default is modelled as an exogenous variable via intensity processes that eliminate the need to have the default depending explicitly on the issuer's capital structure. Structured models give a more precise idea of the impact of true economic factors underlying the pricing of bonds, whereas reduced-form models can handle more complex structures in an easier way and can provide for better fitting of historical data, without always capturing the non-linear interdependence of the variables concerned.

In section 1.2 some key facts on convertible bonds (CBs) and their terminologies and capital structure characteristics are presented. Section 1.2 serves also with a little survey on pricing models for CBs and the risk management and arbitrage techniques of single CBs as well as portfolios of CBs. Convertible arbitrage is a classical strategy in trading & sales, but since the introduction of more exotic products such as credit default swaps and volatility swaps in the late 90's, these strategies comes more and more in the light of capital structure arbitrage strategies. Simply the reason to hedge the credit risk of a convertible bonds using credit default swaps has a capital structure characteristics such in strategies including CDS versus equity options or cash equities. Because of the fact that volatility changes of the underlying stock is frequently positively correlated with credit spreads and negatively correlated with equity prices volatility swaps can be also used to hedge credit-spread risk and to hedge against directional moves in the equity market. This is not a direct or complete hedge and can be seen more as a correlation hedge. However, if the correlation is not significant or not holding anymore a more preferable hedging strategy would be a basket of CDSs with similar characteristics.

Section 1.3 gives a brief overview of the credit derivative sector, where section 1.3.7 concentrates more specifically on CDSs. It is obvious that section 1.3 can only discuss some basic aspects of this important area. First a short introduction about the basics of credit derivatives is presented to clarify the definitions and terminologies used in the following sections and chapters. In the following, an overview of the major credit derivative instruments, their types, structures and applications is briefly presented. Furthermore, a brief overview of the credit derivative market and the use of credit derivatives is given. At the end of this section a comment on completing financial market information by using credit derivatives is presented.

Credit derivatives are a fundamental innovation for many important practical problems such as transfering credit risk. CDSs have become the dominant instrument in the credit derivatives market, and we include a fairly extensive discussion of this instrument within sections 1.3.7 to 1.3.9. Section 1.3.7 concentrates on CDS as they constitute the lions share of the credit derivatives market. Based on the terminologies and definitions made in section 1.3.2, section 1.3.7 will serve with a detailed definition of a CDS and an illustrative example to give a more realistic idea of how CDSs works. Furthermore, the key rule played by CDSs within the credit derivatives market is discussed to point out that the CDS is the main factor in completing financial markets. After clarifying these basic facts on CDSs section 1.3.8 is focusing fairly extensive on the valuation methodologies used for CDS pricing. After presenting an overview of the different methodologies and the involved model parameters including explanations and discussions, the replication method, a structural model and a reduced-form model is considered. The part of the replication method is illustrated with some well-known and important examples initiated by Duffie (1999). The presentation of a structural model by Das (1995) will more brief, where the reduced-from model will serve with some useful fundamental techniques within this methodology. The reduced-form models are most widely used in practice to price CDSs. The implementation of such models is discussed after presenting a recent literature survey for CDS pricing models. At the end of section 1.3

a summary of the main issues of the use of CDSs and their valuation approaches are given.

At the beginning of Chapter 2 the focus in section 2.1 is on the main capital structure arbitrage and hedging strategies. Here only the main strategies are presented including strategies of equity instruments against debt instruments, as well as strategies with equity and credit instruments as well as mixed strategies. Some types of basic debt versus equity trading are quite simple, but more interesting and classical examples are plain convertible or reverse convertible bond's delta-neutral hedging strategies with stocks or equity options. More advanced strategies in this area, more known as convertible aribitrage - however with capital structure characteristics - are the use of convertible bonds with embedded options such as call write or call for take-over protection or covered convertibles. Further well-known strategies in capital structure arbitrage strategies are those where equity instruments are traded against credit instruments. Here the main trading strategies are CDS versus common stocks or CDS versus equity options. These strategies can also be implemented as pure volatility arbitrage trade between the two asset classes where an implied equity volatility is extracted from CDS rates and traded against volatility in the equity market. For mixed strategies every possible trade can be done, but in this section we focus on basis trades which are also a key indicator to quantify arbitrage opportunities, meaning CDS versus the corresponding cash market instruments such as bonds and asset swaps (so called trading basis risk) . Here some applications of the replication strategies in section 1.3.8 are presented and much more important strategies to transfer credit risk such as convertibles versus CDS or asset swaps using convertible stripping and special purpose vehicles (SPV) are discussed.

Section 2.2 serves with a market description of capital structure arbitrage strategies and the players involved. As we show previously in chapter 1 and section 2.1 the major role in those strategies. Therefore we will focus only on CDS-related strategies. We want to make more clear the different rules of banks and hedge funds for such capital structure arbitrage strategies and what the consequences are for the financial markets, for example such that easy arbitrage opportunities will start disappearing faster and the volatility of profit and loss positions in financial institutions will fall.

Chapter 3 presents the empirical part of the thesis. The empirical evidence for the relationship between CDS, implied equity option volatilities and cash equity prices was analysed using cointegration analysis. The empirical analysis will try to assess if there is some long-run equilibrium between the CDS market and the equity options market and also to analyse the mechanism of price discovery between these markets for a specific company. In section 3.1, the rationale for the empirical analysis is outlined and explained. Section 3.1.1 shows a formula to estimate the implied put option volatility depending manly on the credit spread and leverage of a firm. In order to explain the econometrics behind the cointegration theory, section 3.2 presented a brief description of stationary and non-stationary stochastic processes, VAR models, cointegration and error correction model. The data set used was described in section 3.3 and section 3.4 presents the description of the applied tests and the obtained results.

It is not possible to present all details about credit derivatives pricing as in Schönbucher (2003) or practical issues on credit derivatives as presented in JPM (1999), Das (1998, 2001), O'Kane (2001), Tavakoli (2001), and Bowler and Tierney (2000). More detailed information on convertible bonds trading is presented in Calamos (2003). The objective of the thesis was to discuss the main strategies and the needs for capital structure arbitrage. A complete discusssion of the full range of trading strategies including mixed and hybrid instruments including as well as new models which are not completely available now can easily fill an entire book. It is interesting that there does not exist enough public material about capital structure arbitrage strategies such as in the area of credit derivatives at the moment. In fact, the presentation in chapter 2 is based on a large mix of internal documents and interviews with practitioners. The objective in the descriptive part of the thesis was to work out the main key facts in these different areas including practical issues which are relevant for capital structure arbitrage strategies, such as CDS against equity options or cash equitie, which is the most common form. CDSs are relatively new in the literature and play the key rule in capital structure arbitrage strategies, thus we discussed this area fairly deep. Therefore, sections 1.3.7 to 1.3.9 can be seen as a primer for CDS but can not replace standard works such as Tavakoli (2001), Das (1998, 2001), Schönbucher (2003), Cossin and Pirotte (2001), JPM (1999), O'Kane (2001), Bowler and Tierney (2000) or Moore and Watts (2003) which serves with a recent market guide for CDS. For the concepts and the mathematics behind the most important and popular credit risk models see Schönbucher (2003).

This thesis is a contribution to the empirical literature on credit derivatives. In fact, the approach presented here is innovative in terms of variables used such as CDS rates and volatility skew and the choise of the econometric tests applied. Although Hull et al. (2003) used similar variables in their work, the applied econometric test are different. In that work, rank order correlation measures such as Kendall and Spearman measure were used. Another similar work is Skinner and Townend (2002). The authors interpret CDS as put options and regress CDS prices on factors that should influence their price in this framework. Again, this thesis differs from that work in terms of the analysed variables and the method used. Campbell and Taskler (2002) explores the effect of equity volatility on corporate bond yields.

Blanco et al. (2003) is probably the first paper to examine CDS rates in a time series framework. This work has a significant resemblance with this thesis. The authors analysed three main issues. First, whether bond and CDS markets price default risk equally. Second, where credit risk price discovery takes place predominantly. Third, what are the factors are that influence short-run changes in CDS prices and credit spreads. This thesis works out the second and third issues of Blanco et al (2003). The first issue could not be addressed because the available data was not sufficient. The second issue is adressed partially in that the Gonzalo-Granger measure is reported here. The third issue is addressed in the same fashion employing cointegration test, which in our work were based on the variables CDS rates and volatility skew, whereas in the work of Blanco et al. (2003) were based on several variables: the change in long-term interest rate, equity market returns, firm-specific equity returns, change in market volatility, firm-specific volatility, and change in slope of yield curve. It will be shown that a significant relationship between the CDS rates and the corresponding volatility skew exists.

Notation

Part I :

A	protection buyer
A_t	asset value at time t
a	value of default-free annuity in discrete time
B_t	debt value at time t
B	protection seller
$B(t,T)$	default-free zero coupon bond value at time t
$B^d(t,T)$	defaultable zero coupon bond value at time t
C	reference credit
$C(t)$	default-free fixed coupon bond value at time t
$\tilde{C}(t)$	default-free floating coupon bond value at time t
$C^d(t)$	defaultable fixed coupon bond value at time t
$\tilde{C}^d(t)$	defaultable floating coupon bond value at time t
c	Coupon for a default-free fixed coupon bond
c^d	Coupon for a defaultable fixed coupon bond
CB	convertible bond
CBO	collateralised bond obligation
CDO	collateralised debt obligations
CDS	credit default swap
CLN	credit linked note
CLO	collateralised loan obligation
CO	call option premium
DD	distance to default
DDS	digital default swap
E_t	equity or stock value at time t
F	face value of debt
$F(t)$	default-free FRN value at time t
$F^d(t)$	defaultable FRN value
FTDS	first to default swap

J	Cox or time-inhomogeneous Poisson process
K	strike level
L	LIBOR (London inter bank offering rate)
l	leverage of a firm
M	cumulative bivariate standard normal distribution function
N	cumulative standard normal distribution function
N'	derivative of N
P	survival probability
P^{dirty}	dirty price
PD	default probability
PO	put option premium
PV	present value
q	continuous dividend yield
R	recovery rate
r	instantaneous risk-free rate, continuously compounded
s	swap spread
s^{asset}	asset swap spread
s^c	credit spread
s^{cds}	CDS rate
s^{frn}	FRN spread
s^{par}	par spread of defaultable coupon bonds
T	maturity or expiration time
TRS	total return swap
α	scalar
$\alpha_{t,T}$	annuity or numeraire in continuous time
$\beta_{t,T}$	risk free discount factor in continuous time
Γ	option gamma
Δ	option delta
Θ	option theta
κ	option moneyness

λ	intensity or hazard rate
μ	Black-Scholes drift
ν	implied equity put volatility
O	omicron
ϱ	quasi-debt ratio (leverage measure)
σ	asset volatility
σ_E	equity volatility
τ	time to default
Φ	present value of the premium leg of a CDS
Ψ	present value of the protection leg of a CDS

Part II:

$\vec{\beta}$	cointegration vector
GG	Gonzalo and Granger measure
ε_t	white noise process
L	lag operator
θ	moving average coefficient
μ	statistical drift term or mean
σ^2	Variance
γ	speed-of-adjustment coefficient
ϕ	autoregressive coefficient
$\vec{\phi}, \vec{\varphi}$	short-run dynamics among markets
H_0	null hypothesis
y_t	time series

Part I

Descriptive Part

Chapter 1

Theoretical and Practical Background

In this section, the theoretical background of risky debt valuation based on the Contingent Claims Analysis (CCA) is presented. CCA started with the seminal work of Black and Scholes (1973) and later refined by Merton (1974, 1977). Their famous option pricing model was extended to valuing corporate liabilities. The related approach called "reduced-form" firstly introduced by Jarrow and Turnbull (1995) is analysed here as well. We present also key facts about convertible bonds including a brief survey on pricing models and the coresponding hedging and arbitrage techniques. The descriptive part will serve with basics about credit derivatives, their main structures, markets and applications. This includes an discussion on how credit dreivatives can complete financial market information. After this we focus more on credit default swaps, the leading credit derivatives intrument and present here the valuation methodologies, such as replication approaches, a structural and a reduced form model. Within this part a Litrature review for credit default swap pricing models including implementation issues is given as well. Based on the knowledge of the these securitie classes, such as credit default swaps, equity derivatives and convertible bonds, which form the building block for the capital structure trading strategies, the main strategies and the market of them are discussed. The discussion includes the rule of large banks and hedge funds as well as the impact of capital structure arbitrage strategies to the financial market.

1.1 Bond Pricing Models based on Equity Price Behavior

The link between the values of equity and debt of a firm has many important applications, such as:

- Trading between shares and debt of a given company (or between equity options and credit derivatives);

- Assessment of the probability of default of a given firm based upon its assets and level of debt;

- Capital structure efficient frontier optimisation;

The probability of default (or credit risk) of a company based on its fundamental (balance sheet) analysis tends to be the focus of most models of risky debt valuation. The classical version of this valuation, known as Contingent Claim Analysis or Merton's model, is due to Black and Scholes (1973) and Merton (1974, 1977). Essentially, default is assumed to occur when the firm's market value of assets falls below the outstanding firm's debt. This model is termed also structural model because it relies upon balance sheet information, such as the capital structure, and based on that the probability of default is derived endogenously. A second approach, called by Duffie and Singleton (1999) "reduced-form", models the time of default exogenously, eliminating the dependency on the firm's capital structure. Reduced-form models assume the probability of default as a perfectly unpredictable event.

1.1.1 Structural Models

The key insight of Black and Scholes (1973) and Merton (1974) is that both equity and debt can be regarded as derivative securities on the value of the firm's assets. The model makes it possible to analyse and measure the impact on credit risk spreads of a change in the parameters of an option, such as volatility of the underlying, volatility of interest rates, etc. The method points out that equity is considered a call option on the market value of the firm's total assets with a strike price equal to the book value[1] of the firm's debt. The value of corporate debt can be determined by solving for the equity value and using the accounting identity, expressed in market terms, *total firm assets = total debt + total equity*. Debt could also be considered as a put option on the market value of the firm's total assets with a strike price equal to the book value of the firm's debt. We could also express debt as a combination of a default-free loan and a barrier put option implicitly sold to the firm.

Some assumptions are required to derive Merton's model and its simple methodology. The objective is to have a pricing formula of a straight bond issued by a defaultable firm for a given period of time.

> **Assumption 1**: There are no transactions costs or taxes. Assets are perfectly divisible and are traded continuously. Nor are there short-selling restrictions. Borrowing rates are equal to lending rates.

[1] The book value is the current value of an asset as it appears on the balance sheet.

Assumption 2: There are sufficiently many investors with comparable level of wealth such that they can buy or sell as much as they want at a given market price.

Assumption 3: The instantaneous risk-free rate, r, is known and constant over time. This implies a flat term structure of risk-free interest rate and the value of a riskl-free zero-coupon bond paying \$1 at time T will be $B(0,T) = exp[-rT]$.

Assumption 4: The value of the assets of the firm, A, follows a diffusion-type stochastic process with stochastic differential equation

$$\frac{dA}{A} = (\mu - C)\,dt + \sigma dz, \tag{1.1}$$

where μ is the instantaneous expected rate of return per time, C is the total cash outflow by the firm per unit time, σ the volatility of the return on the underlying firm's assets per unit time, and dz is a standard Wiener process.

Assumption 5: The Modigliani-Miller theorem holds in the sense that the value of the firm is invariant to its capital structure. The total value of the firm is financed by equity, E_t, and a zero-coupon noncallable debt, B_t, maturing at time T with face value F,

$$A_t = B_t + E_t \tag{1.2}$$

where the subscription t means per unit of time.

Assumption 6: Debt holders receive whatever asset value remains in the event of default and default can occur only at the maturity of the debt.

Assumption 7: Bankruptcy protection: Firms cannot file for bankruptcy except when they cannot make required cash payments. Perfect priority rules govern distribution of assets to claimants at the time of liquidation.

Assumption 8: Dilution protection: Unless all existing non-equity claims are eliminated, no new securities other than additional common equity can be issued. Equity holders cannot negotiate arrangements on the side with subsets of other claimants.

Assumption 9: Perfect liquidity: Firms can sell assets as necessary to make cash payouts with no loss in total value.

Consider a firm with one class of equity, one class of zero-coupon debt with face value, F, and no dividend payments. Using the assumption 6, the value of the bond at maturity is

$$B_T(A, T) = \min(A_T, F). \tag{1.3}$$

The value of the equity is given by

$$E_T(A, T) = \max(0, A_T - F). \tag{1.4}$$

The payoff of the equity is exactly the payoff of a European call option on the firm's value; the payoff of the bond is either its face value or whatever is left of the firm's value A if it is below the face value of the debt. The payoff of debt value is showed in Figure 1.1 and the payoff of equity value is shows is Figure 1.2.

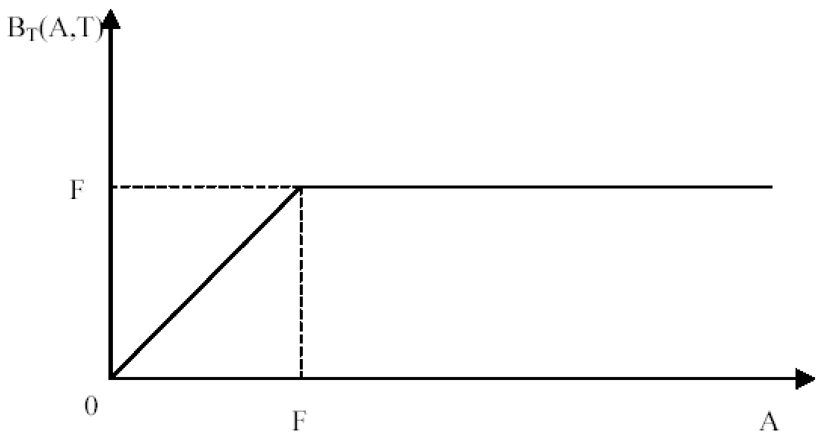

Figure 1.1: Debt value at maturity as a function of the assets value of the firm

Alternatively, the debt holders are said to have lent money without risk with face value F and to have written (sold) a put option on the assets of the firm with an exercise price of F. To see this, rewrite equation 1.3 as

$$\min(A_T, F) = F - \max(0, F - A_T). \tag{1.5}$$

The value of the equity is given by the partial differential equation (PDE)

$$0 = \frac{1}{2}\sigma_t^2(A)A^2\frac{\partial^2 E_t(A)}{\partial A^2} + rA\frac{\partial E_t(A)}{\partial A} - rE_t(A) + \frac{\partial E_t(A)}{\partial t}, \tag{1.6}$$

subject to

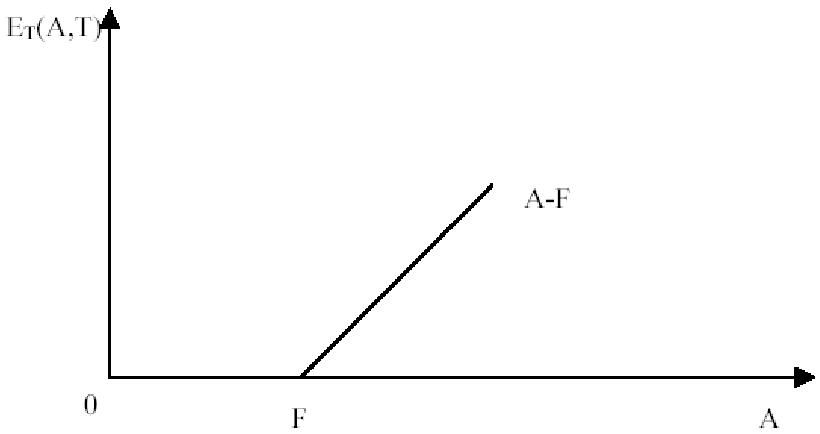

Figure 1.2: Equity value at maturity as a function of the assets value of the firm

$$
\begin{aligned}
\frac{E_t(A)}{A} &\leqslant 1, \\
E_t(0) &= 0, \\
E_T(A) &= \max(0, A_T - F).
\end{aligned}
$$

In the case of constant volatility, the well known Black and Scholes relation give the pricing formula of the equity:

$$E_t(A, T, \sigma, r, F) = A_t N(d_1) - F e^{-r(T-t)} N(d_2), \tag{1.7}$$

with

$$
\begin{aligned}
d_1 &= \frac{\ln\left(\frac{A_t}{F}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln\left(\frac{A_t e^{r(T-t)}}{F}\right)}{\sigma\sqrt{T-t}} + 0.5\sigma\sqrt{T-t}, \\
d_2 &= d_1 - \sigma\sqrt{T-t},
\end{aligned}
$$

where as N is the cumulative normal distribution function, $N(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{u^2}{2}} du$. The value of the debt satisfies the same PDE, but with a different boundary conditions:

$$0 = \frac{1}{2}\sigma_t^2(A) A^2 \frac{\partial^2 B_t(A,T)}{\partial A^2} + rA \frac{\partial B_t(A,T)}{\partial A} - rA_t(A,T) + \frac{\partial B_t(A,T)}{\partial t}, \tag{1.8}$$

subject to

$$\begin{aligned}
\frac{B_t(A,T)}{A} &\leqslant 1, \\
B_t(0,T) &= 0, \\
B_T(A,T) &= \min(A_T, F).
\end{aligned}$$

From the accounting identity we know that the value of a risky zero-coupon bond is equal to the value of the firm less the value of the equity (which is calculated as a call option). By equation 1.2, the risky bond is equal to the risk free bond less the value of the credit risk put option. Assuming again constant volatility and applying the Black and Scholes pricing formula, the debt value is given by

$$\begin{aligned}
B_t(A,T) &= A_t - E_t \\
&= A_t - A_t N(d_1) - Fe^{-r(T-t)} N(d_2) \\
&= A_t N(-d_1) + Fe^{-r(T-t)} N(d_2),
\end{aligned}$$

or

$$\begin{aligned}
B_0(A,T) &= Fe^{-rT} - \text{European put} \\
&= De^{-rT} - \left[-N(-d_1) A_0 + De^{-rT} N(-d_2)\right] \\
&= A_o N(-d_1) + De^{-rT} N(d_2),
\end{aligned} \tag{1.9}$$

where d_1, d_2 and N($\cdot$)are defined as above.

Given the basic Merton framework, we are ready to derive analytical expressions for the yield to maturity, the probability of default, the credit spread, and the discounted expected recovery value.

The yield to maturity, y, of a bond in continuous time is the solution to

$$B_t = Fe^{-y(T-t)}, \tag{1.10}$$

which is given simply by

$$y_t(T) = -\frac{\ln\left(\frac{B_t}{F}\right)}{T}. \tag{1.11}$$

Substituting 1.11 in 1.7 and knowing that the credit spread, $s_t^c(T)$, is the difference between the yield to maturity and the risk free rate,

$$s_t^c(T) = -\frac{1}{T}\ln\left[N(d_2) + \frac{A_t}{Fe^{-rT}}N(-d_1)\right]. \tag{1.12}$$

It is worth noting that the credit spread is a function of only a measure of leverage $\varrho = \frac{Fe^{-rT}}{A_t}$ (sometimes called quasi-debt ratio[2]) the volatility of the firm value, and the time to maturity.

Using Ito's lemma, derive an expression of the standard deviation of the bond, which corresponds to the default risk over the next trading interval:

$$\sigma_B = \frac{A_t}{B}\frac{\partial B}{\partial A}\sigma = \frac{N(-d_1)}{N(-d_1) + \varrho N(d_2)}\sigma. \tag{1.13}$$

Despite the fact that both equations 1.12 and 1.13 depend on the same variable, they measure different risks. Credit spread is the promised risk premium over the life of the bond and the standard deviation of the bond measures the risk of the rate of return over the next period.

The risk-neutral probability of default, PD, is the probability that the shareholders will not exercise the call option to buy the assets of the firm for F at time T. Crouhy and Galai (1997) and others show that

$$PD = N(-d_2). \tag{1.14}$$

To get the expected discounted recovery rate, it is useful to rewrite equation 1.9 as (at time $t = 0$),

$$B_0(V,T) = Fe^{-rT} - N(-d_2)\left[Fe^{-rT} - \frac{N(-d_1)}{N(-d_2)}A_0\right], \tag{1.15}$$

where the fraction $\frac{N(-d_1)}{N(-d_2)}$ in brackets is the recovery rate, see later section 1.3.2.

Similarly to equation 1.13, because the equity value is a function of the asset value, the instantaneous standard deviation of equity is a function of the asset volatility and is given by,

$$\sigma_E = \sigma\frac{\partial E}{\partial A}\frac{A}{E}, \tag{1.16}$$

where σ is the standard deviation of the asset return ot the firm and $\frac{\partial E}{\partial A}$ is the partial derivative of the value of equity with respect to the asset value of the firm. This means that

$$E_0\sigma_E = N(d_1)A_0\sigma, \tag{1.17}$$

[2]later we use $l = \varrho^{-1}$ as a measure for the leverage.

according to Hull et al.(2003), where E_0 and A_0 are the equity value at time 0 and the asset value at time 0 of the firm.

Bohn (2000a) makes a link between the probability of default given by equation 1.14 and the CAPM framework. A relationship between the expected return on the firm's assets and the overall expected return for the market is given by

$$\mu - r = \frac{Cov\left(r_A, r_M\right)}{\sigma_M}\frac{\mu_M - r}{\sigma_M} \tag{1.18}$$

where r_M is the return on the market, r_V is the return on the firm's assets, μ_M is the expected return on the market, and σ_M is the volatility of the market. After some calculus, the risk-neutral probability of default is given by

$$PD = N\left(\frac{\ln\left(\frac{F}{A_0}\right) - \mu T + 0.5\sigma^2 T}{\sigma\sqrt{T}} + \rho\frac{\mu_M - r}{\sigma_M}\sqrt{T}\right), \tag{1.19}$$

where $\rho = Cov\left(r_A, r_M\right)$.

Essentially, the actual probability of default is adjusted upwards to reflect the compensation necessary to motivate risk-averse investor to buy an asset with price sensitivity to overall market risk and time to maturity. $\frac{\mu_M - r}{\sigma_M}$ is determined by the entire market and can be interpreted as the reward per unit of market risk taken (an overall market Sharpe ratio). ρ derives from the sensitivity of the firm's assets to the overall market risk.

See Table 1.1 for a categorization of several representative structural models published in the finance literature as shown in Bohn (2000a)[3].

Extensions of Merton's model

The basic framework of Merton's model can be extended to more complicated debt securities by reflecting a security's payouts and indenture provisions in the model definitions. Some extensions of this basic modelling include modifying the asset value process, the default-risk-free rate process, and the conditions that could trigger default (including both the default barrier and the assumptions governing the reasons for default).

Relaxing assumption 6, Black and Cox (1976) introduced an absorbing barrier to reflect the presence of net worth or safety covenants. In this way, the asset value can be modelled such that it can be absorbed into the default barrier (which is defined as the value at which the

[3]The table includes notations original from the articles. For the detailed drift and diffusion terms as well as specific dynamics we refer to the original articles.

Reference	Asset Value and Interest Rate
Black and Scholes (1973); Merton (1974)	$dA = \mu A dt + \sigma A dz$ $dr = rdt$
Black and Cox (1976)	$dA = (\mu - \delta)Adt + \sigma Adz$ $dr = rdt$
Leland (1994); Leland and Toft (1996)	$dV_A = (\mu(A,t) - \delta)dt + \sigma Adz$ $dr = rdt$
Shimko, Tejima, and Van Deventer (1993)	$dA = \mu Adt + \sigma_1 Adz_1$ $dr = \kappa(\gamma - r)dt + \sigma_2 dz_2$
Kim, Ramaswamy, and Sundaresan (1993)	$dV_A = Adt + \sigma_1 Adz_1$ $dr = \kappa(\gamma - r)dt + \sigma_2 dz_2$
Longstaff and Schwartz (1995)	$dA = \mu Adt + \sigma_1 Adz_1$ $dr = (\gamma - \kappa r)dt + \sigma_2 dz_2$
Briys and de Varenne (1997)	$dA = rAdt + \sigma_1(\rho dz_2 + Adz_1)$ $dr = \kappa(t)(\gamma(t) - r)dt + \sigma_2(t)dz_2$
Zhou (1997)	$dA = (\mu - \lambda\delta)Adt + \sigma_1 Adz_1 + (\Pi - 1)dJ$ $dr = (\gamma - \kappa r)dt + \sigma_2 dz_2$

Table 1.1: Categorization of Structural Models

firm can no longer meet its contractual obligations). Valuation becomes a first-passage-time problem, which determines the probability of the first time the asset value passes through the default barrier.

Geske (1977) models defaultable coupon debt as a compound option on the firm's value. Default occurs at the coupon dates when the firm's value is insufficient to pay off the coupon. His model gives a closed-from solution for a defaultable bond with one intermediate coupon. However, for a higher number of coupons the compound options become difficult to be represented in simple integrals of the normal density.

As default cannot occur by surprise in the traditional Merton's framework, it would be expected as time to maturity of the debt goes to zero, credit spreads should also approach zero. In practice, non-zero credit spreads are observed for nearly all-corporate debt regardless of maturity. Zhou (1997) presents one solution to this problem. He incorporated in the firm's asset process a jump process to periodically shock the asset price process. In this way, short-dated risky debt can be shown to require a significant credit spread. This model assumes investors demand a price for both general credit risk and credit risk arising from jumps.

Relaxing assumption 3, Shimko et al. (1993) combined the basic structural model with a stochastic process for the default-risk-free interest rate using the mean-reversion model of Vasicek (1997). An important characteristic of this model specification involves the correlation between the asset value factor and the risk-free interest factor. The importance of this extension is not only that of a more sophisticated model but it is also an extended reasoning that shows how far the structural model might further serve for the monitoring and the management of the financial funding activity.

The main drawback of Merton's model and Shimko et al. (1993) is that the firm can only default at the maturity of the debt and default occurs only when the assets of the firm are exhausted. Longstaff and Schwartz (1995) developed a two factor models with an exogenous threshold value at which financial distress occurs. They used the approach of Black and Cox (1976) where a down-and-out barrier is added to the Black and Scholes framework in a way that default can be triggered earlier should this barrier be reached before the maturity of the debt. This allows the firm defaulting by asset insolvency or by cash-flow insolvency. The term structure of interest rate is assumed to follow the Vasicek model. The authors evaluate a risky corporate zero-coupon as a risk free bond minus a value resulting from the loss that can be incurred, times the probability of defaulting, either during the life of the bond or at maturity when the assets value process reaches the barrier. A solution for a floating rate debt is also provided.

Other more complicated characterizations of these models where default can occur prior to maturity (the option to default can be considered a barrier option) are presented in Ericsson and Reneby (1995) and Briys and de Varenne (1997). After simplification, the form of the valuation equation for many of these models resembles the one in equation 1.7.

Another modification of the basic Merton framework concerns about the specification of the firm's asset value that set off bankruptcy. In the majority of the structural models, this value is set exogenous. Leland (1994) makes it endogenous by introducing taxes, bankruptcy costs and bond covenants as determinants of the optimal asset value at which a firm should declare bankruptcy. Leland and Toft (1996) relax the assumption of infinite life debt of Leland (1994) and study the impact of the choice of debt amount on the capital structure and thus on credit spread and also the impact of the maturity of the debt chosen. The model thus gives the value of the overall firm depending on its capital structure. The model can be used as a basis for determining the term structure of credit spreads as well.

See Table 1.2 for a categorization of several representative of the extension of structural models published in the finance literature as shown in Bohn (2000a).[4]

Empirical evidences

The empirical evidence for the Merton's framework identifies the following issues addressed in the recent literature:

- Term structure of credit spreads

[4] The table includes notations from the original articles. For the detailed default barrier and recovery terms we refer to the original articles.

Reference	Default Barrier	Recovery
Black and Scholes (1973); Merton (1974)	D	A_T
Black and Cox (1976)	$LFe^{-r(T-t)}; AB$	$LFe^{-r(T-t)}$
Leland (1994); Leland and Toft (1996)	$A_t^*(\delta, T, \xi, \alpha); AB$	$(1-L)A_t^*$
Shimko, Tejima, and Van Deventer (1993)	F	A_T
Kim, Ramaswamy, and Sundaresan (1993)	$c/\delta; AB$	$min[(1-L(t))P(r,t,c), B_t]$
Longstaff and Schwartz (1995)	$K; AB$	$(1-L)F$
Briys and de Varenne (1997)	$LFP(t,T); AB$	$LFP(t,T)$
Zhou (1997)	$K; AB$	$(1-L)F$
AB denotes an absorbing barrier		

Table 1.2: Categorization of Structural Models - continuation

- Ranking of issuers
- Credit spread movements or prediction of default probabilities
- Pricing accuracy

As far as the shape of credit spreads are concerned, Sarig and Warga (1989) estimated the term structure of credit spreads using a small sample of zero coupon corporate bonds and zero coupon U.S. treasury bonds. They show that the term structure is slightly upwardly sloping for investment grade debt, humped shaped for lower grade debt, and downward sloping for very low-grade debt. Those shapes are consistent with the contingent-claims model predictions. More recently, Wei and Guo (1997) and Bohn (2000b) the results of faveor Merton's model as well. However, Helwege and Turner (1997) report the exception and found positive slope for speculative grade debt for each issuer.

The next set of tests examines whether structural models are able to reproduce a rank ordering of the creditworthiness of different companies at different times. Although admitting that there may be pricing errors, this test checks whether Merton's model is able to distinguish between riskier and less risky issuers. Crosbie (1998) and Crosbie and Bohn (2003) explain the predicted power of the KMV model in estimating the relative creditworthiness[5]. Lardic and Rouzeau (1999) perform a study using corporate bonds issued by French firms. Despite the fact that theoretical prices did not reproduce the risk ranking in the market, the movements of theoretical and empirical spreads seemed cointegrated, which means that the Merton's model was able to pick up changes in the credit quality of the same obligor. Hull et al. (2003c) test the assertion of Crosbie (1998) and Crosbie and Bohn (2003) using different market proxies[6].

The prediction of default probabilities is relevant for hedging purposes. Longstaff and Schwartz (1995) examined credit spreads movements on the aggregate using Moody's corpo-

[5] The KMV model will be explained later on this section.
[6] Hull et al. model (2003) will be detailed later on.

rate bond yield averages. They found that the pairs credit spreads-rating class and spreads-share index are both negatively correlated. The found also that the influence of the share index was less important than the influence of interest rates. Another important result came from Delianedis and Geske (1998). They found that rating migrations (using S&P credit ratings) and defaults are detected months before in the equity markets.

As far as pricing accuracy for credit spreads are concerned, Fons (1994) finds that his model seriously underestimates the spreads he obtains from fitting linear regressions through data within different credit classes. Particularly, his model specification shows difficulty with investment grade bonds. In a broad empirical research, Eon et al. (2003) test five different models: Merton's original framework from 1974, Geske (1977), Leland and Toft (1996), Longstaff and Schwartz (1995), and Collin-Dufresne and Goldstein (2001). They implement the models using a sample of bond prices from firms with simple capital structures during the period 1986-1997. The conventional wisdom that the structural models do not generate spreads as high as those seen in the bond market is confirmed in their work. They found substantial pricing errors in all models, as all five models tend to generate extremely low spreads on the bonds that the models consider safe (usually bonds with low leverage and low asset volatility) and to generate very high spreads on the bonds considered to be very risky. The Merton original model underestimated the spreads by as much as 80%, and even variations in the parameters in not help much to improve the pricing accuracy. Geske's model performed similarly to the Merton model with severe underestimation of spreads. The Longstaff-Schwartz model tended to overestimate the spreads severely for risky bonds and underestimated spreads for investment grade ones. Yet its performance was slightly better than the Merton model. In the Leland-Toft model they found that the coupon size drove much of the variation in predicted spreads. Therefore, in a recent research, Gemmill (2002) found that Merton's model predicts bond yield spreads fairly well when the model is applied to companies with very simple capital structures and transparent values for both assets and liabilities. These companies are UK based closed-end funds that issue zero-coupon bonds.

Recent market applications

A particularly recent field of research is the relationship of the credit default probabilities and the volatility skews or surface of equity options and the estimation of credit spreads from option prices. Hull et al. (2003a) propose a way that the basic structural model's parameters can be estimated from the implied volatilities of options on the company's equity. In fact, the research objective is to provide a rank ordering of creditworthiness for U.S. companies rather than to estimate precisely the level of credit spreads. In order to accomplish this, the authors explore the Merton's model role of explaining equity implied volatilities and the volatility skews that are observed in the equity options. They use extensively the model proposed by Geske (1979) to evaluate compound options. Within Merton's model framework, an option on the firm's equity that expires before the debt matures is a compound one, an option on a European call option. The formula for a put with strike price K and expiry time $\omega < T$ on the equity is

$$P = Fe^{-rT}M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + Ke^{-r\omega}N\left(-a_2\right), \tag{1.20}$$

where

$$\begin{aligned} a_1 &= \frac{\ln\left(\frac{A_0}{A^*_\omega e^{-r\omega}}\right)}{\sigma\sqrt{\omega}} + 0.5\sigma\sqrt{\omega}, \\ a_2 &= a_1 - 0.5\sigma\sqrt{\omega}. \end{aligned}$$

M is the cumulative bivariate normal distribution function, and A^*_ω is the critical asset value at time ω, the value for which the equity value at that time equals K. In fact, A^*_ω is the asset value below which the put on the equity will be exercised. The parameters α and κ are defined as

$$\begin{aligned} A^*_\omega &= \alpha A_0 e^{r\omega}, \\ K &= \kappa E_0 e^{r\omega}. \end{aligned}$$

The parameter α is the scalar multiple of the forward asset value at which the option is at-the-money (referred as the implied strike level). The parameter κ is the ratio of the strike price to the forward equity price (referred as the option's moneyness). Defining ν as the implied put option volatility and assuming that market prices are given by Merton's model, the implied volatility can be determined by solving:

$$F^* M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa E_0 N\left(-a_2\right) =$$

$$= \kappa E_0 N\left(-d^*_2\right) - E_0 N\left(-d^*_1\right) \tag{1.21}$$

where

$$\begin{aligned} d_1^* &= \frac{-\ln(\kappa)}{\nu\sqrt{\omega}} + 0.5\nu\sqrt{\omega};\ d_2^* = d_1^* - \nu\sqrt{\omega} \\ a_1 &= \frac{-\ln(\alpha)}{\sigma\sqrt{\omega}} + 0.5\sigma\sqrt{\omega};\ a_2 = a_1 - \sigma\sqrt{\omega}, \\ F^* &= Fe^{-rT} \end{aligned}$$

Inserting equation 1.21 in equation 1.7 results in[7]

$$lM\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa N(-a_2)\left[\kappa N(d_1) - lN(d_2)\right] =$$

$$= \left[\kappa N(-d_2^*) - N(-d_1^*)\right]\left[N(d_1) - lN(d_2)\right]. \tag{1.22}$$

The authors rewrite equation 1.7 to determine the implied strike level , α:

$$\kappa E_0 e^{r\omega} = A_\omega^*\left[N(d_{1,\omega}) - \left(\frac{l}{\alpha}\right)N(d_{2,\omega})\right],$$

so that

$$\kappa = \frac{\alpha N(d_{1,\omega}) - lN(d_{2,\omega})}{N(d_1) - lN(d_2)}, \tag{1.23}$$

where

$$d_{1,\omega} = \frac{-\ln\left(\frac{l}{\alpha}\right)}{\sigma\sqrt{T-\omega}} + 0.5\sigma\sqrt{T-\omega};\ d_{2,\omega} = d_{1,\omega} - \sigma\sqrt{T-\omega}.$$

Equations 1.22 and 1.23 define an implicit relationship between the implied volatility of an option and the parameter κ, the moneyness, for a set of model parameter values of leverage, l, asset volatility and debt and option maturities. The volatility skew arises from different values of κ and different values of implied volatilities. In summary, the equations 8 and 9 developed above suggest a new way of implementing Merton's model. Having two implied volatilities and assuming a value for T, equations 1.22 and 1.23 can be solved for the leverage ratio and the asset volatility. This in turn can be used to calculate the risk-neutral probability of default by time T using equation 1.14 or the credit spread for a zero-coupon bond maturing at time T using equation 1.12. This approach allows probabilities of default and credit spreads to be estimated directly from implied volatility data. In fact, it is a powerful alternative to

[7] We have modified the results from the Hull et al. (2003a) as there seems to have been a computational mistake in their working paper. A detailed proof is presented in Apendix I.

Merton's model implementation such as Jones et al. (1984) because it avoids the estimation of the instantaneous equity volatility and the company's liability structure into a single zero-coupon bond. More detailed proofs refer to Appendix I.

For the illustration between the implied credit spread (see below) and the implied volatilities the following example and assumptions are considered in Hull et al. (2003a):

- The time to make the payment to the shareholders, T, is one year, $T = 1$.
- The time to exercise the put option on equity, ω, is $\omega = 0.25$
- The asset volatility is given by $\sigma = 15\%$

To calculate the implied credit spread s^c (see A.27) of the debt for each value of the leverage parameter, l, the formula A.27 is used. To calculate the implied volatility of puts on the equity option moneyness, κ, the compound option model of equation A.23 and A.18 is applying. For an option moneyness $\kappa = 1$, an at-the-money put option is considered where the put option delta is approximately $\Delta^p_{\kappa=1} \simeq -0.5$ and for $\kappa = 0.9$ the put option is out-of-the-money with a delta of about $\Delta^p_{\kappa=0.9} \simeq -0.25$. In general Δ^p_κ denotes an put option delta for a given option moneyness, κ.

Given a range of values for the leverage parameter, l, and defining the *implied volatility surface* as the spread between the implied volatility of the out-of-the-money put and the implied volatility of the at-the-money put, denoted as SKEW, see 3.1, the relation between implied credit spread, implied volatilities and and the volatility surface can be easily estimated.

Figure 1.3 shows that there is a positive relationship between the at-the money implied volatility and the implied credit spread. This result is not too surprising and consistent with observable parameters. Figure 1.4 shows that the relationship between the volatility skew and the implied credit spread is a little bit more complex. The volatility skew increases monotonically with the credit spread for relatively low spreads (up to about 3%). For higher spread levels the slope of the skew is almost zero. As Figure 1.5 shows, the relationship between the volatility skew and the level of the at-the-money implied volatility is monotonically increasing. These results were generated using a particular set of assumptions, the general character of them, however, is quite insensitive to the assumptions. Changing the various parameter values or using changes in the asset volatility to generate the variation in implied volatility and the credit spread almost always produces similar relationships between the variables.

The empirical tests of this new methodology use as estimate of the company's credit spreads five-year credit default swap spread[8]. This is done because in theory, as empirically analysed

[8] CDS securities will be detailed in section 1.3.

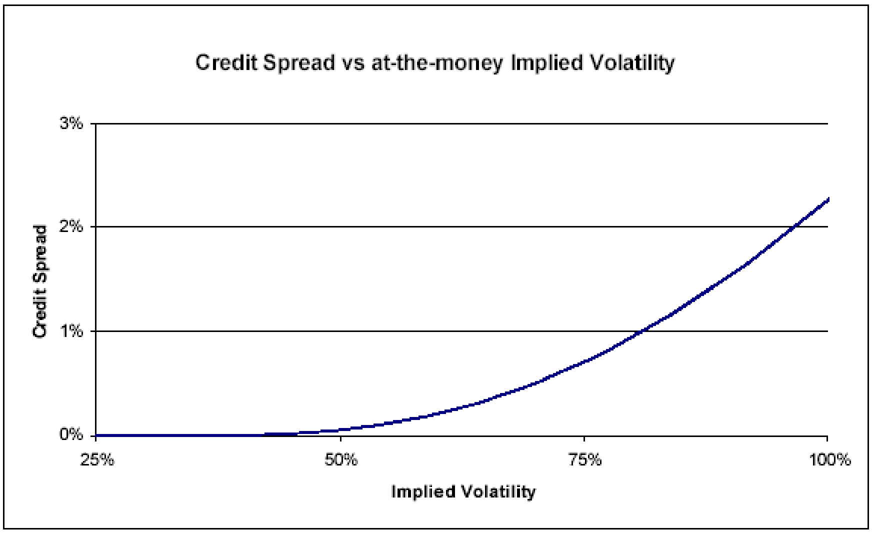

Figure 1.3: Relationship between the at-the money implied volatility and the implied credit spread

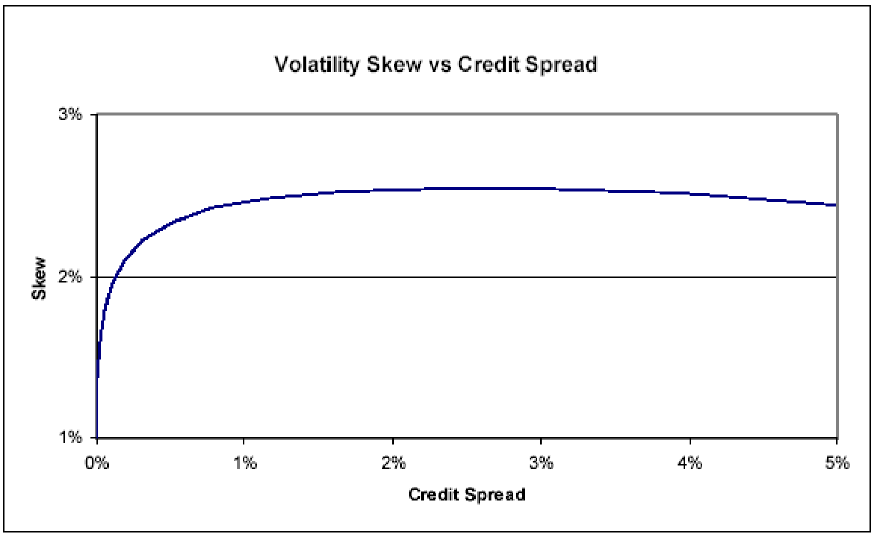

Figure 1.4: Relationship between the volatility skew and the implied credit spread

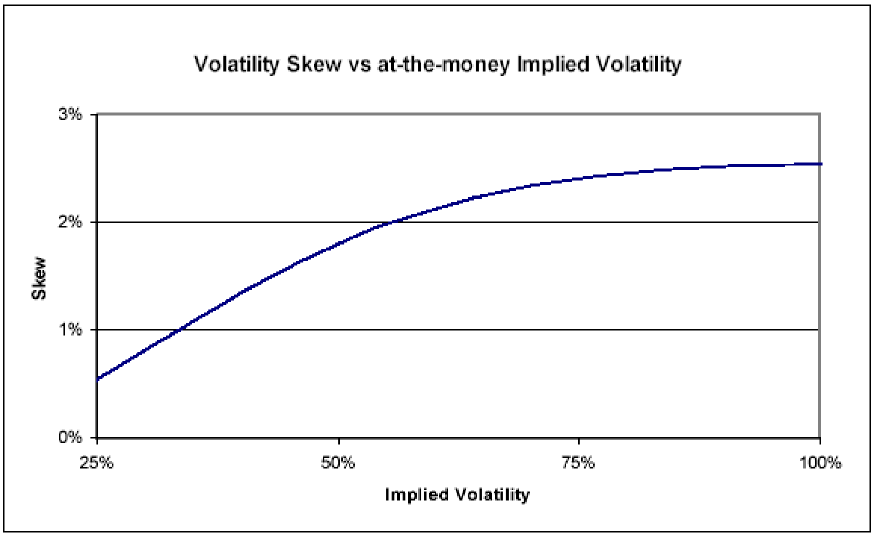

Figure 1.5: Relationship between the volatility skew and the level of the at-the-money implied volatility

in Blanco et al. (2003), CDS spreads are very close to the credit spreads on five-year par yield bonds (despite the fac t that in Merton's model implied spreads are the spreads on five-year zero-coupon bonds). This discrepancy is said to be not important because the objective of this implementation is to test the ability of structural model ranking the creditworthiness of companies. Thus the creditworthiness is measured by the estimate of the company's five-year CDS spread.

The authors test the ranking order ability for their model and compare it to Jones et al. (1984) and reduced form model derived from Duffie and Singleton (1999)[9]. One of the results is the insensitiveness of ranking order to the debt maturity when using their implementation. This shows effectively that their model requires only two implied volatilities. They also find that their implementation works better than the one of Jones et al. (1984) and the reduced form model. In that sense, they suggest that equity implied volatilities could be used (using their implementation) in the place of CDS spreads to estimate the probability of default for given company.

Contrarily to the above research, Zou (2003) is interested in finding a model that predicts credit spreads rather than ranking order using the information from the equity options volatility surface. His goal is to develop a model that does not require additional unknown parameters and does not rely on the log-normality assumption. Using a maximum entropy method to estimate the implied probability density from volatility smile of equity options, it

[9]Reduced models will be explained in the next section.

is possible to estimate the corersponding CDS premium. The maturity of the debt is chosen to be between two to three years (the tests show that CDS is not very sensitive to the maturity within this range) and the recovery rate is set to 40%, as this seems to be the market convention (the model is not very sensitive to the precise level of the recovery rate between 30% and 60%). The results seem to be encouraging and show also that the CDS level could be used as a major factor for forecasting long-term equity volatility.

Commercial Applications based on Merton's Model

A variety of commercial applications have been developed based on Merton's model. Among the most important ones are Moody's KMV model (a Moody's company) and CreditGrades model (a joint venture of Deutsche Bank, Goldman Sachs, JP Morgan and RiskMetrics Group).

Moody's KMV has created an approach for estimating the default probability that follows three steps essentially, according to Crosbie and Bohn (2002):

1. Estimation of the asset value and the volatility: market value and volatility of the firm are estimated from the market value of its stock, the volatility of its stock, and the book value of its liabilities.

2. Distance-to-default (DD): DD is calculated from the asset value and asset volatility (estimated in step 1) and the book value of liabilities. It is the number of standard deviations the asset value is away from default.

3. Default probability: the default probability is determined directly from DD and the default rate for given levels of distance-to-default.

The market value and volatility of assets can be determined directly using an option-pricing based approach, such as the ones presented above. Given the more complex balance sheets structures observed in practice, KMV proposes using the default point as the sum of the short-term liabilities (coupon and principal payments due in less than one year) and one-half of the long-term liabilities. This choice is based on the empirical evidence that firms default when their asset value reaches a level that is somewhat between the value of total liabilities and value of short-term liabilities.

The probability of default is the probability that the asset value will fall below the default point. This area is called, within the KMV model, as the Expected Default Frequency (EDF). EDF is defined as the relative amount of firms with the same distance to default that have actually defaulted in history. If the future distribution of the distance to default were known, the default probability (or EDF value) would simply be the likelihood that the final asset value was below the default point. However, the distribution of the DD is difficult to measure and the asset returns of the firms may in practice deviate from a normal distribution. As a consequence, the KMV model measures DD as the number of standard deviations the asset value is away from default and then uses empirical data to determine the corresponding default probability. The distance-to-default is calculated as:

$$[\text{Distance to Default}] = \frac{[\text{Market Value of Assets}] - [\text{Default Point}^{10}]}{[\text{Market Value of Assets}] - [\text{Asset Volatility}]}$$

or formally

$$DD = \frac{\ln \frac{A_0}{F_{T^*}} + \left(\mu - \frac{\sigma^2}{2}\right) T^*}{\sigma\sqrt{T^*}},$$

where, $0 < T^* < T$ and D_{T^*} is the book value of the frim's liabilities due at time T^*.

Schöhnbucher (2003) argues that with the calibration to historical data to determine the probability of default, the KMV model leaves the Merton's framework. He states that the KMV model should be viewed as a statistical scoring model with a huge historical database and a very specific definition of the distance to default, which is the key quantity within the used model.

Another commercial application is an open and transparent web-based service of Credit-Grades, which can be accessed at the website *www.creditgrades.com*. It provides analytics on more than 11,000 of North American, European, Asian and Japanese companies. Finger *et al.* (2002) provide the technical discussion of the CreditGrades model. Finger et al. (2002) assume the firm's asset value, A, follows a lognormal random walk and default is defined as the first time A crosses the default barrier. The assumed asset value follows a Brownian motion with zero dirft (pure diffusion prozess) The default barrier is defined as the amount of the firm's assets that remain in the case of default. This quantity is the recovery value that the debt holders receive, $RV \cdot DS$, where RV is the average recovery on the debt and DS is the firm's debt-per-share. The average recovery value RV is assumed to be stochastic and to follow a lognormal distribution with mean $\overline{RV}$ and a percentage standard deviation of the default barrier η. Specifically,

$$\begin{aligned} \overline{RV} &= E[RV], && (1.24) \\ \eta^2 &= \eta Var[\ln(RV)], && (1.25) \\ RV \cdot DS &= \overline{RV} \cdot DSe^{\eta z - \eta^2/2}, && (1.26) \end{aligned}$$

where z is a standard normal distributed random variable[11]. For an initial asset value A_0 default does not occur as long as

$$A_0 e^{\sigma W_t - \sigma^2 t/2} > \overline{RV} \cdot DS e^{\eta z - \eta^2/2}, \tag{1.27}$$

where W is a Wiener process well known as a standard Brownian motion.

After some algebra, the authors derive the following approximation for the survival probability up to time:

$$P(t) = N\left(-\frac{Q_t}{2} + \frac{\log(h)}{Q_t}\right) - h \cdot N\left(-\frac{Q_t}{2} - \frac{\log(h)}{Q_t}\right), \tag{1.28}$$

where

$$\begin{aligned} h &= \frac{A_0 e^{\eta^2}}{\overline{RV} DS}, & (1.29)\\ Q_t^2 &= \sigma^2 t + \eta^2. & (1.30) \end{aligned}$$

In order to implement the survival probability formula 1.28, it is necessary to link the initial asset value A_0 and the asset volatility σ to observable market parameters. For this the model is using a calibration procedure and assumes that for the initial asset value A_0 at time $t = 0$

$$A_0 = E_0 + \overline{RV} DS, \tag{1.31}$$

where E_0 is the initial equity or stock price. This gives

$$\sigma = \sigma_E \frac{E}{E + \overline{RV} DS}, \tag{1.32}$$

which relates the asset volatility to the observable equity volatility. It often makes sense to use a reference share price E^* and the corresponding reference equity volatility σ_E^* (either historical or implied) to determine an asset volatility, σ, and keep it stable for some period of time. In this case, the asset volatility will be given by

$$\sigma = \sigma_E^* \frac{E^*}{E^* + \overline{RV} DS}. \tag{1.33}$$

Now, equation 1.28 can be calculated using only observable market parameters as follows:

[11] $E[\,]$ and $Var[\,]$ denote the Expectation and the Variance of a random variable.

$$d = \frac{E_0 + \overline{RV}DS}{\overline{RV}DS} e^{\eta^2}, \tag{1.34}$$

$$Q_t^2 = \left(\sigma_E^* \frac{E^*}{E^* + \overline{RV}DS} \right)^2 t + \eta^2. \tag{1.35}$$

The debt-per-share DS is taken from balance sheet data. Firstly, all liabilities that are part of the financial leverage of the firm are calculated. This calculation includes the principal value of all financial debts, short-term and long-term borrowings and convertible bonds. Additionally, this calculation includes quasi-financial debts such as capital leases, under-funded pension liabilities or preferred shares. On the other side, non-financial liabilities such as accounts payable, deferred taxes and reserves are not included. Thus debt-per-share is the ratio of the value of the liabilities to the equivalent number of shares. The equivalent number of shares is calculated as the common shares outstanding, as well as any shares necessary to account for other classes of shares and other contributors to the firm's equity capital. The mean, $\overline{RV}$, is set to a historical average recovery rate of 0.5 and the percentage standard deviation, η, are set to a historically plausible recovery rate volatility of 0.3, following Hu and Lawrence (2000). The equity price E is taken from the market value and σ_e is the 750-1000-day historical average volatility.

1.1.2 Reduced-form Models

As already stated, in reduced form models the time of default is modelled as exogenously defined, in fact an intensity process that eliminates the need to have the default depending explicitly on the issuer's capital structure. In fact, the assumption is that default is an unpredictable event governed by an intensity-based or hazard-rate process. It is worth noting that since the default process can be endogenously derived, the structural model can be recast as a reduced form model making the structural modelling approach a special case of the reduced form approach. The strength of this modelling lays in the ability of modelling default without much information about why the issuer defaults. However, the model weakness is this separation between the possible economic factors explaining default.

One of the first works on this modelling is Jarrow and Turnbull (1995) where the default or stopping time is exponentially distributed and the loss given default (LGD) is constant. The distribution is parameterised by a hazard rate or the intensity of default, here equal to l. Furthermore, they assume that the default-free process, the LGD function and hazard rate process are mutually independent. In this approach, they model risky bonds as foreign currency bonds denominated in promised dollars. If default has not occurred the exchange rate equals 1 and while it equals the recovery rate in case of default. Despite the fact that this framework allows a variety of specifications for the default risk-free process, the assumption of constant default intensity is unrealistic. This specification implies default as a Poisson process and makes the model easier to estimate. However, companies might be characterized by different default intensities depending on the time horizon considered.

The previous model was extended by Jarrow, Lando, and Turnbull (1997) in assuming the default time following a continuous-time Markov chain with k states (where the states were associated with credit ratings) and default occurring the first time the chain hits the default (absorbing) state. The advantage of this model rests in its great flexibility to calculate the parameters from observable data and to use it for many purposes: pricing and hedging of bonds with embedded options, OTC vulnerable derivatives, pricing of credit derivatives and credit-risk management. This flexibility greatly increases the number of parameters to be estimated. The authors resolve this problem by suggesting the use of S&P transition probability matrices and the default process is modelled as a finite state Markov process in the firm's credit ratings. Lando (1997) affirms that without any doubt a formulation involving credit ratings is necessary for the pricing of instruments whose contractual terms explicitly involve ratings changes of the issuer. An earlier application of the above Markov model , modified to have random recovery rates, can be found in Das and Tufano (1996).

Duffie and Singleton (1999) represent one line of reduced-form models that were originally inspired by the initial research of Mandan and Unal (1993). In Duffie and Singleton (1999) default is viewed as an unpredictable event governed by a hazard rate process. The difference is on the continuous-time specification of the claim in case of default. The advantage of this model is that currently available term structure models of interest rates (such as the Heath, Jarrow, and Morton (1992)) can be applied to the modelling with small adjustment, such as refining it for an additional credit risk spread.

Duffie and Lando (1997) showed how to formulate a structural model such that it can be estimated as a reduced form model. The firm's asset value is assumed to follow a diffusion process and there exists a default barrier that marks the asset value at which the firm defaults. They derive a formula for the hazard rate that is a function of the asset value volatility, the default barrier, and the conditional distributions of asset value based on available public information.

The imperfect accounting information is the mechanism that creates the inaccessible default stopping time. With imperfect accounting data, credit spreads are bounded away from zero if the risky security matured.

Cathcart and El-Jahel (1998) discuss another example of a reduced form model that can transformed to a structural model. In the corrsponding framework, default occurs when a signalling process (instead of asset value) hits some lower barrier. The default risk-free rate is assumed to follow a stochastic CIR process (Cox, Ingersoll, and Ross (1985)) and a signalling process. The interest rate process are assumed to be uncorrelated. The authors argue that their model produces credit spread term structures more consistent with observed credit spreads than other approaches.

More about reduced form models later in section 1.3.8.

Empirical evidences

The empirical evidence of reduced form models starts with Duffie (1996a). He adds up three independent square-root processes[12] for the default risk-free term structure to arive at the default-adjusted discount rate and assumes constant recovery rate. Strong evidence of mis-specification is found with the model having notably fail producing a flat term structure of credit spreads for investment-grade bonds with less credit risk and steeper term structure of credit spreads for the same category of bonds with more credit risk. If non-investment-grade bonds are included the evidence of mis-specification is magnified. However, on average the model appears to reproduce investment-grade corporate bond prices reasonably well.

Duffie and Singleton (1999) test the Duffie and Singleton (1997) model on defaultable swap yields. They express the default-adjusted discount rate as the sum of two-independent square –root diffusions processes. One drives credit risk and the other liquidity risk. They subtract the corresponding US treasury zero-coupon yields to arrive at implied, defaultable swap spreads. These spreads are studied in the context of a multivariate vector autoregression with proxy variables for credit and liquidity risk. Liquidity shocks are found to be short-lived while credit shocks have small short-term impact followed by significant long-term impact. The model reproduces well the swaps yields with the exception of the short-end of the term structure.

These two empirical tests assume independence between the interest rate process and credit spreads. More complicated two-factor models (where both interest rate and credit spreads processes are assumed stochastic) introduce a relationship between these variables. Longstaff and Schwartz (1995) report a negative relationship between credit spreads and interest rates, which is characterized by the fact that an increase in the interest rate increases the drift of

[12]For square-root processes see Jamshidian (1996).

the asset value process. As a consequence, the risk-neutral probability of default decreases leading to lower credit spreads.

Duffie (1996b) has found as well time periods where this negative correlation appears. He finds that for bonds with highest credit-quality, changes in credit spreads are generally unrelated to changes in interest rates. However, for lower credit-quality bonds that are still investment grade, credit spreads appear to be negatively correlated with interest rates (liquidity). Another interesting finding is the rejection of the hypothesis that the relationship between treasury yields and credit spreads is driven by changes in credit quality.

1.2 Convertible Bonds

Convertible bonds (CB) are fixed income hybrid securities that lie between straight bonds and stocks. They are typically listed securities issued by companies and traded on secondary markets. CB are bonds that give their holder the right (but not the obligation) to convert the bond into a fixed number of shares of common stock. These shares are generally of the issuer, but they could be of another company as well. The most common terminologies of a convertible bond are:

- **Bond Value (Bond Floor)**: The price of a non-convertible, corporate bond of otherwise equivalent terms. This value is the lower bound because it is the net present value of the fixed cash flows of the convertible, maximised for early redemptions (puts).
- **Conversion Price**: The price at which shares are bought upon conversion; it is the security price implied by the conversion ratio.
- **Conversion Ratio**: The number of common stock shares for which a convertible is exchanged. The ratio is determined *ex-ante* and remains fixed throughout the life of the CB.
- **Conversion Value (Parity)**: The market value of the stock position obtained in case the bond is converted immediately. It represents the equity part of the CB and is calculated by multiplying the conversion ratio by the spot price of the stocks (and by the spot exchange rate, in case of a non-domestic issue).
- **Conversion Premium**: Is calculated as the difference between the convertible price and the conversion value as a percentage of the latter. It is thus the extra amount an investor must pay to own shares via the convertible.

- **Callable Feature**: This feature gives the issuer the option to call back the instrument prior to maturity at a price specified ex ante. The call provision reduces the price of the CB.

- **Hard Call Protection**: The period of time beyond which the convertible cannot be called back by the issuer.

- **Provisional Call Protection**: This protection implies that the bond cannot be called unless the stock trades above a pre-defined level for a certain period of time.

- **Redeemable Feature (Put Provision)**: This feature enables the holder to redeem the security at a specific price prior to maturity. The put option has the effect of increasing the value of the CB.

Due to this hybrid characteristic, a convertible bond presents features of several instruments. It can be seen as debt, as it pays its buyer a regular coupon until maturity and then restitutes its face value. Obviously, in case of default it ranks senior to equity and its value depends on the current interest rates. A convertible bond can be seen as equity as well, as its holder has the right to convert the face value of the bond into shares. As soon as the quoted stock price is greater than the conversion price, the conversion becomes favorable. A convertible can also be seen as an option to exchange a bond against a share. The convertible holder decides if and when he wishes to convert, unless the CB exhibit presents a callable feature.

Convertibles are worth the greatest of the cash redemption value or the market value of the shares into which they are convertible. However, before maturity the features of a convertible are more complicated. Figure 1.6 shows the typical convertible bonds value as a function of the stock price. The value is bounded below by the conversion value and the straight bond value and there is an upside potential arising from the equity component

It can be seen from Figure 1.6 that there are 5 distinctive stages in the life of a CB. When the stock price is very low, the issuer's ability to finance its debt is unsecure and the convertible enters the distressed or junk area. The parity is between about 0 and 40% of the face value. When the stock price is lower than the conversion price, the exposure to the equity upside will remain small and the bond cap is more important. The convertible is said to be out of the money and the parity is between 40% and 80% of the face value. When the stock price equals the conversion price, the convertible is said to be about at the money. The parity is between about 80% and 120% of the face value. In case the CB being of in the money, the stock price is very high and the conversion is very likely. Parity is commonly above 120% of the face value and the equity premium is less than 10%. The discount to parity is the one that parity is greater than the convertible price, where the parity is the market value of the shares into which the bond can be converted at that time.

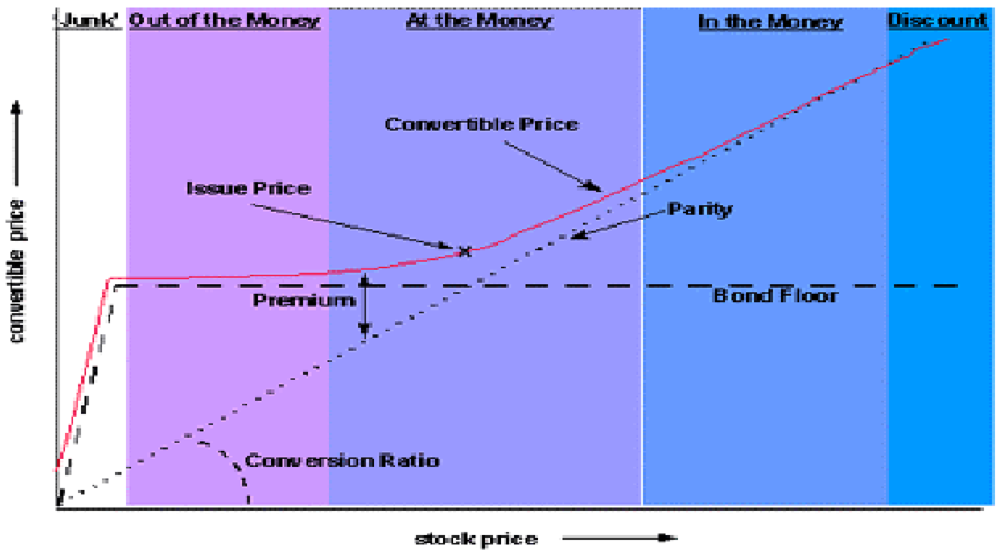

Figure 1.6: Stages of a convertible bond; Source: Deutsch Bank

Companies usually issue convertible bonds because it is a cheaper way of debt funding by implicitly selling an option on their stock, which will only be exercised in case the company financially performs well. Davis and Lischka (1999) state that convertibles help to resolve the problem of asymmetric information on the riskynes of the underlying assets, and in turn reduces agency costs. This is because equity holders have an incentive to increase the risk and return relationship of the assets, which increases the value of equity and decreases the value of debt, when holding the firm value constant. On the other hand, the call feature of a CB increases in value and the total value of the convertible can be made insensitive to changes in risk. A further reason is an indirect and delayed issuing of equity that reduces dilution and avoids circumvents regulatory restrictions.

Essentially, investors hold convertibles for their upside potential with limited downside risks. However, different reasons arise for using convertibles depending on the type of investors dealing with an issuance. According to Aboltina and Skutelis (2001), hedge funds are in particular interested in exploiting the asymmetric link between convertibles and bonds/equities to exploit arbitrage opportunities and profit from volatility trading[13]. Equity funds enhance

[13] Volatility trading is here the trading between different asset class volatilities, such as bond price and equity price volatility.

income via coupons higher than the corresponding stock dividends and increase the universe of available asset classes to invest. Fixed income funds achieve exposure to equity markets at a reduced risk to capital and they manage interest rate cycles. Finally, dedicated convertible investors pursue all of the above strategies.

1.2.1 Brief Survey on Pricing Models

The framework for pricing convertible bonds should incorporate elements of both equity and debt modelling. Andersen and Buffum (2002) argue that there seems to have been confusion and disagreement on how to apply properly and consistently a default-adjusted discount operator to cash flows generated by convertible bonds. Many of the early papers on pricing used an ad-hoc approach to discounting, including McConell and Schwarz (1986), Cheung and Nelken (1994), and Ho and Pfeffer (1996). Some of these models do not model bankruptcy explicitly and to compensate for that shortcomings they apply an arbitrary risky spread to the risk-free discount rate. Recent papers acknowledge that the components of convertible bonds, equity and debt, are indeed subject to different default risk. A study by Goldman Sachs (1994) considers the probability of conversion at every node of a binomial lattice adjusted for the issuer's credit spread. Tsivioritis and Fernandes (1998) split the convertible bond into equity and cash components, with only the latter being subject to credit risk. This modelling is extended in Yigitbasioglu (2001) to include multiple factors. The splitting scheme is analysed also in Ayache et al. (2002). They conclude that this scheme is intrinsically not satisfactory due to the assumption of stock prices being not impacted by bankruptcy.

However, the pricing models for convertible bonds has improved greatly with the researches on reduced-form models that started with Jarrow and Turnbull (1995). One of the developments of this modelling is the inclusion of stock price dynamics that incorporate default events, and the explicit modelling of bond and stock recoveries in case of default as well. As already stated, default is modelled as Poisson process and the process drives stock prices into some low value and coupon bond prices into a fixed percentage of their notional amounts. The works of Davis and Lischka (1999) and Takahashi et al. (2001) incorporate market and credit risk in a similar way in the convertible pricing model. Ayache et al. (2002) discuss in detail how state-of-the-art finite-difference methods can substitute the not optimal binomial and trinomial trees applied in the previous literature. Finally, Andersen and Buffum (2002) discuss the parameterization and calibration of convertible bond models to quoted prices of straight debt[14] and equity options. In fact, the authors attempt to imply parameters from market quotes on actively traded securities.

[14]Sometimes called "plain vanilla bond", which means bonds without embedded or hidden options.

1.2.2 Risks and the Greeks

As already stated, a convertible bond is a hybrid securitiy that is characterized by a fixed-income part as well as an equity part and as a consequence diverse risk factors arise to the holders of a CB. Therefore a CB is exposed to the same or even more risks than its constituents. A brief summary of these risks is:

- **Equity market risk**: As seen in Figure 1.6, at high share prices the CB price approaches the parity line and it behaves like pure-equity and thus shares the benefits of a rising market. At low share prices the CB value falls to a lower rate and flattens out to a constant level and at maturity it is likely the redemption would be invoked rather than conversion. The relationship between equity volatility and CB is that a share with a higher volatility has a higher chance of ending up with a value significantly greater than the conversion price and thus has the potential to be worth more. In fact, the equity volatility risk can be hedged by shorting the underlying stock against the long position convertibles. Such hedging produces a very small beta risk and thus a market neutral position.

- **Interest rate risk**: As for every bond, the price of a CB moves inversely to interest rate changes and its sensibility to these changes depends on how closely it is in relation to the fixed income part of the instrument. Therefore, the embedded option value moves in line with interest rate changes. Commonly, the interest rate risk is hegded with treasury futures or interest rate swaps.

- **Credit risk**: The exposure comes from the long convertible position, because a widening in credit-spreads widening leads to the stock price to decline. Typically this risk is hedged with credit default swaps (CDS) or by shorting a plain bond or another not identical CB from the same issuer.

- **Liquidity risk**: A CB investor is subject to liquidity risk of the long position not beeing liquid as expected, the short equity position being called in or being short squeezed. Liquidity risk can occur as well due to the size of an issue or because of the low credit quality of the issuer. There is no hedging possibility for such risk.

- **Currency risk**: Some CB issuances are overseas and thus foreign currency denominated and to hedge the currency risk the investor usually utilises currency options or forward contracts.

To quantify more precisely the impact of the market variables in the valuation of a CB, it is important to analyze the "greeks" or the sensitivity of the CB to a given market movement.

The measure *delta* quantifies the change in the CB's value CV with respect to changes in the stock price E and can represented as:

$$\Delta^{CB} = \frac{\partial CV}{\partial E} = e^{-q(T-t)} N\left(d_{1,q}\right), \tag{1.36}$$

where

$$\begin{aligned} d_{1,q} &= \frac{\ln\left(\frac{E}{K}\right) + \left(r - q + 0.5\sigma_E^2\right)(T-t)}{\sigma_E\sqrt{T-t}}, \\ d_{2,q} &= d_{1,q} - \sigma_E\sqrt{T-t}, \end{aligned}$$

q is the continuous dividend yield and K the adjusted strike price.

Delta estimates the number of shares of the corresponding stock to short against the long CB position in order to keep a neutral hedge position and it can be visualized in Figure 1.6 as tangent to the convertible price function. Thus it can be seen that in case the CB moves towards the in-the-money position, Δ approaches 1 and Δ approaches 0 if the CB moves towards the out-of-the-money position.

Another important greek measure is *gamma* and it quantifies the changes in delta with respect to the change in the underlying stock price. *Gamma* represents the convexity of the convertible price evolution and it is represented as[15]:

$$\Gamma = \frac{\partial^2 CV}{\partial E^2} = \frac{N'\left(d_{1,q}\right) e^{-q(T-t)}}{E\sigma_E\sqrt{T-t}}. \tag{1.37}$$

Low *gamma* values are found when the CB value is deep in-the-money or deep out-of-the-money, while a CB value cloae to at-the-money possesses higher *gammas* values. *Gamma* is associated with the rebalancing frequency of a delta-hedge portfolio. The higher a CB's *gamma*, the more frequent the hedge needs to be rebalanced. *Vega* measures the change in the convertible price with respect to changes in the implied volatility of the embedded stock option and is calculated as:

$$Vega = \frac{\partial CV}{\partial \sigma} = E\sqrt{(T-t)\, N'\left(d_{1,q}\right)} e^{-q(T-t)}. \tag{1.38}$$

The relationship between *vega* and the stock price is similar as in the case of *gamma* in that CB value in-the-money or out-of-the-money possesses lower *vegas*, while CB values close to to at-the-money presents higher *vegas*. *Vega* risk for instance can be reduced with put options on a stock index. When volatilities decline, so does the market, the put option becomes more valuable while the long position in convertible declines in value.

[15] $N'()$ denotes the first derivative of $N()$ subject to E.

Theta measures the change in the CB price with respect to changes in time. *Theta* should also be associated with volatility in that if implied volatility, meaning that changes in a given position, *theta* should be recalculated. *Theta* is given by:

$$\Theta = \frac{\partial CV}{\partial t} = \left[\frac{-EN'\left(d_{1,q}\right)\sigma e^{-q(T-t)}}{2\sqrt{T-t}} \right] - rKe^{-r(T-t)}N\left(d_{2,q}\right) + qEN\left(d_{1,q}\right)e^{-q(T-t)}. \tag{1.39}$$

When a CB is out-of-the-money it has little *theta* sensitivity until maturity, while an at-the-money CB presents a high *theta* risk, due to the high value of the conversion premium and because of the price of the CB being above the call price or par value. As an in-the-money CB has less conversion premium to lose it is less sensitive to *theta*. The sensitivity of the CB value to changes in the interest rate is named *rho* and, as for all bonds, higher interest rates lead to lower CB prices. A CB has negative *rho* values at almost all the points of its valuation because as interest rates increase, a bond value decreases. When the CB is deep-in-the-money it will have a *rho* value close to zero. The CB reaches its maximum value of *rho* when the it is out-of-the-money and thus its equity component has a very low value. In case the CB moves in-the-money it becomes less sensitive to changes in interest rates and more sensitifity to equity price movements. When CB moves from deep-out-of-the-money to out-of-the-money values it exhibits a very small sensitivity to changes in interest rates.

Omicron is defined by Calamos (2003) as the measure of the change in CB value with respect to changes in credit spread and is calculated as:

$$O = \frac{\partial CV}{\partial OAS}, \tag{1.40}$$

where OAS is an option adjusted spread. According to Calamos (2003), Omicron represents an important measure for many low-grade CB issues trading near or below their exercise price. In general, an out-of-the-money CB is more influenced by *omicron* than to any other greek. In fact, an out-of-the-money CB has a high *omicron* measure, whilst a deep-in-the-money CB has a very low one.

1.2.3 Arbitrage Technique - Delta Hedging

The most popular CB arbitrage strategy is the so-called delta neutral hedge. The basic idea is to buy a convertible and to short the underlying stock at the appropriate *delta*. The position is fully hedged for very small movements in stock price and cash-flow is received from the convertible's yield and from the short's position interest rebate. In this hedge, the CB's equity risk is neutralized but not the interest rate risk (*rho*) and the long volatility risk (*vega*). Interestingly, another denomination for this strategy is long volatility hedge, because of the remaining *vega* risk.

Because of this long *vega* exposure, it is expected that the implied volatility of the CB should increase from its initial value. If the implied volatility level decreases or stays the same, the position will benefit just from the income flow from the convertible's yield and the short's interest rebate. Therefore, the more volatile the implied volatility, the more earnings the position gets. Thus the arbitrageur should identify convertible with low implied volatility level relative to his or her expected level of future volatility. Other criteria used to identify delta neutral hedge opportunities could be:

- A higher value for *vega* than for *omicron*,
- A Convertible's yield higher than LIBOR[16],
- A Low conversion premium,
- Low or no stock dividends on the underlying shares,
- A high gamma,
- A low liquidity risk, and
- Stock short-selling possible.

However, some risks have to be taken into consideration, including:

- A decreasing in implied volatility,
- A widening of credit spreads, and
- A CB beeing called from the issuer.

In more detail, the appropriate number of shares to short against the long convertible determines the accuracy of the hedge. Shorting too many shares can cause the hedge to lose money if the stock price increases, and shorting too few can cause a loss if the stock price decreases. Thus the correct basic hedge ratio is calculated as:

$$Neutral\ hedge\ ratio = Conversion\ ratio * \Delta.$$

Consider the following example, extracted from Calamos (2003): A CB is trading at 105% of par with 19.65% conversion premium that converts into 22.50 shares of stock and possesses

[16] The LIBOR, London interbank offering rate, is considered as a risk-free rate.

a *delta* of 0.594. The investor makes a long investment of $1,050,000 in buying 1,000 bonds. He shorts 16,000 underlying stock based on the current stock price of $39 and the *delta* of 0.594 . If the stock price thus moves up 1% to $39.39 the convertible price moves up to 105.624% of par and the gain of $6,240 from the long CB position is offset with an equivalent loss on the short stock position. Table 1.3, illustrates the initial hedge setup:

	Quantity	Price	Value	Profit/(Loss)
CB long	1,000	1,050.00	$ 1,050,000	
Short stock	16,000	39.00	$ (624,000)	
Stock price moves 1%				
CB long	1,000	1,056.24	$ 1,056,240	$ 6,240
Short stock	16,000	39.39	$ (630,240)	$ (6,240)
			Net P/L	$ 0

Table 1.3: Initial hedge

Theoretically, each time the stock price moves up (down) a few percentage points, the investor increases (decreases) the short position to reflect the increasing (decreasing) *delta*. In the previous example, after the movement of the stock price the new *delta* for the CB is 0.605. In order to maintain the proper hedge ratio, 223 additional shares of stock should be shorted. If not, the original hedge ratio would realize a gain or loss on the next move of the stock price. As shown in table 1.4, in case there is no adjustment to the hedge ratio the CB position gains $6,390 while the short position loses $6,304 for a net gain of $86. As the example demonstrates, the investor must constantly measure the portfolio hedge ratio and compare it to the CB's theoretical *delta*.

The delta neutral position can also capture gains from mispricing in the convertible market and an optimal strategy is one in which the arbitrageur purchases a theoretically undervalued CB, in terms of implied volatility. From the previous example, the long volatility hedge may be theoretically undervalued if the purchased CB has an implied volatility of 35%, yet the

	Quantity	Price	Value	Profit/(Loss)
CB long	1,000	1,056.24	$ 1,056,240	
Short stock	16,000	39.39	$ (630,240)	
Stock price moves up 1%				
CB long	1,000	1,062.63	$ 1,062,630	$ 6,390
Short stock	16,000	39.78	$ (636,544)	$ (6,304)
			Net P/L	$ 86

Table 1.4: Initial hedge after stock move

	Quantity	Price	Value	Profit/(Loss)
CB long	1,000	1,056.24	$ 1,050,000	
Short stock	16,000	39.00	$ (624,000)	
Stock price moves down 1% and volatility rises to 40%				
CB long	1,000	1,055.00	$ 1,055,000	$ 5,000
Short stock	16,000	38.61	$ (617,760)	$ 6,240
			Net P/L	$ 11,240

Table 1.5: Initial hedge after an increase in volatility

investor expects the volatility to rise to 40%. The CB with a current volatility of 35% and stock price at $39.00 is bought for 105% of par, but if priced with a 40% volatility the CB is worth 106.5% of par. As the position is long vega, it is expected to pick up five volatility points. With the equity position hedged, the position is not affected by the stock price moves up or down, but that volatility reverts back to the 40% level. The *vega* for this position is calculated in 0.28.

Considering the previous example again, suppose the stock price declines 1% to $38.61, and the CB increases in value because it trades at the expected 40% volatility level . As a consequence the theoretical CB value increases $ 5 per bond to $ 1,055, as shown in Table 1.5. In fact, the vega influence offset the delta impact of the stock price decline. Therefore, the short position and the long side of the trade return a profit.

In summary, delta neutral hedges are build to get the cash flows from the long convertible position and to profit from the long cheap volatility position. Therefore, the strategy should be dynamically rebalanced in order to keep the neutrality of the hedge or the in terms of greek exposure wanted. The desired amount of volatility sought after in the strategy determines the right time to rebalance the position and the rebalancing frequency of rebalanced is a function of the *gamma* measure and of the stock liquidity.

As already stated, the higher the *gamma*, the more the position has to rebalanced. Surely, static hedging or least infrequent rebalancing could occur in a not leverage portfolio when trading costs are important. Some others CB hedging strategies can be established by means of *gamma*. It is worth mentioning the delta neutral gamma hedge, the bull gamma hedge and the bearish gamma hedge. The way to capture *gamma* is to combine a static hedge ratio with a changing *delta*, regardless of the direction of the stock price move. In fact, the delta neutral hedge is established to account for *gamma* because even with a very small stock price move, *gamma* will be captured before the hedge is rebalanced. Frequent rebalancing can cover incremental gamma, but the smaller the gamma capture, the lower the volatility; and the total return will be significant only for a leveraged position. The neutral gamma hedge strategy works best for a CB with a high *gamma* value and with the price of the underlying stock being highly volatile. A high *gamma* convertible results in a large change in delta when the stock price changes and because of that the CB offers good equity upside potential with low equity exposure on the downside. Differently from the delta neutral hedge, a gamma hedge will generally lead to a slight price direction bet on the underlying stock to maximize returns with an unleveraged hedge. Therefore, to reduce the overall equity market exposure, the hedge portfolio should have both bullish-tilt gamma hedges and bearish-tilt gamma hedges. These positions have a longer expected holding period due to the larger stock price moves necessary before a hedge adjustment is made. The *gamma* hegde will also cause the portfolio to be more volatility in returns because of the higher equity exposure.

The tilt of a *gamma* hedge is measured by comparing its hedge ratio with the ratio of a delta neutral position. The bull gamma hedge is hedged less than required for a delta neutral position, and bear gamma hedge is hedged more. The bull gamma tilt hedge captures upside *gamma* potential with a semi-directional tilt on the hedge. The difference between the delta neutral and the bull gamma hedge ratios is a function of the downside risk tolerance and the expected short-term upward trend in the underlying stock price. Conversely, the bear tilt hedge position is designed for profit in case the underlying stock price declines. The following characteristics help identifying a bull gamma arbitraging opportunity:

- Undervalued or fairly valued convertible;
- A CB with high *gamma* measure - high upside and low downside *gamma* potential,
- A yield advantage compared to the underlying stock,
- A stock characterized by high volatility, and
- A common stock with improving technical ratios, such as improving RSI, increasing EPS estimates[17].

However, some risks have to be taken into consideration, including:

- A decline in Stock price,
- A decline in implied volatility,
- Yield curve shifts, and

[17] RSI: Relative Strenght Index; EPS: Earnings per Share

- Credit deterioration.

The following characteristics help identifying a bear gamma arbitraging opportunity:

- Both securities undervalued or fairly valued convertible,
- A CB with high *gamma* measure - low upside and high downside *gamma* potential,
- A yield advantage compared to the underlying stock,
- A stock characterized by high volatility, and
- A common stock with deteriorating technical ratios, such as declining RSI, decreasing EPS estimates.

However, some risks have to be taken into consideration, including:

- A rise in stock price,
- A decline in implied volatility,
- Yield curve shifts, and
- Credit deterioration.

1.2.4 Portfolio Risk Management

In terms of CB portfolio risk management, it is important to separate the different types of hedge positions to guarantee that the inefficiency arbitraged at individual CB level is not hedged away at the portfolio level. *Delta* and *omicron* (specific credit spread) risks are best hedged at position level because they are non-systematic risks, whilst other systematic risks, such as *vega*, *rho*, and *omicron* (general credit spread), should be hedged at portfolio level. The question of diversification is accounted for at the portfolio level. The diversification should include the industry and sector exposure, credit exposure, and the exposure to the variety of convertible hedges. In relation to arbitrage strategies, it is important that the portfolio level risk management does not eliminate a risk or a hedge that is otherwise acceptable at the position level.

The means of hedging systematic risks within the portfolio level are generally performed with index options, U.S. Treasury or LIBOR futures, variance or volatility swaps, and CDS basket or closed-end funds. One way to hedge the portfolio due to market crashes is to buy out-of-the-money index put options. These options can be puts on indices that mirror the portfolio's securities and they represent a fixed cost insurance that pays off in case of extreme market events. Using index put options has a significant impact on the negative return part of the portfolio's return distribution. The number of put option contracts to purchase is determined as:

$$number\ of\ contracts = \left(\frac{Market\ value\ of\ portfolio *\ adjusted\ beta}{Contract\ strike * 100}\right), \tag{1.41}$$

where the adjusted beta is calculated as the weighted average beta for the underlying stock versus the index times the convertible's expected *delta* in extreme conditions. Generally, U.S. Treasury futures are used advantageously to hedge interest rate risk (*rho* risk) at portfolio level. It is not favorable to do so for an individual position, because *rho* moves quickly in case the underlying stock price moves sharply up or down. Yet at portfolio level, these moves are decreasing by the moves of other CBs, providing a more stable portfolio *rho* measure. The short-term interest risk, or *rho*2, the leverage borrowing cost, is hedged at the portfolio level with LIBOR futures or with a fixed receiver interest-rate swap. The number of futures contracts to short is calculated as:

$$number\ of\ contracts = \left(\frac{\$\ Change\ in\ portfolio\ for\ a\ 1\ bps\ move}{\$\ Change\ in\ futures\ contract\ for\ a\ 1\ bps\ move}\right). \tag{1.42}$$

As already stated, delta neutral strategies are long-volatility hedges, making the CB portfolio exposed to volatility (*vega*) risk. Therfore an increase in volatility should cause many of the individual positions to increase in value and should have some impact on the portfolio's overall value as well. The simplest way to manage *vega* risk at position level is shorting call options against a position when the implied volatility is expected to decline. Furthermore, purchasing put options can reduce this *vega* risk in that the movement of volatility and equity prices is inverse to each other. At the portfolio level, variance and volatility swaps are recommended to hegde volatility risk[18]. The swap allows the portfolio to gain long or short exposure to market volatility. Interestingly, the swap provide also a way to dynamically rebalance the delta-neutral hedge strategy.

It is known that volatility changes are highly correlated with other market factors that must be analysed before a hegde is designed. Rising volatility is commonly associated with increasing risk aversion in the market, which results in changes of *delta*, *gamma*, and *rho*. Moreover, volatility is frequently positively correlated with credit spreads and negatively cor-

[18] For a comprehensive giude of such financial instruments see Demeterfi et al. (1999).

related with equity market indices. Because of these correlations, volatility swaps can also be used to hedge credit-spread risks (*omicron*) and to hedge against equity market directional moves.

The *omicron* hedge with volatility swap is not a direct credit-spread hedge, but a "correlation" hedge in which the positive correlation between credit spreads and volatility is expected to hold. It would thus be preferable to use a credit default swap (CDS) basket with similar characteristics for hedging the long portfolio. The characteristics to match are -among others- credit quality, duration, convexity, and sector exposure. The way the basket CDS hedge works is similar to the individual CDS hedge applied to individual issues, which will be describe in details in section 2.1.3. Another strategy to hedge *omicron* is to short a high-yield closed end fund against the portfolio, which will however generally bring additional duration, convexity, and credit rating mismatches into the hedge.

1.3 Credit Derivatives

This section gives a very brief overview of the credit derivative market, where the next sections concentrate more specifically on credit default swaps (CDSs). It is obvious that this section can only discuss some very basic aspects of this important area. First a short introduction about the basics of credit derivatives is presented to clarify the used definitions and terminology in the following. Then an overview over the major credit derivative instruments, their types, structures and applications is briefly presented. Furthermore, a short overview of the credit derivative market and the use of credit derivatives is made. At the end of this section a brief comment on completing financial market information by using credit derivatives is presented. For an introduction see Das (1997), more complete works are Das (1998, 2001), Tavakoli (2001). Very helpful works including a wide range of practical issues are JPM (1999), O'Kane (2001), and Bowler and Tierney (2000).

1.3.1 Basics about Credit Derivatives

Before we point out in detail in the next section what a CDS exactly is and their characteristics we start with some basics about credit derivatives and present the common definitions and terminologies for credit derivatives which we will use throughout the thesis.

Conventional market theory describes two main risk categories: market or price risk and credit risk. Market risk refers to general risks and instabilities inherent in the market, such as inflation, interest rates, and the production of goods. To protect themselves against changes in these areas, investors mostly enter in long positions, forwards, futures and options on exchange rates or prices for assets. But while a variable rate protects the investor against market risk, he still may not receive the entire return on the bond, as the bond issuer may not be able to make all its coupon payments, and thereby defaults. This is the simplest manifestation of credit risk. The derivative market is a lucrative one which aims to structure and price the market and credit risk respectively to hedge against these risks. Economic theory tells us that market and credit risk are related to each other and not separable.

Dealing with over-the-counter (OTC)-derivative financial instruments bears a *counter-party-risk* (see 10, 1.3.2, below). Instead of exchange traded futures and options, the mostly used derivative instruments in corporate treasury activities and financial institutions are interest rate swaps or currency forwards and other structured fixed income derivatives. These financial instruments are traded over-the-counter and therefore entering in these contracts bears the risk of the default of the counterparty. Credit derivatives are OTC derivative financial instruments whose payoff depends on the credit quality of a certain issuer. This credit quality can be measured by the credit rating of the issuer or by the credit spread of his defaultable bonds over the yield of a comparable default-free bond (see 12,1.3.2, below). They represent a diverse and heterogeneous group of transactions, which are principally concerned with the isolation of credit risk as a separately traded market-variable. The different products essentially are focussed on structuring financial instruments to allow trading in this attribute in varied formats to allow hedging or risk assumption by market participants.

Credit derivatives are simply a mean of protecting against credit risk. They come in many shapes and sizes to protect against different kind of credit risk. Essentially, a credit derivative is a security with a payoff linked to credit related event (see 'Terminology and Definitions', 4., below), such as default, credit rating downgrade, or structural change in a security containing credit risk. Credit derivatives can make large and important risks tradeable. They form an important step towards market completion and efficient risk allocation, and can further bridge the traditional market segmentation between corporate loans and bond markets.

For detailed discussion of credit derivative structures, instruments, markets and application we refer to Das (1998, 2001) and Takavoli (2001). Moore and Watts (2003) provides a very informative introduction about the credit derivatives business.

1.3.2 Terminologies and Definitions

One of the attractions of credit derivatives is the large degree of flexibility in their specification. Here we should recall in mind on the complexity of specifications in OTC derivatives transactions. Additionally the credit derivative market is relatively new in Finance. Standardized contracts do not exist for a long time compared to the fixed income and equity derivatives market. Therefore, some of the terminologies used in credit derivative market comes from the interest rate swap market and is mixed with some new terminologies. First we need to clarify the common features of most credit derivatives. The list below is selective rather than being comprehensive.

1. *Credit derivative*: A credit derivative is a derivative security that has a payoff which is conditioned on the occurrence of a credit event (or so called contingent on the occurrence of a credit event, see below). The credit event is defined with respect to a *reference credit* (or several reference credits), see below, and *the reference credit asset(s),* see below, issued by the reference credit. If the credit event has occurred, the default payment has to be made by one of the counterparties. Besides the default payment a credit derivative can have further payoffs that are not contingent on default.

2. *Reference Credit* (sometimes called credit entity or reference entity): One (or several) issuer(s) whose defaults trigger the credit event. This can be one (typical) or several (basket structure) defaultable issuers. We use throughout the thesis party **C** for the reference credit.

3. *Reference Credit Asset (or Reference Asset)*: A set of assets issued by the reference credit. The instruments on which credit risk is traded. They are needed for the determination of the credit event and for the calculation of the *recovery rate* (which is used to calculate the default payment), see below. The definition can range from any financial obligation of the *reference credit* to a specific list of just a few *bonds* issued by the reference credit. *Loans* and liquidly traded bonds of the reference credit are a common choice. Frequently, different assets are used for the determination of the *credit event* and the *recovery rate.*

4. *Credit Event*: A credit event is the default or the down grade of a firm (also called *credit risk*). A precisely defined *default event* (or so-called *default risk*), which is usually defined with respect to the *reference credit(s)* and *reference credit assets.* Throughout the thesis we will not distinguish between credit risk and default risk. Possible definitions for credit events include:

   - payment default (typically a certain materiality threshold must be exceeded or simply the failure to pay),
   - bankruptcy or insolvency; protection filing; ratings downgrade below given a threshold (i.e. ratings triggered credit derivatives),
   - changes in the *credit spread*, see below,

- repudiation/moratorium[19], and
- restructuring.

The definition for a credit event can further include events that go as far as armed hostilities, social unrest or earthquakes for sovereigns or mergers and takeovers for corporates. A default event: can significantly extend or limit the scope of protection in the case of bankruptcy, when a credit event upon merger, in cross acceleration, in cross default, in a rating downgrade, currency convertibility, governmental action, market disruption on one of several reference assets.

5. *Recovery value*: When the credit event occurs, the payoff of the credit instrument will depend on the recovery value of the asset at the moment of default. This value is rarely zero. Usually some positive amount will be recoverable. Hence, the *protection buyer*, see below, needs to buy protection over and above the recoverable amount. Determination of the recovery value needs to be done before the settlement. Major rating agencies such as Moody's and Standard and Poor's have *recovery rate* tables for various credits that are prepared using past default data.[20]

6. *Default Payment*: Sometimes called contingent default payment. These are payments which have to be made if a credit event has happened. The default payment is the defining feature of most credit derivatives.

7. *Protection buyer/seller*: Most credit derivatives have a default-insurance (or so called protection) feature. In naming the counterparties we will use the convention that counterparty **A** will be the insured counterparty (i.e. the counterparty that receives a payoff, in form of *contingent payments*, see below and pays in exchange to that a *premium payment*, see below). **A** is long the credit derivative, the *Protection buyer* which buys a credit derivative instrument, and counterparty **B** will be the insurer or *protection seller,* which sells the credit derivative instrument (the one who has to make contingent payments and receives a premium). Party **C** will be the reference credit.

8. *Contingent payment:* These are payments based on a defined credit event to occur with respect to the underlying reference asset. If a default happens we called *contingent default payment*, see *default payment.* Otherwise we called periodic payments (sometimes called termination payments or contingent periodic payments)

[19]Means a reference credit (a) disaffirms, disclaimes, repudiates or rejects, in whole or in part, or challenges the validity of, one or more obligations, or (b) declares or imposes a moratorium, standstill or deferral, whether de facto or de jure, with respect to one ore more obligations (see also 1999 ISDA Credit Event Definitions)

[20]Usually the recovery rate is defined as a percentage of the par value.

9. *Premium payment*: Payment (e.g. for default protection) which is typically made periodically, although it may be paid as an up-front fee for short-dated transactions.

10. *Counterparty risk*: Type of risk which creates defaultable payment obligations in OTC derivatives transactions or simply risk in terms of the default of a counterparty. The counterparty defaulting means that there is a loss incurred by an investor or lender, such as failure to make scheduled principle or interest payments. Given the long term and large notional amount of some of these transactions the counterparty risk may be significant, even if netting procedures are in place.

11. *Notional amount*: The principal balance underlying a swap transaction, and the amount used to compute swap payments e.g. for an interest rate swap is called the notional principal. The two counterparties to a swap agreement exchange periodic cash flows not the notional amount.

12. *Credit spread*: In general credit spreads represent the margin relative to the risk-free rate designed to compensate the investor for the risk of default on the underlying security. The credit spread itself is calculated as: Credit spread = yield of security or loan-yield over the corresponding risk-free security. Two general formats of credit spreads are used: (1) credit spread relative to the risk-free benchmark (the absolute credit spread) and (2) credit spread between two credit-sensitive assets (the relative credit spread).

1999, the ISDA has published a standard specification for credit default swaps. This contributed much more to the transparency of the market. For some more specific standardized definitions see also ISDA (International Swaps and Derivatives Association) 2002 Master Agreements.

1.3.3 Credit Derivative Types

The classification of credit derivatives differs from the one of other types of financial derivatives Even the difference between forward and option-type contracts being clear-cut for other financial derivatives is less clear in credit derivatives (where credit default swap may look as much as an option on swaps; more about this later). There are currently three main classes of credit derivatives:

1. *Credit event instruments*: These are instruments that make payments depending on the occurrence of a mutually agreeable event. The CDS is the most common example here. CDS offer users protection against a credit event pre-specified in the contract by

having a payoff contingent on this event (here the default event). Other instruments within this class are, for example, credit event swaps, credit linked notes (CLN) and credit default options (sometimes classified as spread instrument in terms of credit spread options). The most common credit events with this instruments are linked to the migration and the default risk.

2. *Spread instruments*: The payoff of spread instruments depends on movements in credit spreads. An example is a Credit Spread Option (mostly puts) that makes a payment in case a credit spread moves beyond a *strike level* K. Credit spread forwards are also common instrument in this class. This kind of derivatives allows users to take positions on the future spread between two financial assets, with one of them of stable credit risk as reference, such as government bond or an interbank rate. There is not yet a liquid market for such instruments, while event instruments have gained significant liquidity.

3. *Total return instruments*: The most popular within this class are total return swaps (TRS, some times called total rate of return swaps, TRORS, where the payoff depends on the behavior of spreads as well as on events such as defaults. We will use TRS as notation throughout the thesis. They allow an investor to transfer the total LIBOR plus a spread, mostly a absolute spread (see above *credit spread)*. Another common type is a loan swap. Some would not consider these being true credit derivatives, though, as they do not isolate credit risk, while the other two classes of instrument do. The main characteristic of these products is that they are instruments written on both credit and market risk. The two risks are bundled together and furtheron sold to clients.

Only the credit default swap and the credit default option truly separate credit risk from market risk. Nonetheless, a TRS, as well as credit linked notes or credit spread option have a significant market risk component as well, and thus can be considered as products with embedded credit risk derivatives. For example, a CDS differs from a TRS in that the investor does only the risk of default and not take price risk of the reference asset.

Current credit derivatives are very flexible financial contracts. Their payouts can be derived from loans or bond values, default or credit event, credit spreads or credit ratings. These reference assets, in turn, can be associated with single-names, baskets or indices with *cash settlement* or *physical delivery* of a relevant underlying asset or portfolio of assets. For an extensive analysis and classification of the characteristics of credit derivative instruments, we refer to Das (1998, 2000) and Tavakoli (2001).

Credit default instruments

The following subsection will focus more on credit default instruments. Prior to doing this some words about the differences. Credit risk can be broadly grouped into two different categories. On the one hand there is *credit migration risk*[21]. A downgrade, or a warning by a rating agency are two explicit examples. But changes in credit spread can also serve as an indicator of credit migration risk. The second element of credit risk is *credit default risk*[22]. This latter is far from being identical to credit migration risk, although it is certainly correlated with it. Credit default instruments trade default risk, by separating it from credit migration risk. Credit default instruments share the properties of instruments that exist already for a long time. For example, banks have issued letters of credits, guarantees and insurances for a long time. The major distinguishing characteristic of these traditional instruments is that they transfer credit risk only. On the other side they do not transfer market risk or the risk of credit migration. Essentially, a payment is made when default occurs. With these products no compensation is exchanged in case of credit migrates. New credit default products share some the properties of these traditional instruments. There are two mainly kinds of such instruments:

1. CDS, where a periodic fee is paid until a credit event occurs, triggered by which a potential is received.

2. *Credit default options* (or *credit spread options)*. These are similar to CDSs, the fee being however paid up-front. Essentially both instruments involve swapping a fixed fee against a contingent payment in case of default.

The characteristics of credit default instruments will be dealed with when we present credit default swaps in more detail (see section 1.3.7). For credit spread options we refer to Schönbucher (2003).

It is worth stressing that default swaps are not options. By academics and practitioners credit default swaps are sometimes referred to as credit default options. This may have come about because the transaction involves payment of a fee by the protection buyer in return for a payoff in case default should occur. However, a key characteristics of options is their asymmetrical payoff and price performance as the price of the underlying changes. By contrast, the price of the default swap varies directly with changes in the credit spread of the

[21] The credit risk based on a multi-state credit event process associated with rating migrations from transition probability based models are called *credit or rating migration risk*.

[22] The credit risk based on the probability of default associated with a default process is called *credit default risk*.

underlying, in a similar way as the price of an interest rate swap varies directly with changes in the underlying interest rate.

We discuss the pricing of credit default swaps below in subsection 1.3.8. Credit option products - so called credit spread options or specifically credit default options are well presented in Schönbucher (2003). Frequently when default swaps are called credit default options, people become confused as to whether the product under discussion is indeed a credit default swap and not a credit default option, as there are different instruments.

Spread instruments and total return instruments are products that trade credit migration -changes in credit worthiness as perceived by the market. For a more comprehensive discussion on this types of instruments we refer to Schönbucher (2003), Tavakoli (2001), Bowler and Tierney (2000), O'Kane (2001) and Das (1998, 2001).

1.3.4 Credit Derivative Structures and Applications

In this subsection Asset Swaps, Total Return Swaps and Credit Spread Options will be briefly presented which are beside the Credit Default Swaps the most important instruments for completing market information in such that the credit worthiness of a firm might be deduced from market parameters. In the next section 1.3.7 a fairly detailed discussion of thee structure and applications is given. For more complete presentation of credit derivatives structures and applications we refer to Bowler and Tierney (2000) and O'Kane (2001).

An **Asset Swap** is a synthetic floating-rate note. By this we mean that it is a specially created package that enables an investor to buy a fixed-rate bond and then hedge out almost all of the interest rate risk by swapping the fixed payments to floating. The investor takes on a credit risk that is economically equivalent to buying a floating-rate note issued by the issuer of the fixed-rate bond. For assuming this credit risk, the investor earns a corresponding excess spread known as the *asset swap spread*.

Asset swaps are playing various roles in the structured credit market. At one level, the investment objectives and economics of asset swap transactions are recurring themes in the structured credit market, and as such asset swaps can be viewed as one of its basic building blocks. Indeed the development of many structured credit products has been driven by the shortcomings of asset swaps. But asset swaps are also an important structured credit product in their own right.

While the interest rate swap market was born in the 1980s, the asset swap market was born in the early 1990s. It continues to be most widely used by banks, which use asset swaps to convert their long-term fixed-rate assets, typically balance sheet loans and bonds, to floating rate in order to match the maturities of their short term liabilities, i.e. depositor accounts. During the mid-1990s, there was also a significant amount of asset swapping of government debt, especially Italian Government Bonds. Figure 1.7 below shows a typical asset swap transaction.

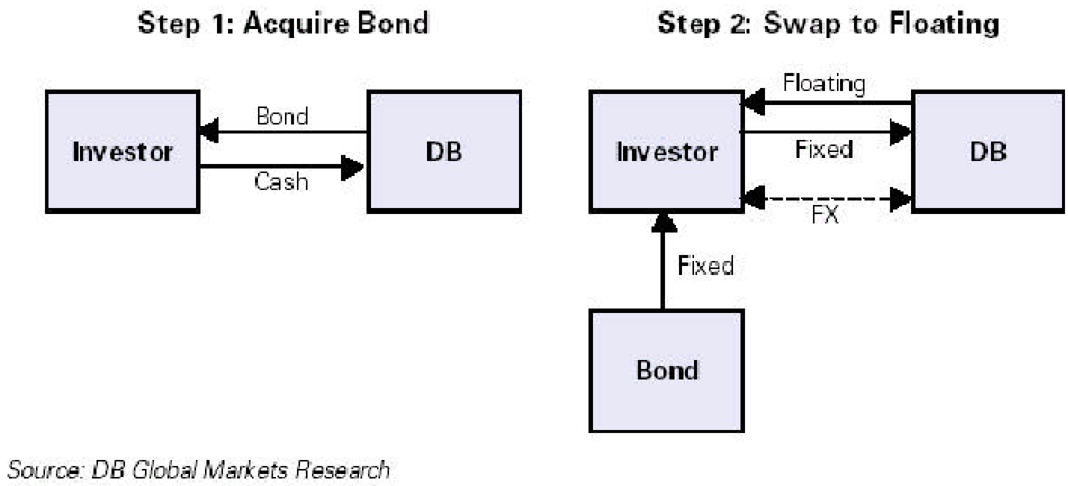

Figure 1.7: Asset swap transaction

The main reason for doing an asset swap is to enable a credit investor to take exposure to the credit quality of a fixed-rate bond without having to take interest rate risk. For banks, this has enabled them to match their assets to their liabilities. As such, they are a useful tool for banks, which are mostly floating rate based. Asset swaps can be used to take advantage of mispricings in the floating rate note market. Tax and accounting reasons may also create an incentive for investors to buy and sell non-par assets at par through an asset swap.

A **Total Return Swap (TRS)** is a contract that allows investors to receive all of the cash flow benefits of owning an asset without actually holding the physical asset on their balance sheet. As such, a total return swap is more a tool for balance sheet arbitrage than an outright credit derivative. However, as a derivative contract with a credit dimension - the asset can default - it usually falls within the scope of the credit derivatives trading desk of investment banks and thus becomes classified as a credit derivative. Before discussing why this product may be of interest for investors, we describe the mechanics of the structure, which is shown in Figure 1.8.

At trade inception, one party, the total return receiver, agrees to make payments of LIBOR plus a fixed spread to the other party, the total return payer, in return for the coupons paid by some specified asset and the changes in value of the asset itself. In case of default, the TRS-receiver faces a loss which encompasses the coupons as well as the usually dramatic loss

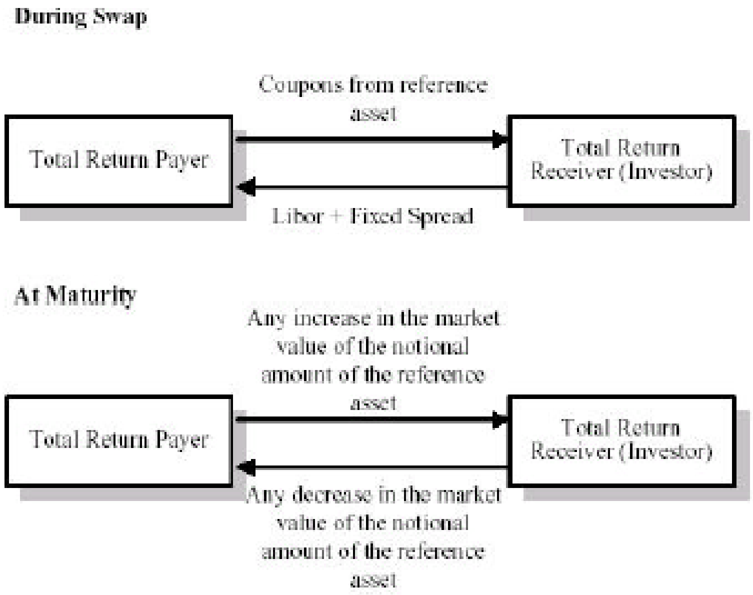

Figure 1.8: Total return swap mechanics; Source: Lehman Brothers International (Europe)

in value of the asset itself. The asset is delivered or sold and the price shortfall paid by the receiver. In some instances, the total return swap may continue with the total return receiver posting some collateral.

There are several reasons why an investor might wish to use such a total return structure: In terms of funding/leverage:

- Total return swaps enable to take a leveraged exposure to a credit.
- They enable investors to obtain off-balance-sheet exposure to assets to which they might otherwise be precluded for tax, legal or regulatory, or other reasons.

In terms of trading/investing:

- Total return swaps make it possible to short an asset without actually selling the asset. This may be useful from the point of view of temporarily hedging the risk of the credit, deferring a payment of capital gains tax, or simply gaining confidentiality regarding investment decisions.
- Total return swaps can be used to create a new synthetic asset with the required maturity. Credit maturity gaps in a portfolio may therefore be filled.

A **Credit Spread Option** is an option contract in which the decision to exercise is based on the credit spread of the reference credit relative to some strike spread. This spread may be the yield of a bond quoted relative to a Treasury or may be a LIBOR spread. In the latter case, exercising the credit spread option can involve the physical delivery of an asset swap, a floating-rate note, or a default swap.

This reference asset may be either a floating rate note or a fixed rate bond via an asset swap. As with standard options, one must specify whether the option is a call or put, the expiry date of the option, the strike price or strike spread, and whether the option exercise is European (single exercise date), American (continuous exercise period), or Bermudan style (multiple exercise dates). The option premium is usually paid up front, but derivatives with a schedule of regular payments exists as well.

A call on the spread (put on the bond price), expressing a negative view on the credit, will usually be exercisable in the event of a default. In this case, it would be expected to be at least as expensive as for the corresponding default swap. For a put on the spread (call on the bond price), expressing a positive view on the credit, the option to exercise on default is worthless and, hence, irrelevant. Figure 1.9 below shows the mechanics of a call option on an asset swap

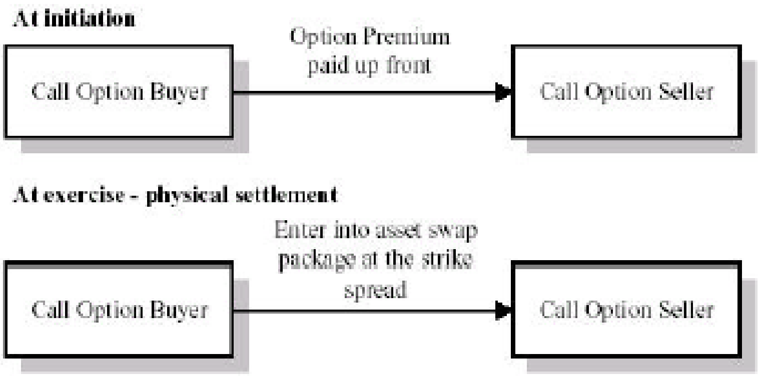

Figure 1.9: Mechanics of a call option on an asset swap; Source: Lehman Brothers International (Europe)

Credit spread options present an unfunded way for investors to express a pure credit view. Unlike options on fixed rate bonds, which we discuss in the next section, the decision to exercise has no dependency on interest rates. It simply depends on where the credit spread of the reference credit is relative to the strike spread.

For example, the value of a 1-year option to enter into a 5-year asset swap is determined by the 5-year asset swap spread one year from today. It is, therefore, a strategy based on assumed movements in the forward asset swap spread and so can be used to take a view on the shape of the credit curve. The more volatile the credit spread, the more time-value the option will have and thus more the option will be worth. Furthermore, if the investors hedge the option by trading the underlying, they will be long volatility. As a result, credit spread options allow investors to express a view about spread volatility separate from a view about the direction of the credit spread.

Buying an out-of-the money put option on a bond is similar to a buying protection with a default swap with one advantage: it can be exercised even when the credit deterioration is significant but a default event has not yet occurred.

An extension of credit-spread options is the exchangeable asset swap option. This gives the purchaser the right but not the obligation to swap one asset swap package for another asset swap package linked to different credit. This makes it possible for the purchaser of the option to take a view on the difference between two underlying asset swap spreads. Investors can use credit spread options to assume a position in credit spread volatility.

Applications

Some of the applications of credit derivatives have already been mentioned when they were specific to the credit derivative discussed. General fields of application common to most credit derivatives are:

1. An important role of credit derivatives on portfolio level is the fact that they enable to choose investments between the two extremes of bond or loan portfolios without credit risk (all assets being hedged by corresponding credit derivatives) and -on the other side- assuming credit risk without any direct exposure at all (pure derivatives portfolio)

2. Applications in the management of credit exposures: These include the reduction of credit concentration (through basket structures), easier diversification of credit risk and the direct hedging of default risk.

3. In trading, credit derivatives can be used for the arbitrage of mispricing in defaultable bonds (through the possibility of long and short positions in credit risk) and the general possibility to trade a view on the credit quality of a reference credit (usually through credit spread products).

4. The largest group of credit derivative users are banks who use credit derivatives to free up or manage credit lines, manage loan exposures without needing the consent of the debtor, manage (or arbitrage) regulatory capital or exploit comparative advantages in costs of funding (sometimes called 'funding cost arbitrage'). Another important application in this context is the securitization of loan portfolios in form of collateral

loan obligations CLOs[23]. There are some additional benefits to booking the transaction *off balance sheet*[24] from a tax or accounting point of view (see chapter 7 in Tavakoli (2001)).

5. The specification of the credit derivatives can be adjusted to the needs of the counterparties: Denomination, currency, type of coupon, maturity or even the general payoff need not match the reference asset. This is especially useful in the management of counterparty exposures from derivatives transactions.

1.3.5 The Market and the Use of Credit Derivatives

Credit derivatives and structured credit instruments are relatively new compared to derivatives related to market risk only and have gone trough a major revolution during the last few years, and are still subject to a continuous. In such an environment where most of these developments come directly from the industry we want to present briefly an overview of the market and the use of they instruments.

The interest in this new market lies not only in the large margins that banks may raise with exotic, difficult-to-price instruments but more importantly in the fact that the birth of this market has truly "completed" the financial market, see subsection 'Credit Derivatives can complete market information', creating both arbitrage opportunities on a previously poorly priced credit risk and allowing corporates and financial intermediaries to manage their *credit risk exposure*[25]. Credit risk is everywhere, in any contract involving uni- or bilateral flows or claims on future cash flows.

The possibility of trading credit risks, buying or selling to a counterparty independently

[23]In general, the purpose of a cash flow Colletral Loan Obligation, so-called CLO, is to move a portfolio of loans off the balance sheet of a commercial bank. This is done in order to free up the regulatory and/or economic capital that the bank would otherwise be obliged to hold for risks arising from these loans. This allows banks to use this capital to fund other higher-margin business, new product lines, or share repurchase plans. It furthermore transfers the credit risk of these loans to the buyer of the CLO, thereby reducing the bank's concentrations of credit risk. The main purpose of cash flow CLOs is to move credit risk off the balance sheet of a bank for the purpose of regulatory capital reduction.

[24]Off balance sheet means, obligations that are contingent liabilities of a bank, and thus do not appear on its balance sheet. In general, off balance sheet items include the following: direct credit substitutes in which a bank substitutes a own credit for a third party, including standby letters of credit; irrevocable letters of credit that guarantee repayment of commercial paper or tax-exempt securities; risk participation in bankers' acceptances; sale and repurchase agrements; and asset sales with recourse against the seller; interest rate swaps, OTC options and Credit Derivatives, except when embedded in structured notes.

[25]The *credit risk exposure* can consist of a couple of components such as current counterparty exposure or potential future exposure and others depending on the implemented methodologies. The *counterparty exposure* is related to the market value of of the portfolio of OTC derivative positions with a counterparty that would be lost if the counterparty were to default. The *current exposure* is the current market value of the exposure to a counterparty. The p*otential future exposure* is a measure on how the exposure might develop in the future with some degree of statistical confidence.

of the underlying bond itself is one of the attractive feature of credit derivatives. The credit derivative market (sometimes called structured credit market) encompasses a broad range of capital markets products designed to transfer credit risk among investors through over-the-counter transactions. These include a variety of off-balance sheet products such as credit default swaps, total return swaps, first-to-default baskets[26], and credit spread options; and on-balance sheet customized structured products such as credit-linked notes, repackaged notes, and standard or synthetic collateralized debt obligations (CDOs)[27]. These types of transactions is what makes the credit derivative market and in particular the CDSs so attractive. The risk replication of a CDO via a portfolio of CDSs is one of the key transactions and one of the major reasons why the CDS market remains more liquid then other credit derivative instruments.

The credit derivative products have developed over the past decade, but it has only been in the past couple of years that this sector has started attracting more attention and experienced accelerated growth. The British Bankers' Association estimates that from a total notional amount of $180 billion in 1997, the credit-derivatives market grew more than tenfold to $2.0 trillion by the end of 2002. Furthermore, the British Bankers' Association (BBA) forecasts that the total notional amount of credit derivatives will reach $4.8 trillion by the end of 2004. In a November 19, 2002 speech before the Council on Foreign Relations, Federal Reserve Chairman Alan Greenspan praised the credit derivatives market for its role in allowing banks and other financial institutions to hedge credit risk. The nearly-explosive growth of this market, however, has also been accompanied by controversy[28]. In particular, concerns have been raised recently about whether credit protection is priced fairly in the credit-derivatives market. For example, it was claimed that hedge funds have artificially driven up the price of credit protection in an effort to induce the ratings agencies to downgrade specific firms (the Wall Street Journal on December 5, 2002).

[26] In a first-to-default basket, or so-called first-to-default basket default swap, we have several reference credits with their respective reference credit assets. The credit event is triggered by the first default in the basket.

[27] A collateralized debt obligation (CDO) is a structure of fixed income securities whose cash flows are linked to the incidence of default in a pool of debt instruments. These debts may include loans, revolving lines of credit, other asset-backed securities, emerging market corporate and sovereign debt, and subordinate debt from structured transactions. When the collateral is mainly made up of loans, the structure is called a Collateralised Loan Obligation (CLO), and when it is mainly bonds, the structure is called a Collateralised Bond Obligation (CBO).

[28] For example restructuring as a credit event in 1999 ISDA definitions has been a bone of contention in the credit derivatives market for quite some time - see "ISDA resolves restructuring issue" (29.5.2001).

Credit Derivatives - a successful Financial Innovation

The growth to date has bee n driven by several factors. Banks, under increasingly pressure to improve financial performance, have turned to credit derivatives to more actively manage the concentration and correlation risk inherent in their loan portfolios as well as the economic and the regulatory capital required to support their operations.

The Asian financial crisis that began in August 1997, with devaluations by Thailand and Korea and various corporate defaults, initiated a new wave of interest in credit derivatives. Investors who had become more aware of about credit risk began to focus more on using credit derivatives to hedge or lay off credit risk. But it was in the period following the Russian default in August 1998 that the credit derivatives market seemed to come of age. On the one hand, the Russian crisis highlighted a number of documentation and administration problems in the market, on which market participants have taken decisive steps to address[29]. These difficulties, however, were relatively minor. In most respects the credit derivatives market worked the way it was supposed to, and the Russian crisis illustrated to participants and non-participants alike how these products transfer credit spread risk and default risk between counterparties. Before that event, many investors had viewed credit derivatives as a curious but highly specialized and exotic corner supplementing the bond market. When suddenly faced with the prospect of deteriorating credits and bond market illiquidity worldwide following Russia's default, investors could see in a very tangible way the attractions of a market where one could "buy protection" to reduce risk and "sell protection" to diversify a concentrated portfolio.

Another source of growth has been the presence of significant credit arbitrage opportunity across different market sectors (e.g., loans and bonds) as well as across different countries. Structured credit entails applying financial engineering techniques to leverage these opportunities and create customized financial products for investors, including credit-linked notes and repackaged notes. Many investors are unable to participate directly in the credit derivatives markets due to regulatory constraints, investment policy guidelines, or a lack of analytic tools to evaluate, price and effectively use credit derivatives in their portfolios. However, in many cases they are able to acquire structured credit investments incorporating credit derivatives or repackaged assets that would have otherwise been inaccessible in conventional cash form. In the emerging markets front, investors often have been unable to invest directly in these securities due to a variety of non-credit risk factors, including administrative, custodial, legal, tax and other issues. As the Asian crisis gave rise to extremely discounted valuations throughout

[29] See also "LTCM crisis" detailed explained in Dunbar (2000).

the emerging markets, investors sought structured products that isolate credit risk from other factors, which helped broaden the investor base for emerging market securities and credit derivatives. Credit derivative trading operations were established to help clients meet risk management needs and develop credit arbitrage techniques to create many new investment products.

A further key factor for the huge growth is the standardized credit derivatives definitions introduced by the ISDA. This project established a new and standardised ISDA confirmation form for credit derivatives, thereby simplifying the documentation and approval process of credit derivative transactions. Even more importantly, transactions based on the new definitions will be based on a common set of definitions. Previously, credit derivatives transactions were based on ISDA documents introduced in early 1998, but different firms and countries had developed minor variations on the standard ISDA long form confirmation. This gave rise to potential legal issues, and as a result, some participants were selective about who they would deal with, and other conservative firms probably did not participate in the credit derivatives market at all. The new definitions, by eliminating this documentation/legal risk, are a significant factor in broadening the investor base for credit derivatives and improving market liquidity. Fore more details see 1999 ISDA credit derivatives definitions.

The credit default swap has become the dominant product in the credit derivatives market. As in previous surveys from BBA, single-name credit default swaps continued to be by for most popular credit derivatives product in 2001, represented 45% or nearly half of market share in terms of notional. The percentage of Portfolio Products/CLOs has substantially increased over the last few years and constituted 22% of market share in 2001. It is predicted to increase even further to 26% of market share by 2004. None of the other products captured more than 8% of market share in 2001 and are not predicted to do so in the near future. Table 1.6

Head	**1996**	**1997**	**1999**	**2002**	**2004(est)**
Credit Default Swaps	35	52	38	37	34
Portfolio/CLOs	NA	NA	18	18	23
Asset Swaps	NA	NA	12	11	9
Total Return Swaps	17	14	11	10	8
Credit linked Notes	27	13	10	11	11
Basket Products	6	5	6	7	7
Credit Spread Products	15	16	5	6	8

Table 1.6: Market share of credit derivatives in percentage

Regulatory issues

A significant issue for the credit derivatives market has been a less than accommodating regulatory environment. Credit derivatives, being relatively new, by and large do not fit into the existing regulatory framework for banks or insurance companies, and face ambiguous accounting treatment. Progress is being made on this sector but for the most part appears to be slow.

Documentation – The new ISDA credit derivatives definitions represents a major step forward for the credit derivatives markets. This project was undertaken largely in reaction to the documentation problems that arose after the Russian default in August 1998. However, its true significance lies in greatly simplifying the administrative process of closing credit derivative transactions and eliminating documentation inconsistencies across different market participants. This will lead to a broadening of the investor base for credit derivatives and substantially improved liquidity.

Banking industry – Many bank regulators consider credit derivatives as being development of enormous potential importance for helping banks manage the credit risk inherent in their lending activities. At the same time they acknowledge that the existing regulatory capital framework provides little incentive to encourage banks to develop and use these tools. According to Bowler and Tierney (2000):

> "*We review the guidance that has been issued to date in six countries. While each country tried to respect the spirit of the original Capital Accord, there are significant differences and inequities among them. On a positive note, bank regulators in some countries are becoming more comfortable with credit derivatives, and have shown willingness to consider and approve credit derivative strategies on a case by case basis.*"

The Basle Committee on Bank Supervision issued a discussion paper in 1998 that proposed a major changes to the existing risk based capital framework, but the proposed framework for credit risk mitigation strategies (i.e. credit derivatives) remains conservative.

Insurance industry – Insurance companies in many countries can use derivatives only for hedging purposes. However, they can invest in CLNs and repackaged notes[30]. Some countries provide strong incentives or even require that insurance companies invest in principal-protected structures. In the US, the industry has been working with regulators to develop

[30] Repackaging vehicles such as repackaged notes are used to convert or create credit risk structures in a securitized form that is accessible to a broad range of investors. They can be used to convert existing credit derivative products into the cash form required by many investors. They can be used as well to increase liquidity and to trade risks that do not currently exist in a traded format.

a replication framework. This would allow insurance companies to create synthetic positions using cash instruments and derivatives that replicate otherwise permitted investments.

Accounting – Derivatives, including credit derivatives, will are be subject to mark-to-market accounting under new accounting standards issued by the Financial Accounting Standards Board in the US and the International Accounting Standards Committee in London. These new rules established a new hedge accounting models for derivatives. Credit derivatives and their applications raise a variety of issues that were not included in current accounting standards. The Derivatives Implementation Group (DIG) will address these issues on a case by case basis.

For detailed discussion on regulatory issues about credit derivatives we refer to Bowler and Tierney (2000) and O'Kane (2001).

1.3.6 Credit Derivatives can complete Financial Market Information

The credit derivative will take more importance when regulators will admit standard reductions in capital requirements when credit risk is mitigated, diversified away, or in particular transferred through credit derivatives. Why is it important? Because it is linked to the main deterrent of sustainable financial markets: the illiquidity or the scarcity of liquidities. Theoretically markets are efficient, they can be volatile as they could be, but times of distress are always recovered. In reality, there exist some barriers under which the market will not be able to recover which might be seen analogous to stop loss rules for traders on the exchange[31]. In periods of recession, the need for cash as well as the cost of available cash can be very high. The lack of available inflow can include large selling blocks, falling prices and rolling distress that can drive the whole market into a crash. It is not obvious that credit derivatives will really help to manage the risk of a breakdown of the market, or a substantial number of its members due to illiquidity.

1.3.7 Credit Default Swap Basics

As mentioned before, credit derivatives are a fundamental innovation that might serve to solve many important practical problems such as transfering credit risk. CDSs have become the dominant instrument in the credit derivatives market, and we include a fairly extensive discussion of this instrument within this and the next section.

[31] For example a stop loss rule for stocks is the customer sell order to triggers a sale in case the stock price falls below a pre-defined value. A stop-loss order therefore will protect profits that have already been made or prevent further losses if the stock drops.

This section concentrates on CDS as they constitute the lions share of the pure credit derivative market. Based on the terminologies and definitions made before this section will provide with a detailed definition of a CDS and an illustrative example to give a more realistic idea of how a CDS works. Furthermore, the key rule played by the CDSs within the credit derivatives market is discussed to point out that the CDS is a major factor in completing financial markets by using credit derivatives. For more complete discussions see Tavakoli (2001), Das (1998, 2001) or Schönbucher (2003) Moore and Watts (2003) serves with a recent market guide for CDS. The valuation of CDSs is discussed more deeply in the next section.

Specification of Credit Default Swaps

For explanatory purposes, we focus on the 'plain vanilla' single-name CDS, which means that the CDS under discussion is an instrument on a single reference credit asset. This is the most common form of such instruments. Let us further mention that there exist some exotic CDS structures, so-called 'Default Digital Swaps' (DDS), which is a CDS with a constant pre-defined amount as default payment or a 'First-to-Default Swaps' (FTDS), which are examples of CDS written on a portfolio or 'basket' of reference credit assets (i.e., CDS linked to a portfolio of business loans or credit-sensitive securities). Such kind of so called multi-named CDS can be thought of as the building blocks for synthetic collateralized debt obligations (CDOs).[32]

Let's make clear, based on our 'Terminologies and Definitions' we made before, what a plain vanilla single-name CDS (in the following denoted as CDS):

> In a CDS **B** (the protection seller) agrees to pay the *contingent default payment* to **A** (protection buyer) in case a default has happened. If there is no default of the *reference asset* until the maturity of the CDS, counterparty **B** pays nothing. **A** pays a fee (premium payment) for the default protection. The fee can be either a lump-sum fee up front (credit default put option) or periodic fixed payments expressed in basis points per notional until default or maturity of the CDS. A typical CDS structure is shown in Figure 1.10.

CDSs mainly differ in the contract specification. The most important features of such contracts are:

1. The specification of the *credit event*, which is in a CDS formally defined as a default,

[32] CDS are no longer purely used as a form of insurance for lenders or an alternative method of gaining credit risk exposure. Their common features and tradability has allowed people to create new and innovative financial products and was named the 'building block' for a number of other products-mainly for portfolio-based ones such as collateralized and credit-linked instruments (see also Moore and Watts (2003)).

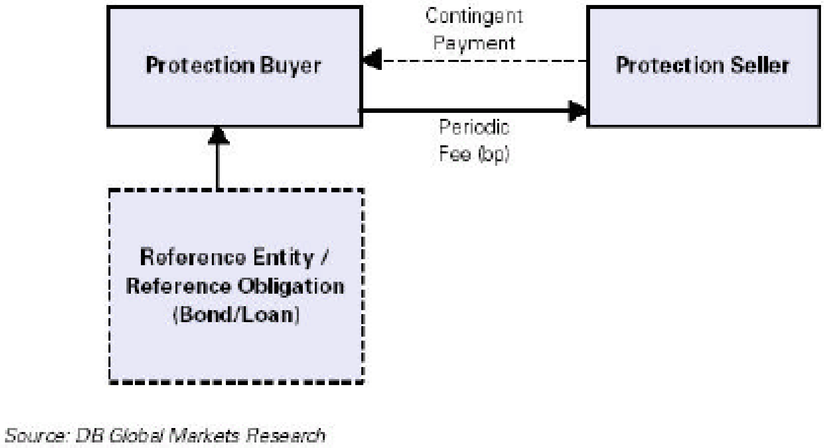

Figure 1.10: Credit default swap transaction

2. The *contingent default payment*, which may be structured in a number of ways. Common alternatives are:

   - An alternative might be the payment of the difference between par and actual value by the protection seller (settlement in cash, so called *cash settlement*),
   - The protection buyer (**A**) may deliver one or several of the reference assets (or other qualifying assets that are equal to or higher in priority of payment) to the protection seller (**B**) in exchange of the corresponding par value (*physical delivery*),
   - It can be set at a pre-determined level of a fixed percentage of the notional amount of the transaction (DDS).

Sometimes substitute securities may be delivered, or an exotic payoff may be specified (e.g. to hedge counterparty exposure, see above, in derivatives transactions).

A CDS allows the separation of the credit risk component of a defaultable bond from its non-credit driven market risk components. The protection buyer (**A**) retains the market risk but is hedged against the credit risk of **C**, while the protection seller (**B**) can assume the credit risk alone. Depending on the credit quality of the protection seller, his counterparty may require him to post collateral on the contract which can range from 5% to 50% of the notional amount, see Schönbucher (2003). The possible payments in default have already been discussed above. Both counterparties can agree to appoint a calculation agent who supervises the price determination and settlement procedures of the CDS in default.

In the context made above it is worth nothing that credit default swaps are not credit swaps or credit event swaps. Sometimes the literature uses the wording default swap, credit swap or credit event swap for credit default swaps. The differences comes from the specification of *periodic contingent payments* which depend, in large part, on the credit quality of the reference asset. Credit swaps or credit event swaps can be settled in cash and also provide for physical delivery. For example, it may involve payment at par by the seller in exchange for the delivery of the defaulted reference credit asset. If the payment is triggered by the default and equals to the difference between the face value of a bond and its market price, a contract is named the CDS.

In *cash settlement* the price determination is involved, Therefore most of the CDSs specify *physical delivery* in default. Cash settlement is only chosen when there may not be any physical assets to deliver (i.e. the reference credit has not issued enough bonds) or if the CDS is embedded in another structure where physical delivery would be inconvenient, as in the case of CLN. Physical delivery is not entirely without problems, e.g. when traders have speculated on the default of **C** by buying credit protection without having an underlying position in the bonds. These traders will have to buy **C**-bonds after a default event in order to deliver them to their respective protection sellers. This demand might push up prices to an artificially high level which may damage the value of the default protection.

One more word on terminology: The asset that is traded in a CDS is *default protection*. This means:

- A *long position* in a CDS is a position as *protection buyer*,
- A *short position* in a CDS is a position as *protection seller*,
- A bid of x *bp*[33] on a CDS means that the bidder is willing to enter in a CDS as protection buyer for a periodic fee (sometimes called CDS rate or CDS spread) of x *bp* at the notional amount, and
- An offer of y *bp* on a CDS means that the offerer is willing to enter in a CDS as protection seller for a periodic fee of y *bp* at the notional amount.

Credit Default Swap Example

Lets look at an example for 'plain vanilla' CDS in which the reference asset is a corporate bond:

Credit default swap on DaimlerChrysler bonds:[34]

[33] bp=basis points. 1bp is 0,01% of an absolute amount.

[34] This example is from Schönbucher (2003) and is here presented with some modifications.

Counterparty **A** and **B** enter in credit default swap on DaimlerChrysler AG. They have agreed on:

- Protection buyer: **A**
- Protection seller: **B**
- Notional amount: USD 50 million
- Reference credit: DaimlerChrysler AG (**C**)
- Reference credit asset: a particular or class DaimlerChrysler bond issues
- Trade date = today ($t = 0$)
- Effective date = Trade date[35]
- Maturity Date: the term of the CDS is 5 years from today
- Payment amount: CDS fee 120 bp per annum
- Day count convention: e.g. act/360[36]
- Default payment and settlements: see below

These are the most important items in the specification of a simple CDS. For other details of contract specifications we assume that **A** and **B** follow the specifications proposed by the ISDA.

The settlement period for the trade is three business days from the trade date, this is when the payment period for the first fee and the default protection begin.

The fee payments: The credit default swap fee 120 *bp* is quoted per annum as a fraction of the notional amount. **A** pays the fee in regular intervals, semi-annually. To simplify the day count fractions we choose $\frac{1}{2}$ such that **A** pays to **B**:

$$120\ bp \times USD\ 50\ million = 600000\ USD\ at\ t_1 = 0.5, t_2 = 1, ..., and\ at\ t_{10} = 5.$$

These payments are stopped and the CDS is unwound as soon as a default of Daimler-Chrysler AG on its payment obligations for the bond in question occurs.

[35] Often the effective start of an CDS or even for a simple interest rate swap could be some days or weeks later than the trading day.

[36] For a comprehensive discussion about day count conventions we refer to Miron and Swannell (1995).

Determining a credit event: The credit events (bankruptcy, failure to pay, obligation acceleration, repudiation/moratorium or restructuring) are defined in the CDS contract with respect to a large set of bonds issued by the reference credit (**C**) DaimlerChrysler. Let us assume these are all senior unsecured EUR and USD-denominated[37] bonds issued by DaimlerChrysler with an issue size of at least USD 10 million or EUR 10 million ($\frac{USD}{EUR} = 1$). Furthermore, let us assume that at the time to default, DaimlerChrysler AG has failed to pay a coupon payment that was due at this date on one of the bonds listed in the contract. This could potentially constitute a credit event according to the CDS contract, if some conditions are met. First the disputed amount must exceed a materiality threshold, and second, it must remain unpaid after a grace period of some days. If these conditions are fulfilled then the protection buyer **A** will notify the protection seller **B** of the occurrence of a credit event and the CDS contract is unwound.

The default payment: First, **A** pays the remaining accrued fee. If the default occurred two month after the last fee payment, **A** will pay $USD\ 600000 \times \frac{2}{6}$. The next step is the determination of the default payment. If physical settlement has been agreed upon, **A** will deliver DaimlerChrysler bonds to **B** with a total notional amount of USD 50 million. The set of deliverable obligations has been specified in the documentation of the CDS contract. As liquidity in default securities can be very low, this set usually contains more then one bond issue by the reference credit **C**. Naturally **A** will choose to deliver the bond with the lowest market value[38], unless he has an underlying position of his own that he needs to unwind. However, even then he may prefer to sell his position in the market and buy the cheaper bonds to deliver them to **B**. This delivery option enhances the value of his default protection. **B** must pay the full notional for these bonds, USD 50 million in our example. If cash settlement has been agreed upon, a robust procedure is necessary to determine the market value of the bonds after default. If there were no liquidity problems, it would be sufficient to ask a dealer to give a price for these bond, and use that price, but liquidity and manipulations are a serious concern in the market of distressed securities. Therefore not one, but several, dealers are asked to provide quotes, and an average is taken after eliminating the highest and lowest quote. This is repeated, sometimes several times, in order to eliminate the influence of temporary liquidity holes. Following this procedure the price of the defaulted bonds is determined, e.g. 450 USD for a bond of 1000 USD notional amount. Now, the protection seller **B** pays the difference between this price and the par value for a notional amount of USD 50 million, in numbers $(1000 - 450)/1000 \times 50{,}000{,}000\ USD = 27{,}500{,}000\ USD$.

[37] USD-denominated means the bond is issued in another currency than EUR, here in USD, but does not necessarily mean the bond is issued outside the EUR-Zone.

[38] Cheapest to deliver.

Whatever settlement procedure is agreed upon, the CDS settles very quickly in a matter of only a few weeks (usually around six weeks) after the credit event is noticed. This is much faster than the determination of the final recovery rate (see recovery value above) through bankruptcy courts.

Tavakoli (2001) presents for the most common structures in the credit default protection market besides many other examples with detailed indicative term sheets which gives an idea how complex a real CDS contract can be. See also Schönbucher (2003) for an illustrative example of a CDS on distressed debt.

Credit Default Swap the leading Credit Derivative Instrument

As we mentioned before the CDS is the most liquid and dominant credit derivative instrument form of a single-named instrument. Even more it is the most common instrument, serving as the building block[39] for many multi-name products. Why is this so? The ability to use CDSs to hedge (or assume) credit spread risk and to create customized maturity products account for large part for the success of the CDS product. CDSs are not static instruments that only perform in the event of default. They are dynamic, market-sensitive products whose mark-to-market performance is closely related to changes in credit spreads. More about this see subsection 'Credit Default Swap valuation' below. As a result they are an effective tool for hedging against (or assuming exposure to) changes in credit spreads as well as default risk. Credit spread option products provide an asymmetric (option) approach to hedge against or assume credit spread risk but the market for this product is not well developed at this time.

Furthermore, CDS have the special feature that they can be used to create credit exposure of different maturities to a reference credit (**C**) that are unavailable in the cash market, which is one of the key benefits of CDSs. For instance, consider an investor who has a negative view on the future credit-worthiness of a given corporation. One strategy for such an investor would be to short the bonds issued by the corporation, but the corporate repo market and other mechanisms for taking short positions in corporates are not well developed for most individual corporate issuers. In buying protection with a CDS, the investor essentially replicates the cash flow of a short position in the corporation's debt. In case the corporation default, the investor is able to buy the defaulted debt for its recovery value in the market and sell it to its counterparty the protection seller (**B**) for its face value. CDSs allow counterparties to buy and sell protection on credit risk inherent in a bond, loan, or guarantee/swap counterparty exposure. According to Bowler and Tierney (2000):

[39] In this thesis we focus principally on the single-named credit default swap, which is commonly described as a 'building block' for a number of other products, mainly for portfolio-based ones such as collateralised and credit-linked instruments, which have become increasingly important products recently. More about this in Moore (2003), JPMorgan (2002), Schönbucher (2003).

> *"... if an investor wants a three-year maturity and duration exposure to an issuer that has only 1.5-year and 10-year securities outstanding, the appropriate exposure can be created by selling a three-year default swap, or by purchasing a credit-linked note with an embedded written default swap* (CDS). *Total return swaps are sometimes marketed as providing customized maturity exposures (e.g., a five-year total return swap on a 10- year reference asset), but it is important to distinguish between maturity and duration exposure. During the term of the swap the total return receiver has full exposure to the market risk and duration of the reference asset, as if it were on the balance sheet."*

To create or hedge maturity exposures, the CDS provides for useful engineering characteristics, in particular, because of the fact that the CDS will fall naturally as the residual from the decomposition of a typical risky bond (we show this later in "CDS pricing via replication" - Example 1 and 2). More precisely we take a bond that has default risk (in the following also called risky bond only) and then show how the cash flows of this bond can be decomposed into simpler, liquid constituents. CDSs fall as a natural constituent of this decomposition. This natural function played by CDS may explain probably mostly their appeal in the credit derivative market and their position as the leading credit instrument.

When an investor's objective is to transfer or acquire credit risk in the derivatives market, the CDS is the most effective and liquid tool available. As we mentioned before the CDSs are often described and offered as financial instruments that provide protection against default risk. While this description is certainly true, it is also limited and does not begin to convey the full versatility of the product.

1.3.8 Credit Default Swap Valuation

This section is focusing fairly extensive on the valuation methodologies for CDS pricing After presenting an overview of the different methodologies and the involved model parameters including explanations and discussions, the replication method, a structural model and a reduced form model is considered. The part of the replication method is illustrated with some well known and important examples initiated by Duffie (1999). The presentation of a structural model initiated by Das (1995) is given in overview form, while more details on reduced-from model will serve with some useful fundamental techniques within this methodology. The reduced-form models are most widely used in practice to price CDSs and were initiated by Jarrow and Turnbull (1995). The implementation of such models is discussed after presenting a recent literature survey for CDS pricing models. Finally we summarize the main issues of the use of CDSs and their valuations. For concepts and the mathematics behind the most important and popular credit risk models see Schönbucher (2003).

Methodologies and Model Parameters

Since a reliable benchmark model for credit derivatives is not yet available, it is common in market practice to value a credit derivative on a stand-alone basis, using a judiciously chosen ad hoc approach, rather than a sophisticated mathematical model. Before we shall review the most widely used methodologies for CDS pricing we quote one statement from the industry: According to Tavakoli (2001):

> *"As one U.S. bank credit derivatives department head said, in talking about some of the more exotic credit default structures:* ***The spread is where it is because that is where market says it is****. What he was referring to is that the models for certain types of credit derivatives either don't exist or require assumptions about unknowable unknowns."*

To give an overview for a classification of market pricing methods for CDSs valuation the following approaches are established in academia and practice:

1. *Replication-based method*: This method prices the CDS by an evaluation the cost of a portfolio replicating the CDS. This method replicates the standard approach to contingent claim valuation in an arbitrage-free set up. The benefit of this approach is the pricing of CDS via replicated well known and more liquid fixed-income instruments from the cash market.

2. *Structural-model-based method*: As we discussed before this method based on the firm value and the share price (capital structure of the firm). The classical credit risk structural models were introduced by Merton (1974) and Longstaff and Schwartz (1995). Based on the balance sheet information of the borrower and the bankruptcy code they derive endogenously the probability of default and the credit spreads based on no-arbitrage arguments, making some additional assumptions on the recovery and the default-free interest rate term structure (see also credit-spread based methods).

3. *Reduced-form-based method*: These models work directly with the probability of default (and the corresponding processes) of risky debt as an exogenous variable calibrated to some data, their name coming from the reduction of the credit economics behind the probability of default. This approach for CDS pricing was mainly initiated from Duffie (1998), Lando (1998) and Duffie and Singleton (1999). For more classical reduced-from models we refer to Jarrow and Turnbull (1995).

4. *Credit-spread-based method*: The valuation using this method is based on a comparison of the yield of the reference credit asset and the yield of a risk-free bond with similar maturities and focus on the credit spread. This method will estimate a credit spread directly using structural models or indirectly by using reduced-from models.

5. *Credit-rating-based method*: This approach based on the available credit ratings by rating agencies such as Standard and Poor's, Moody's KMV or Fitch. To derive default probabilities an estimated Markov transition-matrix is used representing the credit-rating-migration process. The credit-spread-based method can be seen as a variant of the credit-rating-based method[40].

We will focus our discussion within this subsection on the methods 1, 2 and 3. For detailed descriptions of these models we refer to Schönbucher (1998, 2000, 2003), Bielecki and Rutkowski (2000), Duffie (1999) and Cossin and Pirotte (2001). For practical implementations building trees (lattice approach) and Monte Carlo methods are widely used, in some specific cases also closed-form approaches. For closed-from solutions we refer to Hübner (1997), Duffie and Singleton (1999) Jarrow and Turnbull (2000) and Cheng (2001).

The specification of the *model parameters* such as *recovery rate*, *intensity (or hazard-rate)*, *credit spread*, *default probabilities* and *default correlation* within these models is the most difficult part of the valuation of a credit derivative and there underlying risk. The valuation of a credit derivative depends on the measure of the credit risk. Credit risk models usually involve a relatively large number of parameters when compared with any standard model of market risk. Additionally, in many cases the volume of available empirical data related to credit-sensitive assets is insufficient for statistical studies (the scarcity of data might cause problems even for a reliable estimation of the credit-spread curve). Let's now discuss briefly the specifying *model parameters*:

- *Default probabilities*: The notion of a credit event involves a number of various situations related to the *credit quality* of the *reference asset*. It is thus worth to mentioning that in most empirical studies undertaken before 1990 by default probability researchers have meant a probability of defaulting on either interest or principal payment, see also Bielecki and Rutukowski (2000). In more recent studies, it is common to adopt a less stringent definition of default, which can be more adequately referred to as credit distress, see also Bielecki and Rutukowski (2000). In this context, let us

[40] There is a large and - in the light of Basle II - rapidly growing literature on the problem of how to assign a rating to a given obligor and how to build up an internal rating system. We do not consider this problem here.

observe that though the different debts of the same firm encounter credit distress at the same time, it may well happen that senior debt obligations are satisfied in full during bankruptcy procedures, while subordinated debt is paid back only partially. This feature is accounted for in the specification of different recovery rates to individual types of debt of the same firm, according to the debt seniority. Let us stress that observed default frequencies correspond to the actual probabilities of default, as opposed to the risk-neutral probabilities which are used to value derivative securities. In an arbitrage-free setup, the risk-neutral default probabilities should be seen as by-products obtained within the model, rather then the model inputs.

- *Recovery rates*: It is known that the likely residual value net of recoveries depends on the seniority class of the debt, in the case of default. To account for this feature, we may assume that the value of a recovery rate depends not only on the bond credit quality, but also on the seniority classification of the bond, from senior secured to junior unsecured. It is debatable whether it should be represented as a constant or as a random variable. For simplicity, a random recovery rate can be assumed to be independent from other random quantities involved in the used model construction. In practice the recovery can be modelled as recovery *equivalent* in terms of risk-free bonds, as *cash* or as *fraction* of the pre-default value of the defaultable bond. Historically, the first assumption made is that of *equivalent recovery*, introduced by Jarrow and Turnbull (1995) and has been further developed by Lando (1998) and Madan and Unal (1998). Under this assumption, each defaulting security is replaced by a number $0 \leq R \leq 1$ of non-defaultable, but otherwise equivalent securities. One advantage of equivalent recovery is that it allows us to calculate implied survival probabilities (see 'A reduced-form model for CDS pricing' below) from bond prices using closed-form equations for a given recovery rate R. On the other hand, a fixed recovery rate implies an upper bound on the credit spread. An *equivalent recovery model* uses decomposition of the defaultable bond price into default-free bonds and defaultable bonds with zero-recovery. The strength of this approach is that it allows default-risky claims to be valued as if they were default-free and discounted by an adjusted interest rate. Schönbucher (2000) present a *fractional recovery* model as an extension of the Duffie and Singleton model (1999) for multiple defaults. In this model a default does not lead to a liquidation but a reorganization of the issuer: defaulted bonds loose a fraction of their face value and continue to trade. This feature enables us to consider European-type payoffs in our derivatives without necessarily needing to specify a payoff of the derivative at default. Both the equivalent and the fractional recovery assumptions do not correspond to market conventions for bonds. When a real-world bond defaults, the bond holders recover a fraction R of the bond's principal value (and perhaps of the accrued interest since the last coupon date, but we will ignore this for simplicity). The outstanding coupon payments are lost. In the literature, this convention is sometimes called *recovery of face value*. Similar to the case of equivalent recovery, the recovery rate does impose bounds on credit spreads under recovery of face value. However, the effects are more complex than in the equivalent recovery case and are best analyzed on a case-by-case basis. In general, these constraints only become binding for long maturities or in extreme cases.

- *Intensity (or hazard-rate)*: An intensity-based method is an alternative to the structured-based methods. Because of the non-existence of a firm value process within an intensity-based model another technic must be introduced to model the payment in case of default. Therefore an intensity-based modelling is focusing on the default process. The most simple form of the time to default with intensity is a exponential distributed random variable. The characteristic of the default process is described through its intensity. These methods following an insurance-based approach which is based on martingale methods. For an introduction to these methods we refer to Bielecki and Rutukowski (2000).

- *Credit spreads.* The knowledge of credit spreads represents a prerequisite of the credit-spread-based approach. To be more specific, we need to examine not only the credit-spread-curves, but also credit-spread volatilities, and, if in advance several distinct assets are modelled simultaneously, the credit-spread correlations. Due to the relative scarcity of data, the estimation of the credit-spread-curve is more problematic than the estimation of the risk-free yield-curve. This is especially difficult to overcome when one deals with the risky debt issued by a particular firm. In such a case, one might use the rating-specific credit-spread-curve as a proxy for the unobservable firm-specific credit-spread-curve (see Fridson and Jónsson (1995)). The difficulty in collecting sufficient empirical data will be probably lessen in the future, with the further development of the sector of credit derivatives. The same remarks apply to the estimation of credit-spread volatilities, which in principle can be statistically inferred from the observed variations of the credit-spread yield curve, see for example Fons (1987, 1994) or Foss (1995). An alternative and perhaps more promising, approach would be to focus instead on volatilities implicit in market prices of the most actively traded option-like credit derivatives. In practice, the construction of a credit-spread-curve will be mostly described via the term structure of discount factors for a risky issuer under the zero recovery assumption. Inputs for the credit-spread-curve are typically bonds, whether fixed coupon non-callable bonds or floating rate notes, asset swap spreads, or CDS spreads.

- *Default correlations* are simply the correlations between two or more defaults of single named issuer. Empirical analysis of default correlation is limited by the lack of default events. One study, Lucas (1995), which computed the default correlation between assets in different rating categories, has made two particularly interesting observations. The first is that a default correlation increases as we descend the credit rating spectrum. This has been attributed to the fact that lower rated companies are more vulnerable to an economic downturn than higher rated companies and are thus more likely to default together. The second observation is that a default correlation is horizon-dependent and it has been postulated that this may be linked to the periodicity of the economic cycle. Computing industry-industry default correlations is difficult due to the shortage of default events. In practice, default correlation is often approximated using some other quantity such as credit-spread correlation or stock price correlation. While this may appear a reasonable assumption, strictly speaking there is no model-independent mathematical relationship that can link the two together.

Credit Default Swap Pricing via Replication

Although complex mathematical models (see section 1.3.8) should ideally become the way of pricing credit derivative instruments, the complexity of the contracts and low reliability of fitting such models have made more simple methodologies predominant in pricing credit derivative instruments. The best alternative, when possible, is to price these instruments via simple replication. For some of the basic credit derivative instruments there exist simple approximative hedge and replication strategies. The cash-and-carry valuation methods are often very important and popular in financial engineering. This methods are based on the fundamental principle of no-arbitrage and the concept of dynamic hedging[41]. Therefore we must use the risk-neutral probabilities for this valuation approach. The risk-neutral probabilities are extractable from prices of traded securities.

The replication approach provide upper and lower bounds and hedge strategies that cover much of the risks involved in the credit derivative instruments. The results are robust because

[41] The opposite of no-arbitrage is arbitrage which is simply the combination of trades in several instruments which yields in a riskless return without investing additional money. In a perfect market are no arbitrage opportunities, meaning that the market prices are related in absence of arbitrage. If there were arbitrage opportunities, the market would immediately realise such discrepancies and adjust the prices so that there are fair.

they are independent of any specific pricing model. the CDS and also the TRORS are usually priced this way. Applying the replications approach we assume in the following that the real market is perfect. In reality, markets -especially in the CDS market- show significant differences related to the theoretical relations. In this market the liquidity plays a major rule and has to be taken into account for an adequate pricing methodology.

A CDS is designed such that the combined position of a CDS with a defaultable bond issued by **C** is very well hedged against default risk, and should therefore trade close to the price of an otherwise equivalent default-free bond. This is the intuition behind the cash-and-carry arbitrage pricing of CDSs. The cost of setting up the replicating portfolio gives the pice of the CDS.

Before we start with important replication examples for CDS we need to make some *notations and definitions* for underlying and hedge instruments:

- *Loans* are the oldest and well known type of payment obligation, and they are still one of the most important ways to raise investment capital. They are bilateral contracts between the borrower and the lender, which is usually a bank. Basically a loan consists of a sum, the principal or the notional amount, which was originally lent by the creditor to the obligor and which the obligor has to repay to the creditor at the maturity date. Additionally, the obligor has to make regular interest payments in the meantime.

- *Bonds* are the securitized version of loans, the major difference to loans is that they are tradeable in small denominations in the secondary market. Tradeability has the advantage that it opens access to a much larger number of potential lenders. The lenders can lend smaller amounts and do not need to remain invested for the whole borrowing period, because they can easily sell the bond before its maturity. The defining characteristics of a bond are its issuer, a notional coupon size, frequency of coupon payments and maturity date. We consider here standardized plain bonds. For a detailed discussion of standard and structured bonds we refer to Fabozzi (2001).

In the following we consider *default-free bonds* and *defaultable bonds* with fixed and floating coupons as well as *default-free* and *defaultable zero-coupon bonds.*[42]

- *Default-free bonds*: These are default-free coupon bonds issued by **C** with fixed and float coupons. *Fixed-coupon bonds* are the most common type of default-free and

[42] As we described before *convertible bonds* are also typical underlying instruments (see also section1.2). In addition to the conventional coupon and principal payment of a plain fixed coupon bond, the holder of a convertible bond has the right to convert the bond into shares.

defaultable bonds. The coupon amounts are fixed in advance, and we assume that the notional amount is only and fully repaid at maturity. $C(t)$ denotes the time t price of a default-free coupon bond with a coupon of c. *Par floater or floating coupon bonds* also have full prepayment at maturity but their coupon amounts are linked to a benchmark short-term interest rate, usually *Libor*, denoted as L_t for the time t *Libor*. The *floating coupon bonds* has a price $\tilde{C}(t)$ at time t.

- *Defaultable bonds*: These are defaultable coupon bonds issued by **C** with fixed and floating coupons. The default fixed coupon bond carries a coupon of c^d and has a price $C^d(t)$ at time t. We also consider defaultable floating coupon bonds which have also full prepayment at maturity as well whose coupon amounts, however, are linked to a benchmark short-term interest rate plus a constant spread, $L + s^{par}$, and a price $\tilde{C}^d(t)$ .The spread is chosen such that the price of the par floater is initially at par.

- *Zero-coupon bonds* do not have any coupon payments during the lifetime of the bonds and to compensate the investors they are issued at a significant discount from par. They are rarely seen in the corporate bond market but they are very convenient *building blocks* for credit risk modelling. These results from applying structural models such as Merton (1974). At any time t there are default-free zero coupon bonds of all maturities $T > t$. We use $B(t,T)$, for the time t price of a *default-free zero-coupon bond* with maturity T and $B^d(t,T)$ for the *defaultable zero-coupon bond.*

- *Interest-rate swaps*: $s(t)$ denotes the *swap rate* at time t of a plain vanilla fixed-for-floating interest rate. The *forward interest-rate swap rate* contracted at t for the time interval $[t_n, t_N]$ is
$$s := s(t) = \frac{B(t,t_n) - B(t,t_N)}{a(t;t_n,t_N)}, \tag{1.43}$$
where $a(t;t_n,t_N) = \sum_{i=n+1}^{N} \delta_i B(t,t_i) =: a(t)$ is the value of an *annuity* paying δ_i at each date t_i starting from the first date t_{n+1} after t_n and δ_i are the day count fraction for the time intervals $[t_i, t_{i+1}]$, which are assumed equidistant. The maturity is $t_N = T$. If the swap start immediately, we have $t_n = t$. In this case the annuity value is exactly the value of 1 basis point and therefore called 'PV01'. Some of the hedge instruments may not always be available. Most obligors only issue fixed-coupon bond, if they issue traded at all. In many cases the only bonds available carry call provisions or are convertible into equity which makes them unsuitable for the simple hedging strategies outlined here. On the other hand we can assume the existence of interest-rate default free coupon bonds and interest-rate swaps of all maturities. The floating rate L_{t_i} of the interest rate swap and the swap payment are assumed to be

default-free.

- *Recovery*: In the following we assume that the event premium is the difference of par and the value of a specified reference credit asset after default. The value after default is the *recovery value* which defined by a *recovery rate* R as a fraction of par (see also the item 'Recovery rate' before).

For simplicity we need to make some *assumptions*:

We assume that the coupon payments of coupon bonds and the fixed leg of the swap payments occur on the same dates which are denoted by $0 \leq t_0, t_1, t_2, ..., t_N$. In reality this will not always be the case but the relevant adjustment should be straight forward. We also abstract from the day count conventions that have to be used to adjust payments occurring at regular intervals. Therefore, we write $s(t_i)$ for the payment of the fixed leg of an interest-rate swap at times $t_i, i \geq 1$, where it in reality should be $s\,(t_i)\,\delta_i$, where δ_i is the day count fraction for the time period $[t_{i-1}, t_i\,]$[43]. Furthermore we consider assume zero recovery.

Let's now start with some examples to create a CDS via replication. In order to simplify the discussion of financial engineering aspects we need to make some additional assumptions on the payoff of a CDS in default.

1. We assume that the payoff takes place at the time of default. The time delay through grace periods, dealer polls, etc. is ignored.

2. We ignore the delivery option that is embedded in a CDS with physical delivery. We will frequently consider portfolios that contain a defaultable bond which is protected by a CDS, and in this case the defaultable bond is the only deliverable bond of the CDS.

3. We assume that the CDS is triggered by all defaults of the reference credit and only by defaults of the reference credit. In particular we ignore the possibility of technical default, i.e. events that trigger the CDS default definition while they do not constitute a real default of the reference credit such as "restructuring of the debt". Furthermore, we ignore the possibility of legal, documentation and specification risk, i.e. the risk that a real default does not trigger the CDS. In tabular representations of the payoff streams of a trading strategy we abstract from day count conventions and set for all. Otherwise we assume that the same day count conventions and payment dates apply to all the securities involved.

[43] For standard technics of swap pricing and hedging see Miron and Swannell (1991) (the best book available for pricing and hedging plain-vanilla Swaps and the most expensive one as well).

These assumptions apply throughout the rest of the following examples. They are common assumptions, although they are explicitly stated.

Example 1: ***Synthesizing a risky bond via static replication***

We present here a very intuitive approach to synthesize a risky bond with a CDS and other liquid financial instruments as a technology to replicate a CDS in a simple way. The presentation of this example based mainly on our class notes, see Neftci (2003).

Consider a defaultable bond purchased at time t_0. The bond does not contain any call or put options and pays a coupon c^d annually over 3 years. This means there are coupon payments, c^d, in t_1, t_2, and in t_3. For simplification we make two assumptions. First we assume that, in case of default, the recovery value will be equal to zero, and secondly that the default only occurs in t_3.

Considering the cash flows of this bond and assuiming that the bond is initially purchased for 100, three coupon payments are made and the principal of 100 is returned if there is no default. On the other hand in case there is default (in period t_3 only), the bond pays nothing. There is an optionality due to the possibility of default at time t_3. At this time period, there are two possibilities of default and no-default and the claim is contingent on these. How do we replicate these cash flows in a way the constituents can be converted into liquid financial instruments? We answer this question in steps.

Step 1:

By introducing a useful trick that will facilitate greatly the application of static replication methods to defaultable instruments. Remember that we would like to isolate the underlying default risk using a single instrument. This task will be much simplified if we add and subtract the amount $c^d + 100$ to the cash flows in the default case at time t_3. Note that this does not change the original cash flows in any way.

Yet it is very useful in isolating the inherent CDS. In fact the bond contains three different types of cash flow structures.[44]

[44] Drawing cash-flow diagrams as in Miron and Swannel (1991) can help to illustrate these facts as well as further cash-flow algebtra throughout this example, which we haven't done in this work.

Firstly there are the three coupon payments on the dates t_1, t_2, t_3. Of course, by assumption, the third coupon payment h as default risk, but the trick of adding and subtracting $c^d + 100$ in the cash flows of time t_3 , permits considering this as if there were a guaranteed payment at time t_3. Therefore -although- the last coupon payment is risky, we can extract three default free coupon payments from the bond cash flows due to the trick we used.

In fact, to get the default-free third coupon payment we simply pick the positive c^d at time t_3. Note, that this leaves the negative c in case default occurs.

The second type of cash flow structure is the initial and final payments of 100. Again the trick of adding and subtracting 100 is used to obtain a default-free receipt of 100 at time t_3.

As a result of these two cash flow structure the negative payment of 100 in the default state of time t_3 remains. The remaining cash flow structure consist of the negative cash flow $c^d + 100$ that occurs at time t_3 default state.

Step 2:

The next step is to convert these three cash flow structures into recognizable and preferably the most liquid contracts available in the markets. Remember that in order to do so we add and subtract any arbitrary cash flow to the three chash-flow-structures above as long as the following conditions are met:

- For each addition of a cash flow we need to subtract the same amount (or its present value) from somewhere else in the cash-flow-structures.
- These cash flows should be added so that the resulting instruments become liquid instruments.
- As a consequence when added back together the now modified cash-flow-structures should give back the same cash flows as the one we initially started. In other words, we should be able to recover the cash flows of the defaultable bond.

To convert the first cash-flow-structure above into a recognizable instrument we subtract a floating Libor payment, L_{t_i} at times t_1, t_2, t_3. The resulting series of cash flows looks like a *fixed-receiver interest rate swap.* This is convenient because swaps are very liquid instruments. However, there is one additional modification required. The fixed receiver swap rate is usually a rate $s(t_0)$ such that

$$s(t_0) \leq c^d,$$

the difference, denoted by s^c,

$$s^c = c^d - s(t_0), \tag{1.44}$$

being the *credit spread* over the swap rate. This is how much a credit rated AA- and lower has to pay over and above the swap rate due to the possibility of default. Note that we are defining the credit spread as a spread over the corresponding swap rate and not over the treasury rate. This is in fact market practice and the "correct" way to handle.

Therfore in order to have the cash flows being labelled a receiver swap, we need to subtract s^c from each coupon receipt c. It follows that the fixed receipts will equal the swap rate

$$c^d - s^c = s(t_0) \tag{1.45}$$

and, the resulting instrument becomes a true interest rate receiver swap.

The construction, however, leaves an important question to be answered. Where do we add and subtract the cash flows s^c and L_{t_i} that we just introduced? After all, unless the same cash flows are entered somewhere else with opposite signs, they will not cancel out and the resulting synthetic instrument will not reduce to a risky bond.

Let's start with the Libor payments. The three Libor payments need to be compensated by three Libor receipts that need to be added somewhere else in the whole cash flow structure, preferably at the same times. Adding the corresponding Libor receipts in the second type of the cash flow structure convert this into a *default-free money market deposit*. This deposit will be rolled over at the going floating Libor rate. Note that this is also a very liquid instrument. Alternatively we can call this a Floating Rate Note (FRN).

The final adjustment is how to compensate the reduction of c^d by the credit spread s^c. We add the s^c to the remaining cash flow structure and the result is a new instrument. This is the critical step since we have now obtained a new instrument that has fallen naturally from the decomposition of the risky bond. Essentially this instrument has three receipts s^c dollars at times t_1, t_2, t_3. Yet, if default occurs, the instrument will makes a payment of $c^d + 100$. According to this, in case of default at time t_3, the next payment becomes $c^d + 100 - s^c$. Doing this once can see that indeed the vertical sum of all cash flows replicate exactly the cash flows of the *defaultable bond*. What is the instrument? How can we call it?. In fact this instrument is equivalent to *selling protection against default risk* of the bond. The contract involves collecting fees equal to s^c at evert t_i until the default occurs. Then, the protection buyer is compensated for the loss $c +$ *100*$- s^c$. On the other hand, if there is no default, the fees are collected until the expiration of the contract and no payment is made. We call this instrument a Credit Default Swap (CDS) and in fact the defined credit spread above is equal the CDS spread or so-called the CDS rate ($s^c = s^{cds}$).

The step by step discussion above shows that a *defaultable bond* could be replicated with a portfolio made of a *fixed receiver interest rate swap*, a *default-free money market deposit* and a *CDS*. Here, the maturities of the bond, swap and the CDS would be the same. Hence using these instruments we can write down the following contractual equation:

Defaultable Bond (on credit reference asset) = Receiver Swap + Money Market Account (default free deposit) + CDS (on reference credit asset),

which is algebraically equivalent to

CDS (on reference credit asset) = Defaultable Bond (on credit reference asset) - Receiver Swap - Money Market Account (default free deposit)

and another possible replication following the same logic above is

CDS (on reference credit asset) = Defaultable Bond (on credit reference asset) + Payer Swap - default-free Money Market Loan.

Furthermore, the prices of defaultable bonds contain extremely important information about the market's assessment of the issuer's credit risk. Defaultable bonds are natural hedge instruments against default risk exposures and the price of defaultable bonds and credit default swaps are closely linked, as we will also see in the following examples.

Example 2: *Hedge strategy with fixed-coupon bonds*

Lest's consider the following two portfolios:

Portfolio 1:

- A long position in a defaultable fixed coupon bond: cost at $t = 0$ is $C^d(0)$, coupon is c^d and maturity is t_N
- A short position in a CDS: CDS spread s^{cds}

Portfolio 2:

- A long position in a default-free fixed-coupon bond: this bond has the same payment dates as the defaultable fixed coupon bond, maturity t_N and the size of the coupon is $c^d - s^{cds}$. The cost at $t = 0$ is $C(0)$

Table 1.7[45]

Time	**Portfolio 1**		**Portfolio 2**
	defaultable fixed-coupon bond	CDS	default-free fixed coupon bond
$t = 0$	$-C^d(0)$	0	$-C(0)$
$t = t_i$	c^d	$-s^{cds}$	$c^d - s^{cds}$
$t = t_N$	$1 + c^d$	$-s^{cds}$	$1 + c^d + s^{cds}$
$t = \tau$	R	$1 - R$	$C(\tau)$

Table 1.7: Replication strategy with fixed-coupon bonds

Portfolio 1 is fixed after a default and the default-free fix coupon bond is sold in case of default. The idea is the following: $portfolio\ 1$ can be seen approximately as a synthetical default-free fixed-coupon bond. Defaultable bonds are natural hedge instruments against default risk exposures because the price of a defaultable bond and of the corresponding CDS are closely linked (see example 1). In absence of arbitrage and if the payoffs of both portfolios are the same, provided the payoffs in default also coincide, the initial cost of both portfolios should be the same. This can be seen from Table 1.7. Otherwise, a risk-free profit can be

[45] See later in subsection 1.3.8 an exact definition for the "*time to default*", denoted here as τ.

made by buying the cheaper portfolio of the two, and short selling the more expensive one. Doing this and taken the payoffs in default in account, we must yield

$$C^d(0) = C(0) = B(0, t_N) + a(0)\, c^d - a(0)\, s^{cds}. \tag{1.46}$$

Now the default-free fix-coupon bond price is expressed as the sum of values of the final principal repayment and the regular coupon payments. The equation above can be solved for the 'fair' CDS rate (spread) s^{cds} for any given term structure of default-free interest rates. Considering the payoff in default we realize that the payoffs of portfolio 1 and 2 do not coincide exactly. This is the CDS gives us the right to put the defaultable fix coupon bond to the protection seller (**B**) at par, so that the value of *portfolio 1* will be the notional value in the event of default. The value of the equivalent default-free fix-coupon bond will depend on the term structure of the default-free interest rate, and almost certainly it will differ from par. The difference at default between portfolio 1 and portfolio 2 will be

$$1 - C(\tau). \tag{1.47}$$

There are several reasons why $C(\tau)$, the value of the default-free fix-coupon bond at time to default will differ from 1. It may have already been off par at time $t = 0$, $C(0) \neq 1$, or, if the term structure of the interest rate is stochastic, the value of $C(\tau)$ will move stochastically as well and there is no reason to believe that it will be close to par, except at its final maturity. Additionally, there is the issue of accrued interest. At the coupon payment dates t_i the dirty price of the bond will drop by the coupon payment amount $c^d - s^{cds}$, and in the following it will be increase again until the next coupon payment date.

The resulting price path will have a typical saw-toothed pattern that is common to all coupon-paying bonds. These facts illustrate that the replication method based on the cash-and-carry arbitrage is only an approximate arbitrage relationship. The reason for that lies in the unknown value of the default-free fix-coupon bond at a random point in time. For further discussions on this issue we refer to Schönbucher (2003).

The example presented here mainly based on Schönbucher (1999, 2003). The following example is also based on descriptions in Schönbucher (1999, 2000, 2003). Nevertheless all replication examples presented here and in the recent literature are based on the initial work of Duffie (1999).

Example 3: ***Hedging strategy with floating rate notes and floating rate coupon bonds***

I) *Floating rate notes:*

A CDS on a defaultable floating rate note (FRN) can be used to set up a perfect hedge even for stochastic recovery rates and correlation between spreads and interest rates. We use this property to give a characterization of the CDS rate in terms of the credit spread of defaultable FRNs. Buying protection via a CDS is functionally equivalent to shorting a FRN with the same maturity, issuer, credit rating and seniority in case of bankruptcy and buying a corresponding AAA FRN. In equilibrium, the value of the CDS should be equal to the replicating portfolio.

Assume the following payoffs:

Portfolio 1:

- The defaultable FRN pays the floating *Libor* rate, L_{t_i}, plus a constant spread s^{frn} per time.

Portfolio 2:

- The default swap rate is s^{cds} and the default swap pays the loss to par in default.

The default-free FRN pays the floating rate *Libor*. Both floaters have a final payoff of 1. The payoffs are shown in Table 1.8

Time	**Portfolio 1**	**Portfolio 2**	
	defaultable FRN	CDS	default-free FRN
$t=0$	1	0	1
$t=t_i$	$L_{t_i-1}+s^{frn}$	s^{cds}	L_{t_i-1}
$t=t_N$	L_{t_N-1}	s^{cds}	$1+L_{t_N-1}$
$t=\tau$	R	$-(1-R)$	1

Table 1.8: Replication strategy with FRNs

If the defaultable FRN trades at par at $t=0$, i.e. $F^d(0)=1$[46], then initially (at $t=0$), at default ($t=\tau$) and at maturity ($t=T=t_N$), we have

$$defaultable\ FRN + CDS = default\text{-}free\ FRN. \tag{1.48}$$

[46]Lets denote the time t price for the defaultable FRN by $F^d(t) := F^d(t, t_i)$

As both the spread, s^{frn} for the defaultable FRN, and the CDS rate s^{cds} are constant they must coincide. Therefore we get

$$s^{frn} = s^{cds}. \tag{1.49}$$

Assume there is a defaultable FRN that trades at par and has a spread of s^{frn} over the default-free rate . Then $s^{frn} = s^{cds}$ is the fair swap rate for a CDS of the same maturity. For a deeper discussion on this see Duffie (1999) and Schönbucher (1999, 2000, 2003). This argument only uses a simple comparison of payoff schedules, it does not use any assumptions about the dynamics and distribution of default-free interest rates or credit spreads or about the recovery rates. If the FRNs pay coupons only at discrete time intervals this relationship is only approximately valid, with an exact fit at the coupon dates, because only at these times the default-free FRN is worth exactly 1. Finally, we can say that the price of a CDS is closely related to the price of a defaultable floating rate note. Although the defaultable FRN is not a derivative, its pricing is nevertheless not trivial, because it does not always have to trade at par like default-free FRNs.

II) *Floating Rate Coupon Bonds:*

The replication strategy with $parfloaters$ is quasi equivalent to the FRN replication strategy. (Remember that a par floater with value $\tilde{C}^d$ is a defaultable bond with a floating rate coupon of $\tilde{c}^d = L_{t_i} + s^{par}$, where the par spread s^{par} is chosen such that at issuance the par floater is valued at par, see above.)

Considering the following portfolio strategy:

Portfolio 1:

- One defaultable floating coupon bond (defaultable par floater) with spread over Libor. The coupon is $\tilde{c}^d = L_{t_i} + s^{par}$
- One CDS on this bond with CDS rate s^{cds}

Portfolio 2:

- One default-free par floating-coupon bond with the same payment dates as the defaultable par floater and coupon of Libor flat $\tilde{c} = c = L_{t_i}$

The payoffs are shown in Table 1.9

Time	**Portfolio 1**		**Portfolio 2**
	defaultable floating-coupon bond	CDS	default-free floating-coupon bond
$t = 0$	-1	0	-1
$t = t_i$	$L_{t_i-1} + s^{par}$	$-s^{cds}$	L_{t_i-1}
$t = t_N$	$1 + L_{t_N-1} + s^{par}$	$-s^{cds}$	$1 + L_{t_N-1}$
$t = \tau$	R	$1 - R$	$C^d(\tau)$

Table 1.9: Replication strategy with floaters

The bond is sold after default and matching the payoffs in default and the initial cost we see that the payoffs before default differ only by the difference of the par spread to the CDS rate. These payoffs, however, must coincide, too, in absence of arbitrage. This means that the CDS rate must equal to the par spread, analogously to what we have seen before with the FRN replication strategy

$$s^{par} = s^{cds} \tag{1.50}$$

We can arrange the replication portfolios to synthesize short and long positions in a CDS as shown in Table 1.10.

Synthetic Position	**Replication Portfolio**	
CDS	defaultable floating-coupon bond	default-free floating-coupon bond
long	short	long
short	long	short

Table 1.10: Replication portfolios for synthetic CDS positions

Example 4: ***Hedging strategy with asset swap packages***

The biggest problem with the hedging strategy with floating-rate coupon bonds is that in most cases these instruments do not exist in the market. An obvious alternative would be use of an *asset swap package* instead, as shown in Table 1.11.

Time	**Portfolio 1**		**Portfolio 2**
	asset swap package	CDS	default-free floating-coupon bond
$t = 0$	-1	0	-1
$t = t_i$	$L_{t_i-1} + s^{asset}$	$-s^{cds}$	L_{t_i-1}
$t = t_N$	$1 + L_{t_N-1} + s^{asset}$	$-s^{cds}$	$1 + L_{t_N-1}$
$t = \tau$	$R+$ *interest rate swap value*	$1 - R$	$C^d(\tau)$

Table 1.11: Replication with asset swap package

An *asset swap package* is a combination of a defaultable fixed coupon bond (the asset) with a fixed-for-floating interest rate swap whose fixed leg is chosen such that the value of the whole package is the par value of the defaultable bond. The asset swap package has at least some of the described properties of the defaultable par floaters. Initially, the asset swap package is priced at par, meaning that it has a value of 100, and prior to default it pays a coupon of Libor plus a spread, the asset swap spread, denoted as s^{asset}. Formally,

$$P^{dirty} - 100 = PV^{swap}\left(\sum c^d_{t_i}\right) - PV^{swap}\left(\sum L_{t_i} + s^{asset}\right), \tag{1.51}$$

where P^{dirty} is the dirty price of the defaultable fixed coupon bond (the asset) at time $t = 0$[47]. Therefore, the only difference in value arises at default. While the value of the par floater is just the recovery value, the value of the asset swap package is the recovery plus the market value of the interest rate swap. The interest rate swap derives its value from movements in the default-free interest rate term structure (and from any initial value that the swap might have had).

Let's assume that the initial value of the underlying defaultable coupon bond is close to par, and that the movements in value of the default-free interest rate term structure are quasi independent of the default. Then we can view an asset swap package as a substitute to a defaultable par floater in the strategies of the previous examples and we get the following approximate relationship between the CDS spread and the Asset Swap spread

$$CDS\ spread\ s^{cds} \approx Asset\ Swap\ Spread\ s^{asset}. \tag{1.52}$$

However, there is something missing in our arguments. Compare to the par floater an asset swap package, which has a different feature in case of default. If the default of the reference credit asset occurs the owner of the package receives the recovery value of the defaultable fixed coupon bond and additionally he is the fixed-coupon payer of the interest rate swap. Depending on the swap curve at the time to default, the interest rate swap has a value that could be negative or positive. This value has an influence of the overall recovery value of the asset swap package.

Independently of this tiny issue, the asset swap rate is a good indicator for a fair credit default swap spread. See Figure 1.11.

The accuracy of the relationship between the CDS spread and the Asset Swap spread depends on the degree to which the following *assumptions* are fulfilled:

[47] Note that $P^{dirty} - C^d(0) = accrued\ interest = dirty\ price - clean\ price$, PV denotes the present value in general and PV^{swap} the present value of the swap.

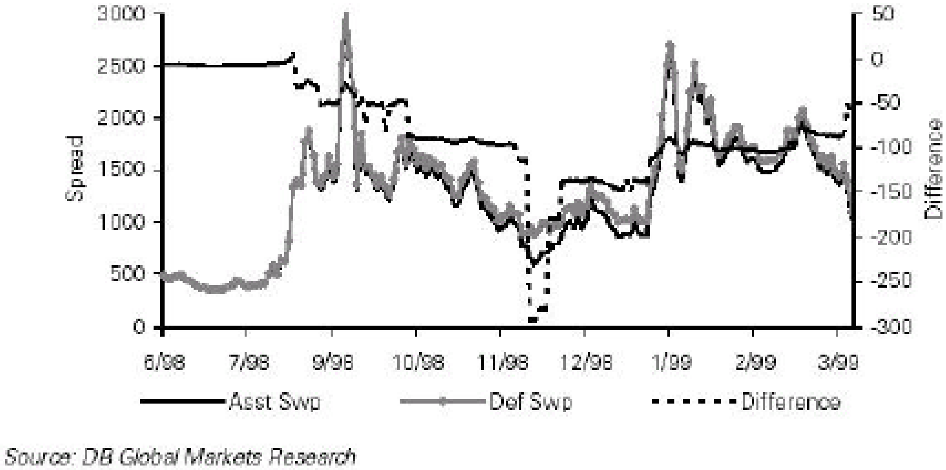

Figure 1.11: Mid-market asset and credit default swap levels on Brazil IDUs

1. The initial value of the underlying bond is at par;
2. Interest rate movements and defaults occur independently;
3. Short positions in the asset swap market are possible; and
4. at default, the default-free floating-coupon bond trades at par.

The first two assumptions ensure that the value of the interest-rate swap does not introduce any bias in the analysis. Assumption 2 makes sure that the expected market value of the interest-rate swap at default is indeed 0. This holds only true for the expectation of the value, while the realization can be different. Assumption 3 is necessary to reach a two-sided bound on the CDS rate, and assumption 4 addresses the slight mismatch at default that arose in the hedge with a par floater.

If short sales of the asset swap package are impossible, then the approximate equality of asset swap spread and CDS is not holding anymore and we get an inequality, see Schönbucher (2003)

$$s^{cds,\, offer} \geq s^{asset,\, offer}, \tag{1.53}$$

where $s^{cds,\, offer}$ is the CDS spread for a CDS offer and $s^{assset,\, offer}$ the asset swap spread for asset swap offer. If on the other hand short sales of the asset swap package are possible, then we have in addition the following inequality

$$s^{cds,\, bid} \leq s^{asset,\, bid}. \tag{1.54}$$

If one analyses quoted spreads in the markets, one can observe that the CDS rate sometimes differ significantly from asset swap spreads even for defaultable bonds that trade close to par. This difference is called the *Basis* between the CDS rates and the spreads in the cash market (bonds, asset swaps, etc.). Theoretically this would tend to show the existence of arbitrage opportunities. The reason for the simply called *Basis*[48] lies mainly in the differences of supply and demand, meaning in fact the liquidity in the different markets. Because of this, the supposed arbitrage opportunities can not be realized in practice. The inequality for the offered CDS rates will in most cases not be violated, but the inequality for the bidding is regularly invalidated.

Sometimes different standards are used in practice to measure the Basis. Usually if the inequality for the bidding is regular, then we have to consider

$$Basis^{*} = s^{cds,\, bid} - s^{asset,\, offer}, \tag{1.55}$$

and if the inequality for the offering is regular we have to consider

$$Basis' = s^{asset,bid} - s^{cds,offer}. \tag{1.56}$$

For an illustration see Table 1.12, which shows differences between CDS rates and asset swap spreads for some selected banks.

Banks	**CDS** bid/offer	**Asset Swap** bid/offer	**Basis** (bid-offer)
Bank of America	48/55	46/43	5
Bank One Corp	60/75	65/60	0
Chase Corp	40/48	35/30	10
Citigroup	38/45	30/27	11
First Union	68/85	66/63	5
Goldman	45/55	46/41	4
Lehman Brothers	70/80	68/63	7
Merill Lynch	40/50	28/23	17
Morgan Stanley	45/55	28/23	22

Table 1.12: CDS and asset swap quotes; Source: Schmidt (2001)

An extreme example for a significant Basis was the telecommunication industry, with 80 bp (calculated as Basis*) for Deutsche Telekom 2001, see Schmidt (2001).

The example (4) above followed mainly the initial work of Schmidt (2001). Schönbucher (2003) presented a similar example.

[48] Basis risk is the imperfect correlation between the reference asset and risk substitution via the corresponding derivative instrument. For example, bond future price versus the theoretical future price of the coresponding bond, or the CDS rate versus the asset swap rate, etc.

Example 5: ***Hedging strategy with the repo market***

A more realistic institutional setup is when the investor wants to replicate the protection through swap and repo market[49]. For this we assume that the defaultable bond is available in the cash market with the same maturity as the CDS. Suppose that an investor sells protection via a CDS on the reference credit asset. The investor can replicate the short position in CDS by the following strategy:

- Purchasing the defaultable bond on the reference credit asset at a spread s^{par} over the risk-free curve for par;
- Paying fixed at a rate $risk\text{-}free + s$ on a swap of the same maturity as the bond against L; and
- Financing the position in the repo market with the defaultable bond on the reference credit asset as collateral at a spread to Libor of $L - s^{repo}$.

See Figure 1.12 for an illustration of the structure[50].

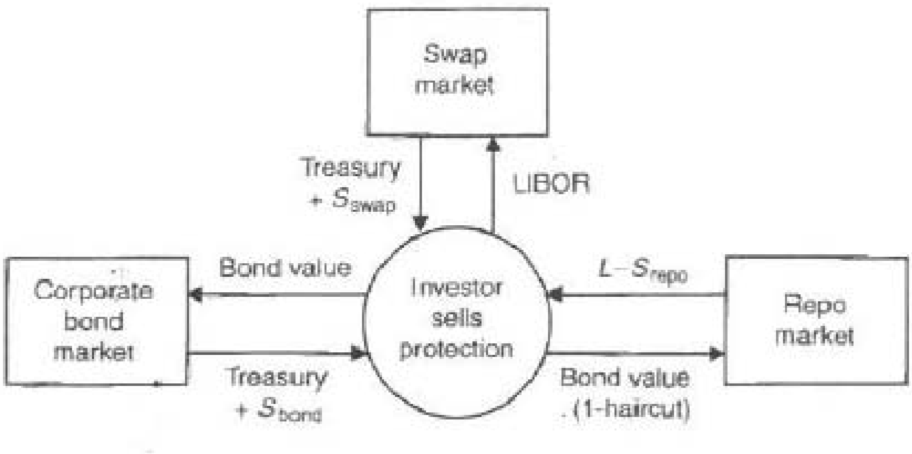

Figure 1.12: Replication of a short position in credit default swap; Source: Cossin and Pirotte (2001)

This strategy exactly replicates a CDS only in the absence of a haircut[51]. In case of a haircut, the price impact of the haircut has to be taken into account. In practice this

[49] As already known by practitioners, government bonds are no longer considered by the markets to be the reference for default free instruments. Swap and Repo rates have taken over this position.

[50] In the picture is Treasry+s_{swap} used instead of *risk-free-rate*+s, as well as $L - s_{repo}$ instead of $L - s^{repo}$, and furthermore s_{bond} instead of s^{par}.

[51] A 'haircut' is an extra amount of cash an investor would have to pay.

replication is used to price CDSs approximately. The net position of the this replication gives as the CDS rate

$$s^{cds} = s^{par} + s^{repo} - s. \tag{1.57}$$

Contract design matters first among instruments that are not fully standardized. Some CDS may require a payment of the CDS rate until maturity even in case of default. This should obviously affect the pricing. For similar discussions see Duffie (1999). Fundamentally, though, the replication described above works well as long as all markets are efficient. Low credit risk may not get good pricing from the swap market, in particular if one cannot access them at all, or even worse, may access them with specific collateral requirements that are not taken into account in the pricing but that do not affect the pricing, see Cossin and Pirotte (2001). One possibility would be to obtain from theoretical models a price modified to take in account the presence of collateral. The swap market remains quite efficient, however the repo markets, especially for high credit risk, may show more inefficiencies. Haircuts may be imposed by collateral, which will also effect the pricing. The counterparty risk itself cannot be neglected as defaults have occurred in the repo market as well. This would also be true for the swap market, but should have less impact on the pricing. More fundamentally, instruments and corresponding rates may not be available to obtain useful replication, i.e. the bond with the correct maturity may not exist and bonds of the same rating are not necessarily good substitutes as rating do not properly take into account recovery risk. Furthermore, the corresponding repo may not available for investors. Worse, and more probable, the repo market may not offer a quote for that specific bond, specially for lowly or unrated bonds. For detailed discussion on this we refer to Duffie (1999), Cossin and Pirotte (2001) and Schönbucher (2003).

Having formal pricing models of different classes of credit derivatives thus represents a viable alternative to replication. We present below models for CDS, or even credit derivatives that can be used when simple replication is not ambiguous, as in subsection 'A Structural Model for CDS pricing'.

Problems with cash and carry arbitrage

The most obstacle to an efficient cash-and-carry arbitrage link between the CDS market and the corresponding defaultable bond market and asset swap packages are problems in implementing short positions in defaultable bonds (i.e. to costly because of possible short-squeezes in bond markets). Furthermore, problems arise when the defaultable bond does not match the CDS in maturity and when coupon and CDS fee payment dates do not coincide. In that case, even in survival, the arbitrageur is exposed to the risk of having to unwind the position in the defaultable bond at time t_N at a disadvantageous price. Let's summarize the problems with cash-and-carry arbitrage as follows:

- The implementation of short positions in a defaultable bond may be difficult or even impossible. Furthermore, the trader is exposed to the risk of changing repo rates.
- The arbitrage is unprecise at default events: The default-free bond price will differ from par; The interest-rate swap of an asset swap package will have a certain market value; CDSs contain a delivery option for the protection buyer in the case of default. This increases the value of the protection offered by a CDS.
- There might be other peculiarities in the settlement of the CDS at default which makes its value hard to predict.
- The only available defaultable bond may not match the CDS maturity or the corresponding coupon payment dates might not coincide with the CDS fee payment dates. Even if similar maturity bonds exist, these may not be very liquid, especially during times of high market volatility. In that case it would be very natural to see deviations between CDS rates and benchmark spreads.
- The tax treatment of corporate bonds and CDSs are different, which will enlarge the gap between the corresponding spread and the CDS rates.
- The method cannot be applied if the reference credit has not issued any defaultable bonds.

All these problems widen the price bounds that can be imposed upon a CDS rate by using pure static arbitrage strategies. The analysis of potential arbitrage strategies enables us to better understand the links between the market for the underlying asset, the 'cash' bond market, and the derivative market for CDS and other credit derivatives. Inefficiencies can occur in both markets but frequently there are more inefficiencies in the cash market than in the credit derivatives market. A liquid CDS market is often a more useful indicator for the price of the default risk of a particular obligor than the underlying cash market.

Nevertheless, cash-and-carry arbitrage is still a very useful instrument for identifying mispricings in the market and for finding advantageous prices, even if they do not constitute pure arbitrage strategies. The strategies only rely on the payoff comparison, therefore they are very robust and unaffected by the model and parameter risk of more complex models.

A Structural Model for Credit Default Swap Pricing

In the previous section we mainly took the discussion of the Das (1995) model from Cossin (2001) and modified some parts. Das (1995) presents an evolution of structural models that allow for pricing of credit derivatives, including CDS as a special case, as a compound option problem[52], credit risk being an option and the credit derivative being an option on the option of credit risk. The model allows for stochastic interest rates as well as stochastic exercise price. Interestingly, the Das (1995) model provides for N-shaped[53] credit term structures by combining stochastic interest rates and endogenous recovery. The classical credit risk structural models, such as Merton (19974) and Longstaff and Schwartz (1995b), give rise to hump-shaped credit spread term structures. The credit spreads rise first and then fall with maturity and tend asymptotically towards a constant in case of the Merton model and towards zero in the Longstaff Schwartz model. For further discussion see Wei and Guo (1997). In case of high credit risk, e.g. junk bonds, these models give a monotonic decrease of spreads with maturity. Fama and French (1987) and Wei and Guo (1997) tend to find that the credit spreads take as N-shape with maturity: they first rise, then decrease, then rise again. Although theoretical discussions of time length considered in the empirical studies would be valid, the empirical fact remains that in the medium to short term, we observe N-shaped credit spread term structures that can not be fitted by classical hump shapes of the common structural models.

The Das (1995) model relies on the Ho and Lee (1986) term structure model and obtains N-shaped credit-spread term structures and prices theoretical *CDS as a compound option.* The Ho and Lee (1986) term structure model has the advantage of being a Markov analytically traceable model. They assume a one-factor Gaussian term structure model for the initial term structure of defaultable bonds. It easily reaches exact fits of the empirical term structure. Its main weaknesses certainly are its lack of mean reversion and the simplicity of the volatility of interest rates in the model.

Consider an option in which the writer agrees to compensate the buyer for pre-specified fall in credit standing (quality) of the underlying bond (Reference Credit Asset), as determined by strike level either in ratings or in yields. The payoff of the option the amount by which the price of the bond at the prevailing default-free rate plus the spread defined by the strike exceed the prevailing market price of the bond. Assuming constant interest rates, it means that $K = F^{-(r+r^*)(T-T^*)}$, where T is the maturity of the bond, T^* the maturity of the option, F the face value of the bond, K the exercise or strike price of the option in the firm value

[52] For the valuation of compound options see also Geske (1979).

[53] N-shaped means here the style of the curve of the credit spread term structure such as a N looks like, first up, then down, and up again.

and r^* the strike rate or strike credit spread. It is further assumed that the value of the firm follows a process

$$dA = \mu A dt + \sigma_1 A dz_1 + \sigma_2 A dz_2, \tag{1.58}$$

where μ is a constant drift term, $\sigma_{1,2}$ two different volatilities and $z_{1,2}$ two different Wiener processes and that the dynamic of the forward interest rate, denoted as $f(t,T)$ is given by

$$df(t,T) = \theta(t,T)dt + \sigma_r dz_{1,}$$

where $\theta(t,T)$ is a stochastic drift term and σ_r the interest rate volatility. The process of the firm value and the process of the forward interest rates are correlated via the Wiener process z_1. Since equity S is a call option on the firm value with maturity being the maturity of the debt T and exercise price the face value of the bond, the bond can be valued as the difference between the firm value and the equity value. Cossin and Pirotte (2001) modified the results of Das (1995) as there seems to have been a computational mistake in Das (1995) and obtain:

$$B_0^d = A_0[1 - N(k)] + FB_0 N(k - \xi), \tag{1.59}$$

where B_0 is the price of risk-free discount bond at 0 maturing at T,

$$k = \frac{\log\left[\frac{A_0}{FB(0,T)}\right] + \frac{1}{2}\xi^2}{\xi} \tag{1.60}$$

and

$$\xi^2 = (\sigma_1^2 + \sigma_2^2)T - \sigma_1\sigma_r T^2 + \frac{2}{3}\sigma_r^2 T^3. \tag{1.61}$$

This simple Ho and Lee structure allows for many different shapes of the credit spread term structure, whicch is a welcome generalization of the previous model using Vasicek and limited to hump-shaped credit spread term structure, see Cossin and Pirotte (2001). Das (1995) provides also a general form for the HJM term structure.

This model takes into account that the shape of the credit spread structure is affected by the choice of three volatility parameters σ_1, σ_2 and σ_r. The measure of the risk-free interest rate volatility,σ_r, tends to affect the long maturity spreads. Here σ_1, the correlation between changes in firm value and interest rate, affects spreads at short maturities, and the volatility, σ_2, of the firm value, which is independent from the interest rate volatility, affects the spread curve across all maturities.

With this structural model, yield curves and spread curves of many different shapes can thus be fitted. N-shaped credit spread curves, as described in Fama and French (1987) and Guo and Wei (1997) can be obtained with this model, see Cossin and Pirotte (2001).

For pricing credit derivatives, once the risky bonds are priced, we can follow Das (1995) in pricing options on risky bonds as compound options. Note here that the risky bond itself is an option on the firm value. The buyer of the option described above receives a compensation in case the date T^* price of the credit-risky bond falls below the price of a bond which is priced at the riskless rate plus r^*, the strike credit spread. The stochastic interest rate leads to a stochastic exercise value of the bond. It can be written, as (see Cossin and Pirotte (2001))

$$K\left(T^*, T, F, r^*\right) = Fe^{\left[-\left(-\left(\frac{1}{T-T^*}\right)\log B_{T^*}+r^*\right)(T-T^*)\right]}. \tag{1.62}$$

Finally the value of the credit derivatives is obtained by taking the expectation under the risk-neutral measure:

$$Credit\ Derivative\ Value = \int_{z_1}\int_{z_2} \max\left[K\left(T^*, T, F, r^*\right) - B^d_{T^*}, 0\right] N'\left(z_1\right) N'\left(z_2\right) dz_1 dz_2. \tag{1.63}$$

For implementation we have to take into account that the double integral takes considerable computing time. Das (1995) proposes a methodology which is based on discrete time approximations and reduces the computating time drastically. Discrete time approximation also allows for easy extensions to coupon bonds, debt with embedded derivatives, such as convertibles or callables. Additionally these framework allows extensions for alternative bankruptcy procedures and stochastic volatilities. The discrete time approach is straight-forward, starting from the expression in continuous time above. The numerical analysis in Das (1995) is based on building binomial trees and shows that such credit derivatives tend to be valued highest at middle maturities, when a combination of both the time value of the credit risk debt and that of the derivative instrument on the credit-risky debt is considered, decrease with the volatility of interest rates and increase with the volatility of the firm value. See also Cossin and Pirotte (2001) for a further discussion of structural models.

Although addressing the problem of the resolution of default[54], available structured models such as described above make the simplification that they involve the presence of one risk-less counterparty against a risky one. Reduced-form models allow relaxation of this important feature. All these models published at present analyse the CDSs value free from any special features such as collateralization or any other credit risk mitigation technique which would

[54] A default process represents a period of time.

considerably reduce or in extreme cases cancel the original exposure.

A Reduced-Form Model for Credit Default Swap Pricing

Drawing from the work of Schönbucher (2000, 2003) and O'Kane and Schlögel (2001), we outline a simple framework for pricing CDSs in a way, which is basically an intensity-based reduced-form model approach originally initiated by Jarrow and Turnbull (1995).

In contrast to structural models, reduced-form credit pricing models do not attempt to explain the occurrence of a default event in terms of a more fundamental process such as the firm value or an earnings stream. The aim is instead to describe the statistical properties of the default time as accurately as possible, in a way that allows the repricing of fundamental liquid market instruments and the relative valuation of credit derivatives. The reduced-form model approach is closer to that of the actuarial sciences than to the corporate finance methods used in structural models, and the pricing techniques are similar to those used in traditional models of the term structure, as opposed to the more equity-like structural models.

We will develop this approach in several steps. First we start with a setup for the model where we give the used *Notations and Definitions*. Than we serve with *basic relationships* and introduce the *inhomogeneous Poisson Process*. After this we work out what needs to be known about *pricing in defaultable claims*. Having done this we *introduce recovery* to come with a framework to *price a CDS*. Finally a brief *remark* on extensions and practical implementations is made.

Setup: Notations and Definitions

The time to default: The recent literature uses two different models for the recovery rate of defaultable bonds, the fractional recovery model and equivalent recovery model (see before the item 'Recovery rates'), the model for the *time(s) of the default* is the same for both. Initiated by Jarrow and Turnbull (1995), we assume that defaults are exogenous events. Therefore we can assume that the random times of default $\tau_i, i = 1, 2, ...$, are generated by a Cox process. Intuitively, a Cox process is defined as a Poisson process with stochastic intensity (or hazard rate) $\lambda(t)$,see Lando (1998). Formally the definition is the following:

J is called a *Cox process*, if there is a nonnegative adapted stochastic process[55] $\lambda(t)$, called the intensity of the Cox process, with $\int_0 \lambda(s)\, ds < \infty \forall t > 0$, and conditional on the realization $\{\lambda(t)\}_{\{t>0\}}$, where $1_{\{t>0\}}$ is the indicator function, see below, of the intensity, $N(t)$

[55] For nonnegative adapted stochastic processes see Jacod and Shiryaev (1988).

is a time-*inhomogeneous Poisson process* with intensity $\lambda(t)$. This definition follows (Lando 1998).

The following assumptions are made:

1. Lets define the *default counting process* as $J(t) := \max\{i \mid \tau_i \leqslant t\} = \sum_{i=1}^{\infty} \mathbf{1}_{\{\tau_i \leq t\}}$, which is a Cox process with intensity $\lambda(t)$,where $i = 1, 2, ...$
   and $\mathbf{1}_{\{\tau_i \leq t\}} = \begin{cases} 1, & \tau_i \leq t, \quad if\ default\ before\ t \\ 0, & \tau_i > t, \quad if\ default\ after\ t \end{cases}$ is the so called *indicator function.*
2. In the equivalent recovery model the time of default is the time of the first jump of J. To simplify notation the time of the first default will be referred to as $\tau := \tau_1$.
3. In the fractional recovery model the times of default are the times of the jumps of J.
4. In the following the time of the first jump is defined as the random *time to default*, τ, and thus, for $J(0) = 0$, we can write
   $$\tau = \min\{t \geq 0 \mid J(t) \geq 1\}. \tag{1.64}$$
   Intuitively, $J(t)$ denotes the number of events that have occurred up to time t.
5. The *intensity* (or hazard rate) can be interpreted as a conditional instantaneous probability of default:
   $$P(\tau \leq t + dt \mid \tau > t) = \lambda(t)\, dt. \tag{1.65}$$
   This equation states that, conditional on having survived until time t, the probability of defaulting in the next infinitesimal instant is proportional to $\lambda(t)$ and the length of the infinitesimal time interval dt. The function λ describes the rate at which default events occur, which is the reason why it is called the *hazard rate* of J.
6. The *implied survival probability* between in the time interval $[t, T]$ is denoted by $P(\tau > T) \equiv P(t, T)$. The cumulative *implied default probability* in the time interval [t,T] is then given by $P(\tau \leq T) \equiv 1 - P(t, T)$.
7. Let's define $\beta_{t,T} := e^{-\int_t^T r(s)ds}$ as the *risk-free discount factor* over the time interval $[t, T]$, where $r(t)$ is the *instantaneous default-free short-interest rate*, and let's use the value of the *money market account* (sometimes called savings or bank account) $\alpha_{t,T} := e^{\int_t^T r(s)ds}$ in continuous time as the *annuity* (sometimes called numeraire).

Basic relationships:

The payoff of a default-free zero-coupon bond is 1 at time T, or formally $1_{\{\tau>T\}}$, where $\mathbf{1}_{\{\tau>T\}} = \begin{cases} 1, & \tau > t, \quad if\ default\ after\ T \\ 0, & \tau \leq t, \quad if\ default\ before\ T \end{cases}$ is the *indicator function.*

Taking into account that the value of a default-free zero-coupon bond is equal to the expected payoff, we can formally write $B(t,T) = E[\beta_{t,T}]$. For the value of a defaultable zero-coupon bond we can than easily write $B^d(t,T) = E[\beta_{t,T}1_{\{\tau>T\}}]$. If there is no correlation between the default and the payoff of the zero-coupon bond (meaning also the independence of interest rates and default intensity), we get

$$\begin{aligned} B^d(t,T) &= E[\beta_{t,T}]E[1_{\{\tau>T\}}] \\ &= B(t,T)E[1_{\{\tau>T\}}] \\ &= B(t,T)P(\tau>T) \\ &= B(t,T)P(t,T), \end{aligned} \tag{1.66}$$

where the *implied survival probability* is the ratio of zero coupon bond prices

$$P(t,T) = \frac{B^d(t,T)}{B(t,T)}, \tag{1.67}$$

with the property that the curve $P(t,T)$ is decreasing in T with $P(t,t) = 1$ and $P(t,\infty) = 0$. $P(t,T)$ will change over time. Typically the survival probability will change over time because of two effects: Firstly, if there were no default in $[t, t+dt]$, this reducing the possible default times, information has arrived via the (non)-occurrence of the default. Secondly, additional default-relevant information could have arrived in the meantime.

Qualitatively the credit default curve, $1-P(t,T)$ is very similar to a curve which consists of discount factors, $\beta_{t,T}$. Formally we have the following slopes

$$\begin{aligned} B(t,t_1) &> \ldots > B(t,t_N), \\ B^d(t,t_1) &> \ldots > B^d(t,t_N), \\ P(t,t_1) &> \ldots > P(t,t_N), \end{aligned} \tag{1.68}$$

and

$$B^d(t,t_i) > P(t,t_i) > B(t,t_i), \ \ for\ all\ i = 1,2,\ldots \tag{1.69}$$

Inhomogeneous Poisson Process:

Given the realization of $\lambda(t)$, the probability of having exactly n jumps is

$$P[J(T) - J(t) = n \mid \{\lambda(s)\}_{\{T \geq s \geq t\}}] = \frac{1}{n!}\left(\int_t^T \lambda(s)\,ds\right)^n \exp\left(-\int_t^T \lambda(s)\,ds\right). \quad (1.70)$$

Using the inhomogeneous Poisson Process[56] we can easily write the probability of no jumps $(n = 0)$ as follows

$$P(t,T) := P[J(T) = J(t) \mid \{\lambda(s)\}_{\{T \geq s \geq t\}}] = \exp\left(\int_t^T \lambda(s)\,ds\right). \quad (1.71)$$

Integration of equation 1.65 gives the same result.

For $\tau > T$, $P(t,T)$ can be interpreted as the survival probability from time t until time T.[57]

Pricing defaultable claims:

Note that the purpose of the model is the arbitrage-free valuation of default-linked payoffs. The probability measure P is therefore a *risk-neutral measure*, meaning that the survival probability under P is not directly related to historical default frequencies, where the default risk can be hedged in the market. Furthermore, the intensity function $\lambda(t)$ governs the behavior of J under P, and must therefore incorporate the risk premium demanded by the market.

Suppose we want to price a defaultable claim that has a payoff $X(T)$ at time T in case no default occurred and 0 otherwise. Under the risk neutral valuation the time t the price of such a claim, in case of a defaultable coupon bond denoted as $C^d(t)$, is given by the risk neutral expectation of the discounted payoff

$$C^d(t) = E_t\left[\frac{X(T)}{\alpha_{t,T}} 1_{\{\tau > T\}}\right], \quad (1.72)$$

where $E_t[]$ is the time t expectation under the risk neutral measure. If we assume that $\frac{X(T)}{\alpha_{t,T}}$, the payoff process and $1_{\{\tau > T\}}$, the default process are independent, we get

$$C^d(t) = E_t\left[\frac{X(T)}{\alpha_{t,T}}\right] P(t,T). \quad (1.73)$$

[56] Inhomogeneous means here a Poisson processes with time-dependent (sometimes called time-invariant) intensity function.

[57] Given $\tau > T$ the probability density of the time of the first default over the interval $[t,T]$ is $p(t,T) = \int_t^T \lambda(u)\,du \exp\left(\int_t^T \lambda(s)\,ds\right)$, see Lando (1998).

Note that in absence of default risk, $P(t,T)$ would be 1 and $\frac{X(T)}{\alpha_{t,T}}$ would be the payoff of a default-free claim. The expected present value of such a payoff would be $C(t) = E_t\left[\frac{X(T)}{\alpha_{t,T}}\right]$, and thus we can write the pricing equation for a default claim as follows

$$C^d(t) = C(t)P(t,T). \tag{1.74}$$

This shows that, under the independence assumption, the price of a defaultable claim is obtained by multiplying the price of the equivalent non-defaultable claim by the implied survival probability in between the time interval $[t,T]$.

Suppose now that the intensity has a constant value λ. We consider a defaultable zero-coupon bond with maturity T under the *zero recovery* assumption, i.e. the bond pays 1 if no default occurs until T and nothing otherwise. The time t price of such a bond is denoted as $B^{d,0}(t,T)$. The survival probability is $P(t,T) = e^{-\lambda(T-t)}$. Assuming the instantaneous default-free short-interest as deterministic, $r(t) = r$, we obtain from equations 1.67 and 1.71 (using that $B^0(t,T) = E[\beta_{t,T}] = e^{-r(T-t)}$)

$$B^{d,0}(t,T) = e^{-(r+\lambda)(T-t)}. \tag{1.75}$$

Therefore, with *zero recovery*, the spread between the yield to maturity on a defaultable bond over that on a default-free bond is given by the intensity or hazard rate of the default process, which represents the credit spread.

Introducing recovery

We can modify the simple pricing model above to allow for *non-zero recovery rates.* To do so let's consider another type of contingent claim which makes a random payment $X(\tau)$ at the time of default, if the default does occur before a given time T, and zero otherwise. Its time s price $D(s,T)$, $s \geq t$, can be written as

$$D(s,T) = E_s\left[\frac{X(\tau)}{\alpha_{s,T}} 1_{\{\tau \leq T\}}\right]. \tag{1.76}$$

To calculate this expectation, we need to derive the probability density of the default time, $P(s < \tau \leq s + ds)$, in order to be able to price such a claim. Using the definition of conditional probabilities, we know from equation (13) that the probability of defaulting in the time interval $[s, s+ds]$ is given by

$$P(s < \tau \leq s+ds) = P(\tau \leq s+ds \mid \tau > s)\, P(s > \tau) = \lambda(s)\, e^{-\int_t^s \lambda(u)du} ds. \tag{1.77}$$

The above equation simply says that the time s probability of a default occurring in some future time interval $[s; s+ds]$ is equal to the probability of surviving through time s, $P(\tau > s)$, times the conditional probability of default in the next time interval.

Let $D(t,T)$ be the time t price of a *default contingent claim*. Again using the money market account as the numeraire, we obtain $D(t,T)$ by integrating over the density of τ, so that

$$D(t,T) = \int_t^T E_s\left[\frac{X(s)}{\alpha_{s,T}}\right]\lambda(s)\, e^{-\int_t^s \lambda(u)du} ds. \tag{1.78}$$

Assuming now that the payoff X is constant and equal to 1, and taking into account that for the default free zero coupon bond holds $E_t\left[\alpha_{\tau,T}^{-1}\right] = E_t\left[\beta_{\tau,T}\right] = E_t\left[e^{\int_t^T r(u)du}\right] = B(s,T)$, equation 1.78 leads to

$$D(t,T) = \int_t^T B(s,T)\lambda(s)\, e^{-\int_t^s \lambda(u)du} ds. \tag{1.79}$$

$D(t,T)$ is merely a weighted average of non-defaultable zero coupon bond prices, where the weights are given by the density of τ.

Assuming again that the intensity and the instantaneous default-free short-interest are deterministic and constant we can write

$$D(t,T) = \int_t^T B(s,T)\lambda(s)\, e^{-\lambda(s-t)} ds = \frac{\lambda}{r+\lambda}\left(1 - e^{-(r+\lambda)(T-t)}\right). \tag{1.80}$$

Consider now a defaultable zero coupon bond. For simplicity we assume again independence between interest rates and the intensity. If the recovery rate is zero, its price $B^{d,0}(t,T)$ is given by

$$B^{d,0}(t,T) = B(t,T)P(t,T). \tag{1.81}$$

Note that we have added a subscript of zero to the bond price to emphasize that this is the price obtained under the *zero recovery* assumption.

We can now price a defaultable zero coupon bond with face value 1 that pays a general recovery rate R, expressed as a percentage of the bond's face value, in the event of default. We can think of such a bond as a portfolio composed by a defaultable bond that pays no recovery and a contingent claim that pays R in the event of default via simple static replication. Using equation 1.80 and 1.75, the price of the defaultable zero coupon bond $B^d(t,T)$ is

$$\begin{aligned} B^d(t,T) &= RD(t,T) + (1-R)B^{d,0}(t,T) \\ &= e^{-(y+\lambda)(T-t)} + R\frac{\lambda}{\lambda+r}\left(1 - e^{-(r+\lambda)(T-t)}\right). \end{aligned} \tag{1.82}$$

This expression can be used for determining the fair value of, for example a corporate bond. In practice though, recovery rates are a major problem in the credit market because of the significant uncertainty surrounding actual recovery rates (see discussion before). More general, if r denotes the default-free short rate, it can be shown (for example see Schönbucher (2000)), that the price of a defaultable zero coupon bond is given by

$$B^d(t,T) = E_t\left[e^{-\int_t^T (r(s)+\lambda(s))ds}\right], \tag{1.83}$$

where the expectation is taken under the risk-neutral measure. This simplifies the modelling process for defaultable bonds, as only the loss rate λ needs to be specified. In particular, the recovery rate does not impose any bounds on the credit spreads. However, knowledge of the default probabilities is necessary for the pricing of credit derivatives, such as digital default swaps. These cannot be directly inferred from defaultable bond prices under the fractional recovery assumption without specifying the stochastic dynamics of λ.

Pricing a credit default swap:

Pricing a CDS means determining the premium or the credit rate, s^{cds}, that will be paid periodically by the protection buyer. Taking s^{cds} initially as given, we first compute the time t value of the premium and the protection legs of a contract with maturity at time T. For simplicity we assume that the premium is paid continuously and that the risk-less rate and the intensity are constant.

The present value of the *premium leg,* $\Phi(t,T)$, is

$$\Phi(t,T) = \int_t^T s^{cds} e^{-(r+\lambda)(s-t)} ds = \frac{s^{cds}}{r+\lambda}\left(1-e^{-(r+\lambda)(T-t)}\right). \tag{1.84}$$

Note that, as in equation 1.75, the premium stream is discounted at the risky rate $r+\lambda$, reflecting the uncertainty surrounding the default event.

To value the protection leg, we note that it is equivalent to a contingent claim that pays $(1-R)$ in the event of default before maturity T. The value of such a claim is given by equation 1.85. Let $\Psi(t,T)$ denote the present value of the *protection leg,* we can write

$$\Psi(t,T) = \frac{\lambda(1-R)}{r+\lambda}\left(1-e^{-(r+\lambda)(T-t)}\right). \tag{1.85}$$

A CDS typically has zero market value when it is set up, and thus pricing such a contract is equivalent to finding the value of the CDS premium, s^{cds}, that makes the two legs of the swap have equal value ($\Phi(t,T) = \Psi(t,T)$). This is given by

$$s^{cds} = \lambda(1-R). \tag{1.86}$$

To provide some intuition about the credit-default swap premium, we note that in the paper, Duffie (1999) shows that the premium equals the fixed spread over the risk-less rate that a corporate floating rate note would need to pay to be able to sell at par. Thus, if both a firm and the risk-free issued floating rate notes tied to the risk-less rate r, the fixed spread between the rates paid by the FRN would equal s^{cds} (see example 3 above). It is important to stress, however, that this result does not extend to the yield spreads between corporate and Treasury fixed rate bonds. Duffie and Liu (2001) show that the spreads on fixed rate bonds can differ from spreads on floating rate securities. To a first order approximation, however, it is often useful to think of the credit-default swap premium as being roughly equal to the yield spread of the reference issue corporate bond over the yield of a risk-free bond with the same maturity date and coupon rate.

Remarks:

Extensions of the above model should take into account that several restrictive assumptions made above, such as continuously paid premiums, constant interest rates and intensity, and zero coupons. All of these assumptions can be relaxed to make the pricing closer to reality.

If we assume stochastic interest rate $r(t)$ and intensity $\lambda(t)$ the present value of the premium leg $\Phi(t,T)$ of a CDS can be expressed as (e.g. Finkelstein (2000)),

$$\Phi(t,T) = E_t\left[s^{cds}\int_t^T e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right]. \tag{1.87}$$

Similarly, the value of the protection leg of a CDS, $\Psi(t,T)$, can be expressed as

$$\Psi(t,T) = E_t\left[(1-R)\int_t^T \lambda(t)e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right]. \tag{1.88}$$

At the time of setting up the CDS we get the following CDS rate, s^{cds},

$$s^{cds} = \frac{E_t\left[(1-R)\int_t^T \lambda(t)e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right]}{E_t\left[\int_t^T e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right]}, \tag{1.89}$$

and the present value of a CDS is then given by

$$PV^{CDS} = -E_t\left[s^{cds}\int_t^T e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right] + E_t\left[(1-R)\int_t^T \lambda(t)e^{-\int_t^s (r(s)+\lambda(s))ds}dt\right]. \tag{1.90}$$

Hull and White (2000a, 2000b) developed a framework contingent on default by multiple reference entities and situations with is counterparty default risk (see also 1.3.8 below). We should also not forget that the liquidity (basis risk) is determining the variation of the CDS spreads, see also 'Summary' below.

Brief comment for practical implementations of reduced-form approaches To value financial instruments related to default risk, e.g. a CDS, we have to set up a credit default curve specified by the default probabilities. The credit default curve is constructed as implied from market prices of liquid instruments, such as bonds and CDSs. Since the pricing of those products involves risk-less discount factors and default probabilities the implied default probabilities are always relative to the specified risk-less curve, i.e. Libor. The steps to price credit derivatives in an intensity-based framework are then mainly the following

1. Identify the payoff function: special care has to be taken in the treatment of defaults
2. Form the expectation and condition on the realization of the intensity
3. Replace direct references to the defaults with references to the intensity
4. The problem is now accessible to standard techniques for continuous processes (change of numeraire, distributions etc.

In this form the pricing issue is much better conditioned for numerical solutions.
Some comments about correlation and counterparty risk

> *Correlation* is primarily an issue at the individual portfolio level and as such it should not affect the pricing of credit default swaps per se in reasonably *liquid markets*. However, in a still developing market, correlation problems may give rise to technical *supply and demand* factors. If correlation in a given portfolio is relatively high or a portfolio has high concentrations that could be highly correlated in certain scenarios, a portfolio manager may be willing to pay a relatively high price (relative to cash market credit premiums) to purchase protection to hedge credit risk. Alternatively a portfolio manager may accept a relatively low premium to sell protection and thereby diversify credit exposure. Over time, however, this activity should actually lead to a more efficient market for default swaps. Using CDS to manage credit risk in a portfolio context has received much attention in the past year, but to date, most credit portfolio managers are either not actively managing their portfolios in this way or have taken only tentative steps. In many organizations, it has been very difficult to gather the data required to develop robust credit models. As more people and institutions become familiar with quantitative credit risk techniques and commercially available models become more robust,

we expect credit derivatives to be used more widely to manage portfolio credit risk and correlation-related issues. This in turn will enhance market liquidity.

Counterparty risk, in a credit default swap transaction, the protection buyer has counterparty exposure to the protection seller. If the protection seller and the issuer of the reference credit default simultaneously, the buyer will suffer the full loss despite having paid for default protection. Thus the protection buyer must ensure that the correlation risk between the protection seller and the reference asset is low. In times of low credit concerns (perhaps in periods of high economic growth), buyers of protection will tend not to focus on counterparty risk and A-rated counterparties may be able to sell protection on equivalent terms as AAA-rated counterparties. In periods of high credit concerns, however, buyers of protection might avoid counterparties rated below, say, A1 at any price.

Literature Survey for Credit Default Swap Pricing Models

The literature on credit derivatives and even more on CDS is growing rapidly. There are two distinct approaches to the modelling of credit risk, structural and reduced-form models, as we pointed out before, but now we focus more on the context of CDS pricing and even more credit derivatives pricing in general. We will present a brief overview of the main theoretical works of structural and reduced-form models and discuss some of them in more detail for those models we not already have considered until now. For a more detailed literature review we refer to Schlögel (2000), Cossin and Pirotte (2001) and Schönbucher (2003).

An overview of structural and reduced-form models for credit risk pricing:

1. Structural approaches:

   Merton (1974): debt as a contingent claim written on the assets on the firm. Others: Bhattacharya and Mason (1981), Black and Cox (1976), Crosbie (KMV 1997), Das (1995), Delianedis and Geske (1998), Geske (1977), Huang (1996), Kim, Ramaswamy and Sundaresan (1989), Leland (1994), Longstaff and Schwartz (1995 a, b), Nielsen, Saa-Requejo and Santa-Clara (1993), and Shimko, Tejima and VanDeventer (1993).

2. Reduced form approaches:

   Early works: Madan and Unal (1994, 1998), Duffie and Singleton (1994), Jarrow and Turnbull (1995), Lando (1994), Jarrow et al. (1997) Das (1995) (solutions that include the real term structure of CDS bearing the default along with a non-flat and non-constant term structure of risk free interest rates were introduced by Duffie and Huang (1996), Li (1996) and Hübner (1997b)).

Newer works: Duffie (1998, 1999), Lando (1998), Schönbucher (1998, 2000), Duffie and Singleton (1999), Jarrow and Turnbull (2000), Hull and White (2000a,b), Jarrow and Yildirim (2001), Das, Sundaram, and Sundaresan (2003), and many others.

Credit-rating based approaches focusing on the default being triggered by a gradual change in ratings driven by a Markov transition matrix were introduced by Jarrow, Lando and Turnbull (1997), Lando (1998), or Das and Tufano (1996).

3. *Intensity-based approaches* were introduced by Duffie and Singleton (1997), Madan and Unal (1994, 1998), Schonbucher (1998). See also Duffie and Huang (1996), Duffie, Schroder and Skiadas (1996), Jarrow and Turnbull (1995), Kijima and Komorobayashi (1998), Kijima (1998, 2000), and Ramaswamy and Sundaresan (1986).

   There are also several recent *empirical studies* of the pricing of credit-default swaps including Cossin, Hricko, Aunon-Nerin, and Huang (2002), Houweling and Vorst (2002), Blanco, Brennan and Marsh (2003), Zhang (2003) and Longstaff, Mithal and Neis (2003).

Let's focus on some main approaches on credit risk pricing, in particular CDS pricing:

In Jarrow and Turnbull (1995) both, a stochastic process of the term structure for default-free zero-coupon bonds and the term structure for defaultable zero-coupon bonds are exogenously specified, as it is the case for any reduced-form model. The main argument for this discrete approach is that many kinds of payoffs, including the decomposition of a CDS, can be reproduced and priced in taking advantage of existing calibration techniques of the arbitrage-free dynamics for these term structures and the risk-neutral valuation procedure.

Scott (1998) developed basically the same simple approach we present in the subsection 'A reduced from model' above in discrete time. He determines the CDS value for all default times τ_i, and then builds the sum over i. See also Brooks and Yan (1998). Furthermore some empirical tests of the model is done for Argentina bonds and the corresponding CDSs with 3,5 and 10 years time to expiration.

Schönbucher (1998) presents a model for the development of the term structure of defaultable bonds based on a multiple-defaults model. Instead of modelling a cash payoff in default he assumes that defaulted bonds are restructured and continue to trade. He uses a Heath-Jarrow-Morton (HJM) approach to represent the term structure of defaultable bond prices and gives arbitrage-free drift restrictions. In its most general version the model is set in a marked point process framework to allow for jumps. For implementation, the extensive machinery of risk-free interest rate modelling can be used.

Duffie (1999) serves with a review of the pricing of CDSs including the pricing with reference to spreads over the risk-free rate of par floating-rate bonds of the same quality and considers model-based pricing. The independence assumption is the same as in Jarrow and Turnbull (1995). His paper is the initial work, which presents the CDS pricing via replication methods. The estimation of hazard rates is discussed and the role of asset swap spreads as reference for pricing credit swaps is also considered.

Duffie and Singleton (1999) presents convenient reduced-from models of the valuation of contingent claims subject to default risk, focusing on applications to the term structure of interest rates for corporate and sovereign bonds. The examples include special cases with exogenous expected loss, the valuation of noncallable corporate bonds with square-root diffusion and more flexible correlation structures, and the valuation of a credit-spread option.

Das and Sundaram (2000) develop a framework for modelling risky debt and valuing credit derivatives based on an expand HJM term-structure model to allow for defaultable bonds. Rather than following the procedure of extracting the behavior of spreads from assumptions conerning the default process, they work directly with the evolution of spreads. The risk-neutral drifts in the resulting model possess a recursive representation that facilitates implementation and makes it possible to handle path-dependence and early exercise features without difficulty. Basically, it is a two factor model on the intensity and the interest rate. In an earlier work, Das and Sundaram (1998) present a model for credit derivatives pricing that is arbitrage-free, includes path dependence, and handles a range of securities, even with American features. The basis of the computer implementation, as the model is notably interesting for its engineering implementation, is presented in the paper.

Hull and White (2000a) provide a methodology for valuing CDS in case the payoff is contingent on default of a single reference entity and there is no counterparty default risk. The paper tests the sensitivity of CDS valuations to assumptions on the expected recovery rate. Furthermore, it tests whether approximate no-arbitrage arguments give accurate valuations and provides an example of the application of the methodology to real data. In a subsequent paper of Hull and White (2000b), the analysis is extended to cover situations where the payoff is contingent on default by multiple reference entities and situations where there is counterparty default risk. It develops a model of default correlations between different corporate or sovereign entities. The model is applied to the valuation of vanilla CDSs where the seller may default and to the valuation of basket credit default swaps. Within this model-based pricing framework the value of the CDS spread is such that the expected present value of the premium must be equal the expected present value of the protection. These expected present values are derived from a theoretical model that incorporates specific assumptions about the behavior of the market interest rate and default rates in order to generate a default probability distribution function for default times τ_i. Instead of using the intensity $\lambda(t)$ they use the default probability density $p(t, T)$.

Bielecki and Rutkowski (2000) provide a survey of the theoretical foundations of intensity-based approaches for credit-rating based and credit-spread-based HJM type models in discrete and continuous time including alternative recovery schemes and the modelling with state variables within a martingale framework.

Cheng (2001) developed a recursive valuation formula for CDS pricing, which implies the most common pricing formulas from different approaches such as the model of Jarrow and Turnbull (1995), Scott (1998), Duffie (1999), Das and Sundaram (2000) and Hull and White (2000a,b).

In any case the most interesting models are already implemented in major banks and not available in public now. They are at the moment the most innovators in pricing credit risk in terms of global market models including capital structure and liquidity issues as well.

Practical Implementation of Credit Default Swap Pricing Models

The practical implementation of CDS pricing models mainly comes with a combination of tree models using the Hull and White algorithm (see Hull and White (1994a, 1994b, 1996)) and reduced-form models. Mostly, the tree models are used to build a recombining trinomial trees for the state variables and processes with Gaussian mean-reverting dynamics such as generalized Vasicek dynamics. The reduced from models are based on the theory of so-called reduced-form default models such as Duffie and Singleton (1997), Jarrow, Lando, Turnbull (1997), Lando (1996, 1998) and Lando and Turnbull (2000).

This class of models takes as key input parameters a stochastic intensity process $\lambda(t)$ and a fractional recovery of bonds. The intensity $\lambda(t)$ can be viewed as the conditional rate of default given no default until t. For a deterministic constant λ, the default time is just the random time of the first jump of a Poisson process. In reduced-from models the default time is unpredictable. A different class of models consists of so-called structural models, where the default time is modelled as the first time a stochastic state variable process (e.g. value of the firm) crosses a certain level. If the state-variable process is continuous these models yield predictable default times in a sense that one can see the default 'coming closer'.

Widely used in practice is a model which combines a one-factor arbitrage free interest rate term structure model with a model for the default intensity. In such a framework both, the risk-less short rate process $r(t)$ and the intensity process $\lambda(t)$ are assumed to be continuous diffusion processes. Following the general theory of arbitrage free pricing a risk adjusted probability distribution is used such that all securities are normalized by the money market account. The implementation of such a model outlined above can be done by means of a two-factor tree. Basically this model is a combination of two correlated one-factor models, one for the risk-less short rate and the other for the default intensity. Monte Carlo simulations can be used to estimate some variables in such frameworks. Combinations with liquidity based approaches could be an extension of such models.

A critical issue is the modeling of *correlated defaults*. In the framework of reduced-form models correlation appears as correlation of the intensity processes. The correlation of the intensity processes is to a certain extend an historically observable variable as correlation of spreads. A first approach to the modeling of correlated defaults would be to apply Monte Carlo methods to sample the correlated default intensity process and to calculate the survival probabilities for the baskets. However, for a model, if the we assume intensity processes to be diffusion processes, it turns out that the impact of correlation on the basket survival probabilities is much less than desired. The conclusion is to use intensity processes that permit jump components which seems also much more realistic. Duffie and Singleton (1998) propose several techniques for the simulation of correlated defaults, which seems to be of particular interest for very large baskets of credits.

For pricing of first-to-default swaps for baskets containing a small number of credits of relatively high credit quality practical experience is that the numbers of samples needed to achieve good results is essential. For example, in case of uncorrelated credits the survival probability for such a basket should be exactly the product of the individual probabilities. Motivated by this, the intention is to depart from any Monte Carlo approach and to develop a reasonable model which allows for closed form expression for the joint distribution of all default times for the individual reference credits from the basket. The starting point for such an approach would be the multivariate exponential distribution.

Multi-issuer credit derivatives such as first-to-default basket and CDOs gain an increasing importance in the market. The pricing of first and second loss products requires the use of models, which induce a default correlation between assets. In such models, the specification of the independence structure is of primary importance, because of the crucial role of correlation parameters when valuing multi-issuer credit derivatives.

1.3.9 Credit Default Swap Summary

In summary, the structured credit market includes a wide variety of capital markets products designed to transfer credit risk to other markets represents an exponentially growing market. This growth is occurring because credit derivatives and structured credit techniques add value by allowing investors and financial institutions to access credit markets and credit instruments that were previously not available and to manage credit risk far more efficiently. The CDSs play the major rule in this growth. A main reason is the rule of hedge funds and their growth in capital structure arbitrage strategies (more about this discussion in 2.2).

Modelling credit is a difficult task for a wide variety of reasons. Nonetheless, credit models have become an essential requirement in the analysis, pricing and risk management of credit. An understanding and appreciation of the advantages and disadvantages of various models is therefore necessary to anyone wishing to apply a more quantitative approach.

In general, and as usual, structured-form models give a more precise idea of the impact of true economic factors underlying the pricing of credit derivatives and are thus more meaningful economically. Reduced-form models, on the other hand, can handle more complex structures more easily and can provide for better fitting of historical data, without generally capturing the nonlinear interdependence of the variables concerned. Within the structural approach, the amount recovered by a bondholder in the event of a default emerges naturally from the model - it is simply the value of the assets of the firm at the bond's maturity. However, within the reduced-form approach, the recovery process must be modelled explicitly. Therefore to completely determine the price process of a security subject to default risk, the payoff in the event of default must be specified in addition to the mechanism describing the occurrence of default events. Structural models were reviewed and shown to be best for performing a risk assessment of publicly traded companies, or where a better understanding of the effect of the capital structure of a firm is needed. Structural models are, however, not the preferred choice for pricing and hedging credit derivatives in a risk-neutral framework. This is where reduced form models are more appropiate since they are powerful enough to price even the more exotic credit derivatives and have the flexibility to exactly reprice the observable market instruments.

The pricing of CDSs is closely tied to the credit premiums on related reference cash instruments and levels in the financing and swap markets. Therefore, liquidity is a major issue for pricing CDS. Because in reality it is fact that some markets are sometimes very illiquid (i.e. emerging markets, specially in the time 1998-1999, telecommunication or aircraft industry in 2001 or in general bonds on corporations in distress). The illiquidity issue, affect not only prices of defaultable bonds but also their derivatives such as CDS. To solve this problem the differences between the market and the theoretical par CDS spreads -meaning the basis- has to be taken into account in the pricing models. Traditional and recent approaches explain the differences mainly by building new implied survival probability curves calculated from market data (see Duffie (1999) and others). It is already known that besides credit risk the liquidity (basis risk) is the driving determinant of the CDS spreads. None of the recent models takes account for this fact properly. Until now there are no approaches for CDS pricing models which valuate a 'liquidity premium' directly.

The problem is widely addressed in the literature such as in Longstaff, Mithal and Neis (2003) or Das, Sundaram and Sundaresan (2003). In case of interest rate swap spreads, Huang, Neftci and Jersey (2003) have shown empirically the significance between liquidity and swap spreads. However, in any case a proper solutions including a liquidity based approach as the one of the main factor (basis risk) in pricing and hedging credit derivatives or capital structure arbitrage strategies is missing. Finally, the following bullets should summarize the major issues:

- *For single-name instruments pricing is well understood*
- *Recovery value definition can have a significant effect on pricing and hedging*
- In *hedging CDS with bonds basis risk cannot be ignored*
- *The distinction between Emerging Markets and Fixed Income credit derivatives gradually disappears*
- *Consistency of pricing and hedging methods becomes more and more important*

Chapter 2

Capital Structure Arbitrage and Hedging

In this chapter, the main arbitrage strategies related to the capital structure of a firm will be analysed. Capital structure arbitrage involves taking long and short positions in different instruments of a company's capital structure. The analysis will focus on the strategies between equity, debt, and pure credit instruments of a given company, sector or industry. This chapter will try to answer why such mispricing occurs and how to profit from it. In the end of this chapter a discussion of the market for these strategies is presented including the rules of banks and hedge funds and the consequences to the financial market such as decreasing margins and increasing hedge fund industry in these strategies.

2.1 Main Strategies

The number of underlying securities and its option leads to a massive number of possible combining trading strategies, not including structured and hybrid products. Therefore describing all existent trading strategies are out of the scope of this thesis. As a consequence, only the most important strategies related to the capital structure of a firm will be presented.

2.1.1 Equity and Debt Market

One example of trading including securities in both equity and debt markets is the convertible bond's delta neutral hedge strategy. Basically, this strategy involves going long a CB and going short the underlying common stock. This example was fully detailed in Chapter 2. However, an interesting *reverse hedge* is an effective way of capturing temporary overvalued CB, so-called the Chinese hedge. In fact, it is the opposite of the traditional delta-neutral hedge in that the investor sells sho rt the CB and buys the underlying stock, making the

	Stock price	**Volatility**	**Credit-spread**
Long convertible	Negative	Positive	Negative
Hedged convertible	No effect	Positive	Negative
Reverse hedge	No effect	Negative	Positive

Table 2.1: Comparison of effects

position short in volatility. The opportunities to set up such a reverse hedge are identified when there is low liquidity in a CB issue, high demand for a particular issue, high implied volatility not sustainable in the short-term, or a combination of a "hot" issue in a favored industry.

The hedge ratio between the short convertible position and the long stock position should be establish to keep a neutral hedge, as to profit from a temporary CB overvaluation and thus to quickly close the short position. In terms of risks, *theta* risk has to be considered, since the reverse hedge creates a negative cash flow carry (CB's yield is higher than stock's yield). Another risk derives from the inverse relationship between volatility and stock price: if a stock price declines, volatility might increase, causing the option component of the CB to increase in value, avoiding the capture of the overvaluation intended. Yet increasing volatility and declining stock prices are generally linked with widening credit spreads, which has the opposite effect on CB values. Therefore, the position's *omicron* risk has to be compared with *vega* risk to ensure that the hedge will pay off on the downside from one or both of these factors. In fact the exposure to positive *omicron* as a result of increasing volatility and declining stock price leads the reverse hedge to be indirectly long volatility. Because of this, the position could provide good returns even in the case it has a greater exposure to *omicron* risk than to *vega* risk. Table 2.1 shows a comparison between the effects of the stock prices, volatility, and credit spreads on the delta neutral and reverse hedges.

Since many types of convertible instruments include embedded call options and contractual or embedded put options, different arbitrage strategies occur between the CB embedded options and the CB issuer's listed or OTC options. The main objective is to sell the expensive option and to buy the cheaper one, although call options can increase the protection of a traditional convertible delta neutral hedge or even enhance the return of a position. Since a CB may be converted into the underlying common stock, a put or call option can be bought or sold against the convertible security. Usually, the investor will short a call option in a covered or partially covered way and/or purchase a put option to get some additional protection. Furthermore, the longer-term embedded CB option's implied volatility is lower than the listed shorter-term call option implied volatility at or close to the same strike price. This suggests that opportunities in the volatility time skews could appear from time to time.

The first example of an option hedge strategy is the *covered (or partially covered) convertible call option hedge*. This strategy is done by selling a call option against a long convertible, and it is considered a full covered hedge if the number of shares the options can exchanged for no more than the number of shares of stock the convertible can be exchanged for. The short call option earns the premium if the stock price remains below the strike price and suffers unlimited loss otherwise. However, the gain of the part long CB's embedded call option offsets or reduces the loss on the call short in case the stock price moves above the option strike price plus the call option premium. Stocks that have recently registered a strong price decline are excellent candidates to hedge because there present high implied volatility and they are not likely to regain their previous value. There are other characteristics that could identify a candidate for this hedge strategy, such as:

- Undervalued CB (*implied volatility* $<$ *expected volatility*),
- CB with higher upside than downside gamma potential,
- Implied call option volatility $>$ expected call option volatility $>$ implied CB volatility,
- Overvalued underlying stock price, and
- No significant reason for upside stock move.

However, some risks have to be considered, including:

- A possible take-over of convertible issuer,
- A stock price moving above call option strike price, and
- A widening volatility time skew.

In fact, shorting an in-the-money call option is an alternative to the traditional (short) stock hedge of the delta neutral strategy presented in Chapter 2. This alternative is put in practice in case a stock borrow is not available, or if the underlying stock pays a high dividend yield. It is worth noting that this strategy will not protect the position if the stock price declines significantly below the option strike price. In fact, a covered call option hedge is not a true risk-neutral strategy, yet it provides an attractive return profile for a wide range of stock prices.

Instead of shorting in-the-money call options, the hedge can also be done with at-the-money and out-of-the-money options. The at-the-money hedge offers the highest risk-reward profile for small stock price ranges around the strike price. The out-of-the-money strike offers better upside returns, but less downside protection. Figure 2.1 shows a covered out-of-the-money call write with a strike price of \$30 established when the stock traded at \$25. At stock price above \$32, the CB's gains do not offset the loss on the call option and the minimum return point is \$35.

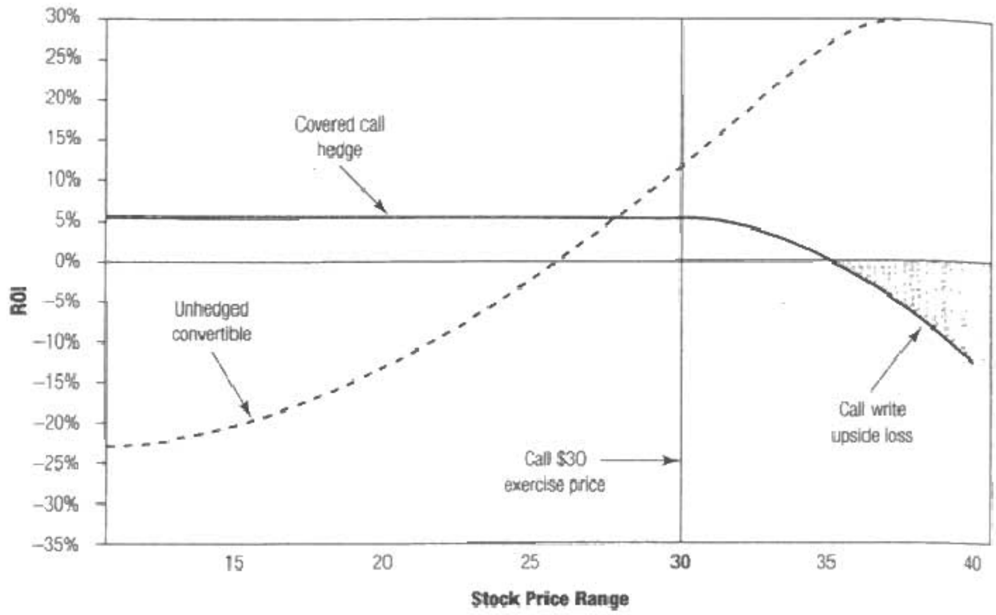

Figure 2.1: Covered call write - upside risk; Source: Calamos (2003)

Even though the neutral delta hedge with stocks offers more downside protection when the stock price declines, covered call writing offers a much better neutral return profile than stock hedging, especially with out-of-the-money convertible bonds, mainly because the income flow from the call write is higher than the income from the short stock position. The covered call write overlay hedge can be set up with protective put options to provide insurance or to raise the bond floor of the CB. Deeply out-of-the-money put options are a means of protecting against downside risk and close-to or at-the-money puts are used to enhance the position return. In either cases, the cost of the puts must be much less than the call option premium received.

Another option hedge technique is the *long convertible stock (or delta neutral) hedge with call option overlay.* This overlaying can significantly improve the total return of the position without compromising the neutral profile. This strategy profits from the changes in the volatility skew of the lower long-term implied volatility of the short stock and the higher short-term volatility of the overlaid covered-call write. The following characteristics help identifying opportunities for this hedging:

- A call option implied volatility above short-term expected implied volatility,
- Difficulties in borrowing additional stock,
- A stock price expected to trade within a small range over the life of the option, and
- A call option implied volatility $>$ the CB's implied volatility.

The risks associated with this hedge strategy are:

- A CB issuer take-over at stock price above option strike, and
- A widening implied volatility skew.

A non-traditional option hedge is the *long distressed*[1] *(or out-of-the-money) convertible with call write and long out-of-the-money call for take-over protection.* In fact, this hedge can be regarded as a synthetic bond in that its entire purpose is to add yield to an already high-yielding security. The risks to consider are related to credit and interest rate risk. *Omicron* is reduced if the CB issuer has a strong balance sheet[2], and *rho* risks is mitigated at the portfolio level, using U.S. Treasury or LIBOR futures. As the convertible is in the distressed or junked area, it is likely that a take-over could happen. Should the CB issuer be taken-over at a stock price greatly above the current stock and option prices, the short calls will generate a huge loss that may not be offset by the CB income and embedded option. That is because a long out-of-the-money call option close to the break-even point should be purchased. The following characteristics help identifying a CB candidate for this strategy:

- A CB with stable or improving credit rating,
- A short call option implied volatility $>$ the expected implied volatility,
- A cheap long call option implied volatility, and
- The CB issuer's industry activity is not in a M&A wave.

Yet some risks have to be considered, such as:

- Widening credit spreads,

[1] Sometimes called "junked" or "busted".
[2] Means here low leveraged companies.

- Interest rates moving up,
- A Further decline in stock price.

Another application for options in the traditional delta neutral convertible strategy is the use of put options to enhance stock hedge positions when the CB's downside risk is high or the bond floor is elusive. Buying put options on the underlying stock improves the hedging risk-reward profile. These options can also be used to hedge the credit risk of low-grade CBs, because of the high correlation between declining stock prices and credit spreads. Put options can also be bought as *catastrophic insurance* instead of CDS. The delta neutral hedge position can be designed with deep out-of-the-money put options that will protect the position should a sharp stock price declining and a credit event occur. The put option strike price should be set at the stock price level at which negative gamma values may occur in the CB. Figure 2.2 shows the return profile of an unhedged, a stock hedged with a put, and a stock hedged without puts, in the event of a declining stock price.

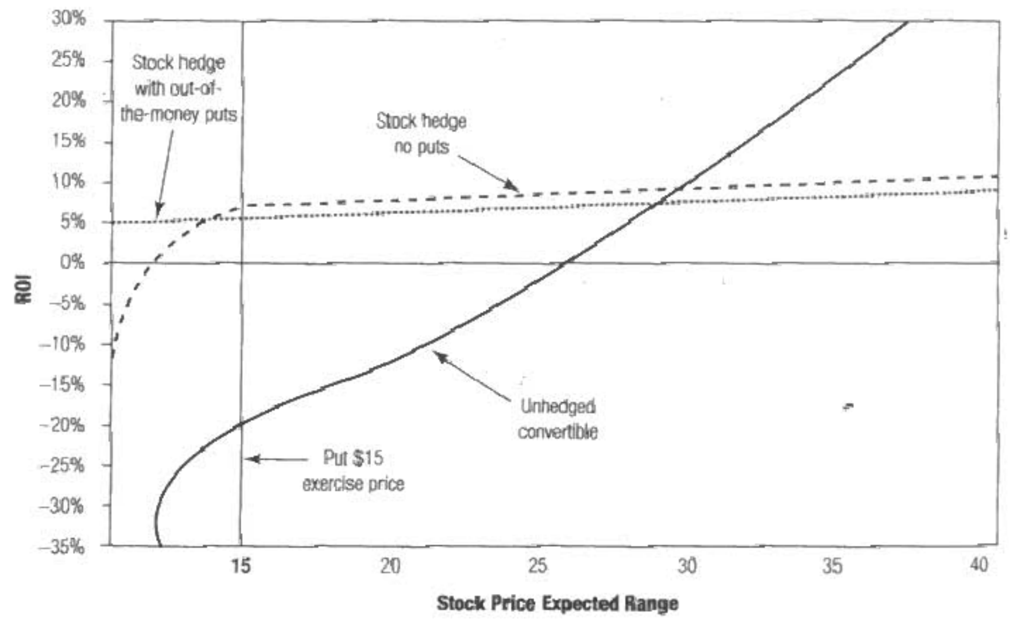

Figure 2.2: Stock hedge with catastrophic put protection; Source: Calamos (2003)

For the stock hedge position without puts, the return starts to enter in negative ground because the CB's delta begins to rise and the stock hedge is insufficient to offset the negative gamma. As seen in Figure 2.2, the profile for the position with puts represents a slightly lower return for all stock prices above $15, because of their cost, but once the stock price declines below $15, the puts preserve the hedge's neutral profile. The following characteristics help identifying CB suitable for this technique:

- A CB with low downside *gamma* potential, high volatility, and high liquidity,
- A low put option implied volatility and price,
- A CB with low credit rating but not trading in the distressed zone, and
- CDS protection expensive or not available.

The following risks should be considered:

- Under-hedging of short stock protection,
- An incorrect setting of negative gamma point, and
- Expensive put options reducing the overall position return.

2.1.2 Equity and Credit Market

The empirical researches of Hull et al. (2003a) and Zou (2003), presented in Chapter 2, show that a direct link between equity and credit market exists. In the former, the ranking order of a company credit worthiness can be drawn given the volatility skew implied by the company's equity option. In the latter, the implied volatility surface of a given company determines its CDS spreads observed in the market. Furthermore, through the analysis of the business cycle of a given company, called leverage cycle, by AXA (2003) it is possible to infer that debt and equity markets follow a regular cycle as shown in Figure 2.3.

In Figure 2.3, when credit quality is good the CDS rate is down and *vice-versa*. The leverage cycle can help explaining the volatility discrepancies of equity and credit. Analysing Figure 2.4, when there is a volatility divergence in the $Debt > Profit$ phase, most likely equity will be ahead of credit, meaning that equity volatility will be higher than credit volatility[3]. When both volatility converges, this could be a bubble burst indicator. In such a situation, both volatilities will increase and they will diverge again in the deleveraging phase (or debt reduction). In this phase, credit will be ahead of equity, meaning that credit volatility will contract more than equity volatility. After the deleveraging phase most likely profits will start again increasing faster than debt and the equity and credit volatilities will stay stable or down until the debt starts growing faster than profits and the cycle starts over again. The leverage cycle and its statistical properties will be empirically analysed in next part of the thesis.

There might be identified at least two trading strategies from the volatility discrepancies explained above, according to Illinski (2003):

[3] Credit volatility means in this context the model implied volatility of the CDS rate.

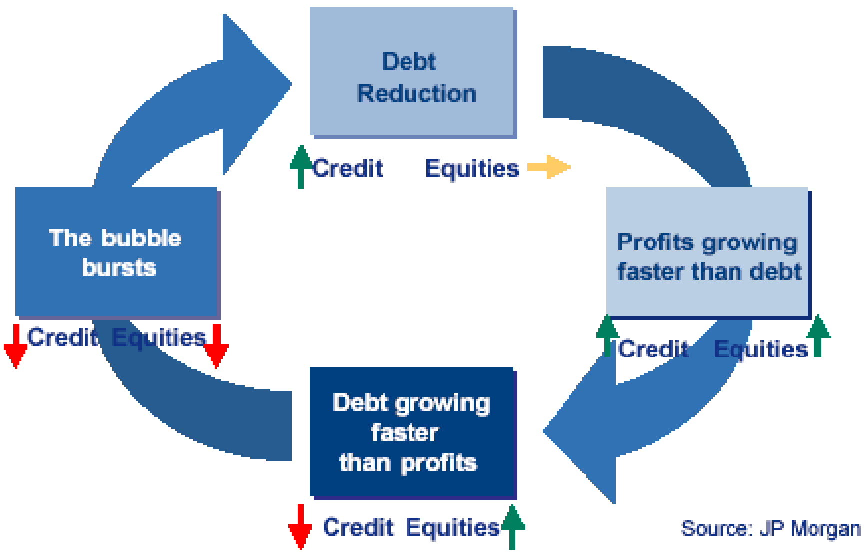

Figure 2.3: Leverage cycle

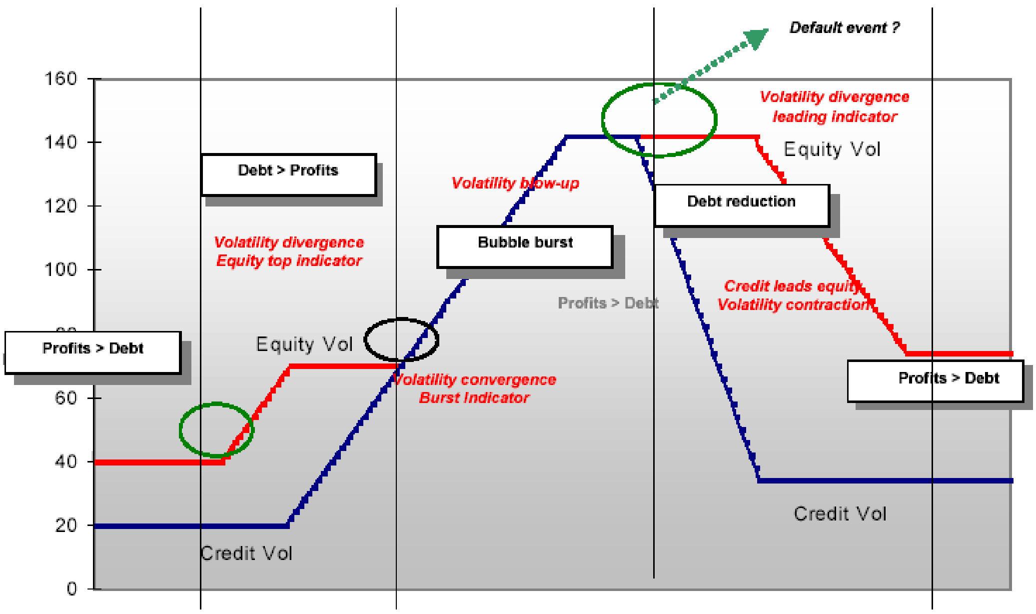

Figure 2.4: Volatility convergence and divergence follow the leverage cycle; Source: AXA (2003)

1. To buy low volatility in CDS market through going long (or buying) a CDS and selling high volatility in equity options through going short (or selling) an option on the equity.

2. To buy low volatility in CDS market through going long (or buying) a CDS and capturing daily volatility spread through daily gamma trading.

The first strategy will arbitrage using the equity and credit implied volatility and as an example, the position can be detailed as:

- Buy CDS,
- Delta-hedge with underlying equity:
  - The result is a long volatility position at $x\%$ for a daily time decay (theta) Z,
- Sell N equity options,
- Delta-hedged with underlying equity:
  - The result is a short volatility position at $y\%$ for a daily time decay (theta) W.

In this strategy, N is calculated in a way that the portfolio is neutral to volatility moves and gamma risks. In fact, the portfolio is delta, gamma, and vega neutral, but theta positive. In case the implied volatilities of CDS and equity markets converge the position is closed. If not, as $W - Z \gg 0$ there exists a positive carry position, which can be carried until expiration to generate positive returns.

The second strategy will arbitrage using equity historical volatility and the matter of factly the entering position can be detailed as:

- Buy CDS,
- Delta-hedge with underlying equity,
- Long volatility at $x\%$ for a daily time decay (theta) Z.

In this strategy, the objective is to capture the daily volatility spread through a daily gamma trading with delta being hold neutral. In this position, the downside is limited with strong upside potential. The exit occurs when the daily spread between realised equity volatility and $x\%$ is captured. In fact, $x\%$ should be as low as possible to benefit from upward volatility moves and to provide a strong downside protection.

These two strategies and the leverage cycle could be combined, as depicted in Table 2.2.

Leverage Cycle	**Equity**	**Credit**	**Strategy**
Profits > Debt	Stock price up Vol. stable or down	Stable or Up	Buy CDS, sell options Delta-hedge
Debt > Profits	Stock price sharply up Vol. up	Stable	Buy CDS, Delta hedge
Bubble Burst	Stock price sharply down Vol. up	Sharply down	Buy CDS, Delta hedge
Deleveraging	Stock price Stable Vol. Down	Up	Butterfly position Buy CDS, sell AtM options Buy OtM call Delta-hedge

Table 2.2: Leverage cycle

Table 2.3 depicts the link between the leverage cycle and possible profits coming from the two strategies analysed above.

Leverage Cycle	**Profit**
Profits > Debt	Theta positive carry
Debt > Profits	Gamma trading
Bubble Burst	Vega position Gamma trading
Deleveraging	Theta positive carry Net short vega position *or* Default *or* Sharp equity upward move

Table 2.3: Leverage cycle and profits

2.1.3 Credit and Debt Markets

As noted by Section 1.3.8, the asset swap rate is a good indicator for a fair credit default swap spread and a positive basis means that the default risk is priced not equally in the debt market and in the credit market. The CDS market considers the default risk to be much higher than the bond market does. Protection sellers demand a high premium through the credit market, whilst bond investors are willing to buy bonds at a rather high price. The bond investors receive a low asset swap spread and a low compensation for the inherent default risk. Thus the arbitrageur could apply the following strategy (sometimes called basis trading), if short sale of bonds are possible:

1. To sell CDS (sell protection) at a high price.

2. To short the bond or the asset swap.

or

1. To sell CDS at a high price.
2. To buy bonds at a lower spread.

The above strategies are the selling side of protection or being long to risk, if an investor wants to short a credit and lock-in term financing, avoiding squeezes and buy-ins, the following strategy is recommended:

1. To buy CDS (buy protection) at low price.
2. To sell bonds at a higher spread.

An example for the above strategy: a bond of a company trades at a spread to Libor of 177 *bps* and a CDS for the same company is offered at 150 *bps*. The investor can thus get short at 27 *bps* cheap to cash. If short sales are not possible due to liquidity, tax, accounting, or other causes, than a possible strategy would be:

1. To issue credit-linked notes (CLN), which are linked to the default risk.
2. To sell these CLN in the asset swap market.

or

1. To buy long dated discount bonds of an issuer.
2. To buy default protection on the same issuer.

For example, suppose the bond is offered at 85.5% of par with a spread of 436 *bps* and a 5-year default protection at 460 *bps*, allowing the investor to purchase a par-put[4] at +24 *bps* to the bond. If the credit defaults within 5 years the investor earns 14.5 *bps*, because in case of default he receives par from the CDS. If credit improves, the higher spread duration on the bonds overcompensates the spread tightening on defaults. If credit deteriorates, the basis between bonds and defaults increases and which helps to offset the higher spread duration of the bonds.

A classical example of credit and debt markets trading strategy is the *Convertible Asset Swaps and Credit Default Swaps*. Basically, a convertible bond is stripped out synthetically in its equity option and fixed-income stub, typically with the fixed-income part being asset-swapped via an interest rate swap or hedged with a CDS. More details about this strategy will be shown in the next section.

[4] Par-put means the investor will "put" the bond to the credit protection seller in case of default and will receive par for the defaulted bond.

Convertible, Asset Swaps and CDS

Convertibles can be stripped in two ways. The traditional approach is through an asset swap, where a fixed income investor purchases the convertible bond and simultaneously enters into a callable asset swap that converts fixed cash flows to floating and effectively monetizes the embedded equity option. The fixed income investor ends up with a position in the credit component of the convertible. Meanwhile equity investors obtain an equity call option that can be held or monetized in the equity derivatives market. A second form of stripping occurs when equity investors purchase convertibles and lay off credit risk by purchasing protection, mostly in form of a CDS.

Convertible Asset swapping One of the most traditional ways of transferring credit risk is via asset swap instruments. Essencially, the asset swap buyer assumes the credit risk for a predetermined return. The asset swap transaction can be easily adapted to the convertible bond market through a call option feature: the convertible investor has the right to call the asset swap at a predetermined rate and thus to reconstruct the CB. Yet the convertible buyer retains the equity option. It can be said that an asset swap provides a way of extracting the mispriced option from the CB. The CB investor initially prefers asset swaps as only holding the equity component commits less capital and allows leverage.

Briefly, a plain vanilla asset swap transforms a fixed income bond to a floating rate synthetic security. To replicate a convertible bond through an asset swap a call option feature is required. In that sense, a basic convertible asset swap strips out synthetically the components of the convertible in its fixed-income and embedded option components. The asset swap components are illustrated in Figure 2.5.

The replication and the mechanism of a typical convertible asset swap can be seen in Figure 2.6. As already stated, the objective of the asset swap is to purchase a cheap or mispriced equity option of the issuer. The description of the mechanism is as follows. An equity investor purchases a CB issue, sells the convertible bond to a broker (in this case Credit Suisse First Boston) and receives an option to repurchase the CB. The broker finds an investor who is interested in the credit feature of the issue. In fact, the broker and the credit buyer enter into an interest rate swap and then exchange floating rate income to fixed rate income. A recall spread is put in place to protect the asset swap credit value against an early call or conversion. That recall spread regulates the price at which the credit seller (equity investor) must repurchase the CB.

If a credit buyer does not want to enter into a swap agreeement, asset swaps can also be structured using a Special Purpose Vehicle (SPV). The broker sells the convertible to the SPV that in turn issues callable floating rate note. Then, the broker enters into an interest rate

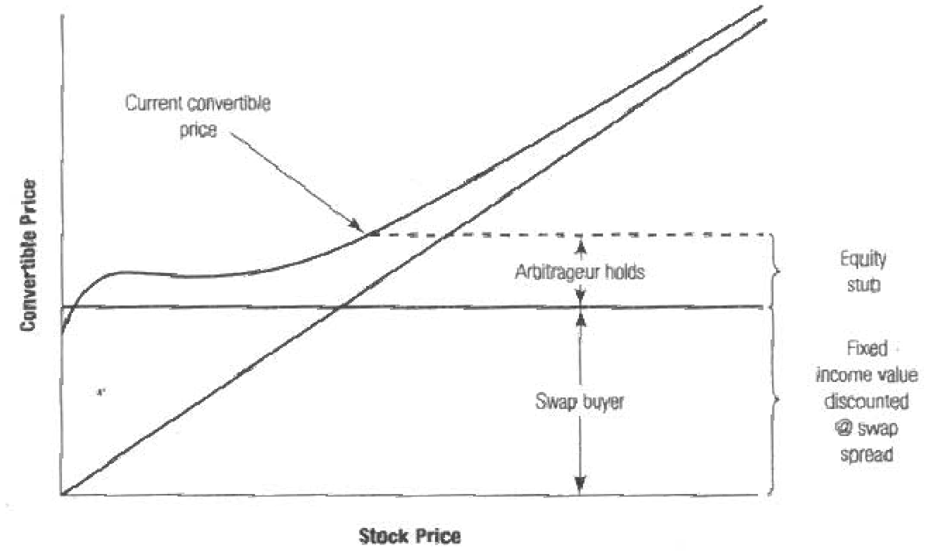

Figure 2.5: Asset Swap Components of a Convertible Bond; Source: Calamos (2003)

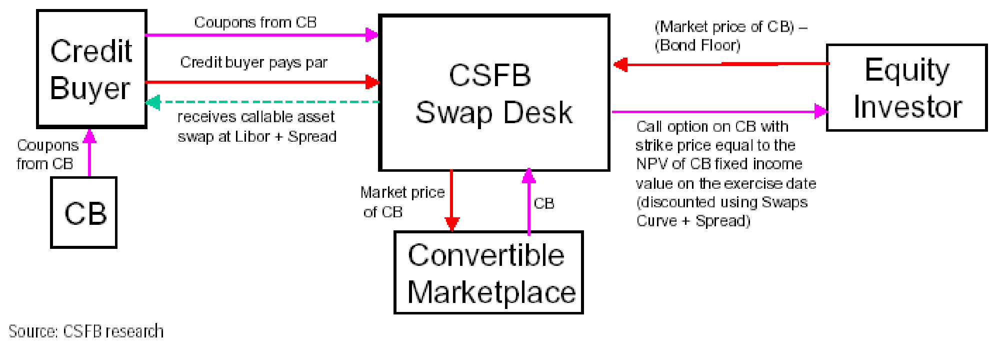

Figure 2.6: Flow diagram for a plain vanilla asset swap

swap with the SPV. The net effect is the same credit risk for the asset swap buyer. Figure 2.7 shows this mechanism.

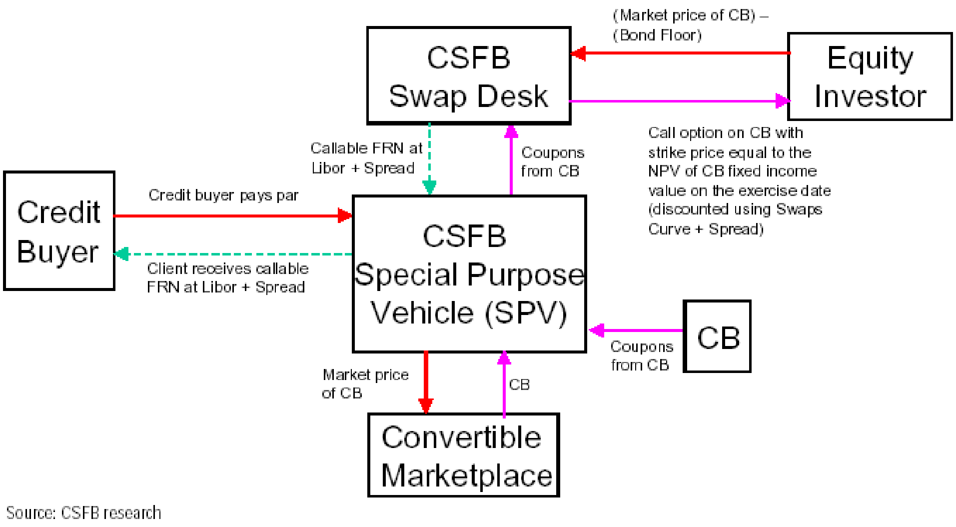

Figure 2.7: Flow diagram for a SPV asset swap

Equity investors may exercise the call option feature of the asset swap for the following reasons:

1. Conversion: the CB investor calls the asset swap and reconstructs the convertible to exercise the bonds conversion option.

2. Issuer calls the bond: due to the issuer's call option prior to maturity, the issuer can force early conversion. The CB investor will thus call the asset swap to exercise the conversion option.

3. Credit spread narrows: the credit buyer can face the asset swap being called back if the credit spread of the issuer tightens and the equity investor simply sells the CB or re-sells the asset swap at the current spread.

To protect the credit buyer from the call, contracts normally offer a "make-whole" period, according to Conway *et al.* (2002), of 6-12 months. If the asset swap is called, then the asset swap buyer receives margin for he next 6-12 months as a compensation. The call feature inherent in the asset swap is an option held by the equity investor, written by the credit buyer. The wider the initial credit spread, the more valuable the option. The credit buyer bears the credit risk until maturity, but can face the rewards being reduced in case the CB is called early. The carried credit risk is greater on the lower credit quality issuers and the risk of the CB being called early. Given the combination of widening credit spreads and the increase in sub-investment grade CB issuance, this call option plays an important role when pricing asset swaps. According to Conway et al. (2002), one solution to protect asset swap buyers is a lengthening of the "make-whole" periods, perhaps to the first call date.

The following characteristics help identifying the opportunity to enter into an asset swap:

- A CB issue has an investment-grade credit rating,
- The CB embedded option is volatile and priced below the expected volatility,
- The structure of the CB is attractive to fixed-income buyers (demand),
- The size of the CB issue is large enough to offer a large enough asset swap to attract fixed income investors (supply),
- An easily-borrowed underlying stock (liquidity),
- The CB call/put terms are long enough in duration to establish a swap, and
- Will an incentive to reduce credit risk and capital employed in hedge.

The following risk should be taken into account when establish a convertible asset swap:

- A considerable reduction of liquidity of the position,
- An increase in counter-party risk,
- A CB issuer calling for early redemption, and
- A position long in rho, vega, and theta.
- Delivery and documentation risk.

The equity component remaining from the bond asset swap can be hedged initially with the underlying stock to establish a delta neutral stock hedge. In that way, the position requires much less capital and provides more leverage opportunities than in a typical convertible hedge, as seen in Chapter 1. The delta-neutral hedging helps decreasing many of the greek risks and further decreasing the capital required. The credit risk is eliminated and the equity risk neutralized. Also, the interest rate or rho risk is significantly reduced as a result of swapping the credit and removing away much of the negative rho presented in a typical convertible bond.

Convertible Bond Credit Default Swap The growth of the CDS market has given CB buyers an alternative to asset swaps. The CDS market provides a way of extracting mispriced options embedded in low-grade CB or also to protect the delta neutral convertible hedge against credit spread widening. The CDS does not involve ownership of the convertible to be transferred. In fact, the CB arbitrageur keeps the bond and also buys credit protection, just like an insurance policy. Contrarily to asset swaps, using CDS commits more capital and reduces the possibility of leverage.

According to Calamos (2003), much of the global CB issuance comes from low-grade companies and this feature makes CDS a useful tool to hedge and manage credit risk. In fact, the CDS provides the CB investor with a way of transfering the credit risk to the swap seller for a specified time period and at a fixed spread over LIBOR. The CB's fixed-income component is valued by the issue's CDS spread. In this strategy, the ownership of the convertible is not transferred: the convertible arbitrageur keeps the bond and additionally buys credit protection, which can be seen as an insurance policy.

This policy gives the holder the right to sell the CB to CDS writer for par, given a predefined credit event. Therefore, the CDS can be viewed as a put option, as it will benefit directly from a spread expansion[5]. The mechanism of a typical CDS and CB strategy is shown is Figure 2.8, as adapted from Tierney (2001). In fact, if the credit spreads widen, the price of credit spread protection increases and the CDS profits as the CB drops in value. Conversely, a CB hedged with CDS protection will not gain from a narrowing in credit spreads, but the convertible itself might gain in value, as it captures some of the benefits of the spread tightening. Another way of hedging credit risk is through put options such as the strategy described in the previous section of this chapter. In fact, to determine if put options offer a better opportunity to protect the hedge is to compare the cost of purchasing puts to cover the difference between par value of the bonds and the recovery rate and the present value of the CDS premiums. In relation to leverage, the CDS does not provide leverage on the position as would the convertible asset swap. Furthermore, it increases the cost of the hedge, thus reducing the amount of leverage used.

This type of stripping has become more prevalent over the past year by convertible bond arbitrage hedge funds, which seek to earn arbitrage profits by purchasing cheap equity exposure through convertibles and shorting the more expensive underlying equity. The problem for these funds to manage the residual credit and credit spread risk associated with convertible bonds, especially when the equity option component is out-of-the-money and the convertible

[5]There is a positive correlation between company specific credit spreads, equity price and option implied volatility.

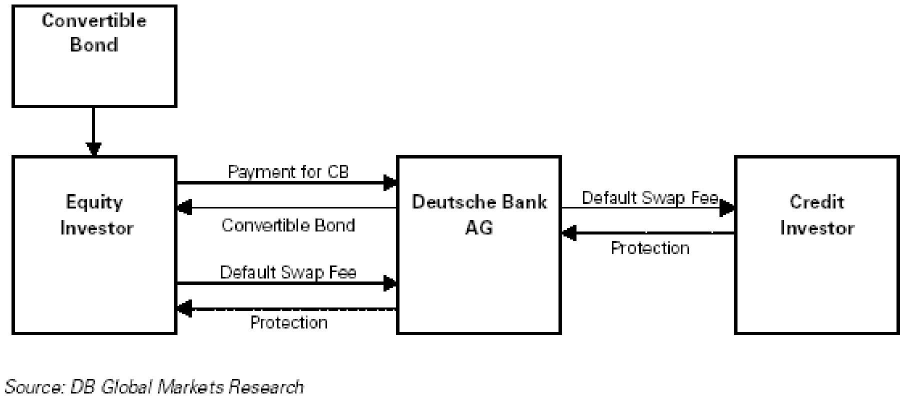

Figure 2.8: Mechanism of a CB being hedged with a CDS

bond is trading more on its credit rather than equity characteristics. Purchasing protection in the credit derivatives market provides an effective hedge against both spread volatility and default risk since the value of default swaps rises (falls) as credit spreads widen (tighten). This allows hedge funds that have neither interest nor expertise in managing credit risk to focus exclusively on the equity component of the arbitrage strategy.

Credit investors, on the other hand, can obtain credit exposure to the company issuing the convertible bond by selling protection (i.e. selling a CDS) or by purchasing a credit-linked note. One interesting consequence of this hedge fund activity is that credit market investors have been able to obtain credit exposure to up-and-coming companies that have issued only equity and convertibles but have no bond market debt outstanding.

The following characteristics help identifying the opportunity to enter into a CDS:

- A CDS exists and it is liquid,
- The CB's embedded equity option is inexpensive,
- The underlying stock easily borrowed, and
- Low counter-party risk.

The following risk should be take into account when establishing a convertible asset swap:

- Narrow credit spreads,
- A deterioration of counter-party credit,
- Documentation and deliverable risks, and

- A CDS was overpriced and returns back to its fair value.

According to Conway et al. (2002), CDS are not suited to dirve a bull credit markets, as the owner of protection cannot gain from a tightening of credit spreads. Conversely, as asset swaps have a pre-specified recall spread, the holder of the equity component (CB investor) will benefit from credit spread narrowing by recalling the original asset swap and re-selling it at the new spread, being thus more suitable for bull credit markets. In fact, in their research there is a very interest scenario analysis that clearly shows those features.

It is worth noting that either swap strategies are useful and the choice depends on the specific market situation and the risk profile of the CB investor. In Table 2.4, adapted from Calamos (2003), a comparison between both strategies features provided.

Convertible Asset Swap	**Credit Default Swap**
Eliminate credit risk	Eliminate credit risk
Reduces interest rate risk	No reduction in interest rate risk
More expensive than CDS	Less expensive than asset swap
Callable	Not Callable
Less liquid market	More liquid market
Provides leverage	No leverage
Non standard contracts	Standard contracts

Table 2.4: Comparing credit risk hedging strategies

2.2 The Market for Capital Structure Arbitrage and Hedging Strategies

This section will serve with a market description of capital structure arbitrage and the players involved. As pointed out in the previous section the major rule in this strategies is played by CDSs. CDS versus equity or equity options are known as the classical capital structure arbitrage strategies, where CDS versus CB is more like a classical convertible arbitrage strategy. Both strategies use the relationship between CDS rates and the implied volatility smile from equity options with detailed knowledge of the leverage of a firm to anticipate on arbitrage opportunities. The implementation of such strategies is simply a convergence trade. Therefore we will focus only on CDS related strategies. We want to make more clear the different rules of banks and hedge funds for such capital structure arbitrage strategies and what the consequences are of the CDS market to the financial market, such as lowering margins.

2.2.1 The Participants of the Capital Structure Arbitrage Strategies Market

We already described the CDS market on instrument level in subsection 1.3.5. According to the 2003 credit derivatives survey by *Risk Magazine,* CDSs with four to six years maturity referenced on investment-grade credits account for around 49% of the total vanilla market, which by self accounts for about 72.5% of the entire credit derivatives market in terms of notional outstanding. Furthermore, it provides a more detailed geographical breakdown over different maturities. It reported that 46,8% of the CDSs in investment grades are linked to North American credits, 42% to European credits and 11,2% to Asian credits. Now we will focus more on the participants of the CDS market.

CDS and in general the wide variety of applications of credit derivatives attracts a broad range of market participants. Historically, banks have dominated the market as the biggest hedgers, buyers, and traders of credit risk. Over time other types of players are entering the market. This observation was echoed by the results of the BBA survey 2000, which produced a breakdown of the market by type of participant. The results are shown in Figure 2.5

Counterparty	Protection Buyer (%)	Protection Seller (%)
Banks	63	47
Securities Firms	18	16
Insurance Companies	7	23
Corporations	6	3
Hedge Funds	3	5
Mutual Funds	1	2
Pension Funds	1	3
Government/Export Credit Agencies	1	1

Table 2.5: Market share of participants

Commercial banks are the largest players in the CDS market. According to figures compiled by the BBA, banks accounted for 52% of the protection buyers market and 39% of the protection sellers market in 2001. The BBA expects both these shares to drop by 2004, to 47% and 32% respectively. That would still make banks dominant in the market for protection buying, but in terms of protection selling they would be overtaken by insurance companies, the share of which will remain steady at 33% in 2004 – identical to their share in 2001, according to BBA forecasts. Hedge funds activities in buying and selling protection was increasing significantly over the last two years as well.

One of the most important *benefit for banks* of credit derivatives in general and in particular CDS can be traced back to 1988 and the publication of the Basel Capital Accord which forced many lenders to reappraise the size of their exposure to corporate borrowers, many of whom would have been long-standing customers. For *lenders*, this represented a serious problem: how could they reduce their exposure (or simply leave their exposure static) to borrowers with whom they had developed close links dating back many decades without seriously jeopardizing those relationships? The CDS market provides a valuable solution to that dilemma. For banks with limits on their credit lines to individual borrowers, the CDS market is an effective means of *transferring risk on outstanding loans without physically removing assets from the balance sheet.* To fit balance sheet reporting intervals of the counterparties, the counterparties exchange payment dates varying from quarterly to annually. Furthermore, banks with high funding costs can effectively achieve Libor funding by sourcing risk through a CDS when they otherwise might pay above Libor. In case of specific collateral agreements the funding for AAA rated banks can be even below Libor, see Cossin (2002).

The most powerful incentive for *lending banks* to use the CDS market as a means of *transferring the risk on their loan books*, however, is that it allows them to do so without the knowledge of the borrower. This in turn allows them to free up additional lending lines for valued customers that may be very important sources of ancillary business in, say, corporate finance. Alternatively, CDSs can be used as a means of a reducing bank's concentration in individual industrial sectors or geographical regions. The use of the CDS market can therefore help banks to promote and maintain their client relationships, allowing them to *open up new credit lines* that might otherwise have remained closed. Note that there is a conflict of interest within commercial banks among the lenders.

> According to Moore and Watts (2003):
>
> "*Another cause for concern about the role played in the CDS market by commercial banks that are active participants in the syndicated lending market is the potential for conflicts of interest among these lenders. Some newspapers have alleged that Chinese walls between banking and trading desks have been broken, with lenders privy to much more comprehensive information about their borrowers than investors in the capital market or sellers of protection in the CDS market.*"

Investment banks are also active participants in the CDS market, both as providers of liquidity for their customers and as proprietary traders. The CDS market can offer a highly efficient means of removing assets from the balance sheets of investment banks, an objective that has become more and more important in recent years as the leading investment banks seek to offer a 'one-stop shopping service' to their corporate clients. Given the relatively *limited size of an investment bank's capital*, the CDS market provides them with a useful means of demonstrating their commitment to corporate clients by supporting *syndicated lending facilities without exerting unsustainable strains on their balance sheets.*

The participation of *Insurance companies*' in the CDS market, predominantly as sellers of protection, is visibly. While many insurance companies will provide protection as writers of single-name CDSs, they are also active in the market as buyers of CDOs and credit-linked notes. It is important, however, to differentiate between the different types of insurance companies active in the CDS market. Life assurance companies, for example, act as an important source of investor demand for ABSs and CDOs. US insurance companies, meanwhile, are important players in the CDS market, often as sellers of credit protection on the senior AAA rated notes in structured portfolio transactions.

Counterparties, which do not face any credit risk linked to a specific area like *hedge funds*, *investment funds* and *insurance companies* might want to take the credit risk from a CDS in exchange for a fixed payment.

> According to O'Kane (2001):
>
> *"Hedge funds are another growing participant. Some focus on exploiting the arbitrage opportunities that can arise between the cash and default swap markets. Others focus on portfolio trades such as investing in CDOs. Equity hedge funds are especially involved in the callable asset swap market in which convertible bonds have their equity and credit components stripped. These all add risk-taking capacity and so add to market liquidity."*

2.2.2 The Rule of Banks and Hedge Funds in Capital Structure Arbitrage and Hedging Strategies market

Hedge funds have come a long way since the days Alfred Jones started borrowing stocks. His simple -yet visionary- idea of adding short positions to a long portfolio with the aim of producing superior risk-adjusted returns, set the path towards the wide-ranging, dynamic industry we see today. For a general introduction of hedge funds we refer to Fung and Hsieh (1999) and Lhabitant (2002). We focus here only on capital structure arbitrage strategies.

Hedge fund managers in these days don't just borrow stocks, they make use of all kinds of financial instruments. The revolution in financial engineering over the last two decades has generated a wide range of trading tools that they can use to participate on arbitrage opportunities. The range of financial instruments includes exchange traded fixed income and equity instruments as well as commodity derivatives, fixed income OTC derivatives, credit derivatives and structured and hybrid instruments.

Since the LTCM crisis, the required repo margins have been large. During this time and even after the equity bubble in 2000 there appears to be a change in the financial markets. An increasing number of credit-dedicated hedge funds are being created.

> According to Patel (2003):
>
> "*We've seen a threefold year-on-year increase in hedge funds' credit derivatives activity over 2001 and 2002, says Barry Bausano, New York-based global head of hedge funds at Deutsche Bank. The number of accounts has doubled during the same period, he adds.*" And furthermore: *"Our credit derivatives business with hedge funds has more than doubled each year for the past years, says Suzanne Cain, New York-based co-head of credit derivatives at Morgan Stanley"*
>
> and according to Currie and Morris (2002):
>
> " *Viswas Raghavan, JP Morgan says, at the beginning of the year, there where only a few serious investors looking at debt-equity strategies, today there are over 30 funds and by the this time next year that figure could be well reach 200.*"

These hedge funds are trading an increasing variety of credit instruments. Predominantly, this increase in business was accounted for by existing users of CDSs becoming more active, funds being launched, old funds pursueing new global arbitrage opportunities, and convertible bond players as well as equity volatility arbitrageurs. Single name CDSs are the most widely-used product among hedge funds applying capital structure arbitrage strategies.

Factors driving these changes include the efforts convertible arbitrage funds have been making to hedge credit risk of convertible bonds, first with callable asset swaps, more recently with CDSs. For example, a lot of convertible arbitrageurs and equity option players took the opportunity to buy protection on the credits of German banks late last year. One striking result was the basis between CDS and the cash bonds of Commerzbank, which was out to a basis of around 90bps. (Source DB research, see also the example of Deutsche Telekom in section 1.3.8 or more illustrative the detailed analysis of France Telekom in chapter3).

The other side of such long CDS protection positions held by hedge funds has typically been either net credit risk of convertible positions or, for equity derivatives players, it has been sold positions in out-of-the money equity puts on the same names. This trading strategies, trading CDS against equity options or convertibles, described in detail above in section 2.2, is very interesting. Trading implied volatilities in the equity options market against default probabilities implied by the credit spreads is taking credit trading beyond simple hedging. These capital structure arbitrage strategies funds, or units within existing convertible arbitrage funds, are leading the way in trading relative value between different claims on the assets of a company.

According to *Risk's* most recent credit derivatives survey, hedge funds account for around 13% of all credit derivatives end-users, as a proportion of trading volumes, see Risk Febuary 2003 page 23 or BBA (2002). In the previous survey, this figure was just 8%. These funds are straight credit players, alongside equity in capital structure arbitrage.

At present hedge funds are facing more difficulties to make money. One reason for this is the unattractive stock market, especially in low volatility periods. They are very eager to look at new areas such as credit derivatives. Other firms are now seeking to challenge JP Morgan Chase and Deutsche Bank, which by the mere of the size of their trading books and early entry into the credit derivative market are the leading counterparties to hedge funds. For more details on this see Patel (2003).

Since 1997, credit derivatives have entered the mainstream of global structured finance as tools in a number of large, high profile synthetic securitization of assets that cannot easily be managed using traditional techniques, see Bowler and Tierney (2000), O'Kane (2001) and JPM (2002). By combining credit derivatives with traditional securitization tools in CLOs or MBS (mortgage back securitization), for example, structures can be tailored to meet specific balance sheet managements goals within *banks* much more efficient. In particular, credit derivatives, such as CDSs have served banks in reducing economic and regulatory capital, preserving a low funding-cost advantage and maintaining borrower and market confidentiality. One can think of the first-to-default basket CDS structure as similar to a senior/subordinated CBO or as a senior/subordinated CLO (see before or Moore and Watts (2003) 'CDSs as a building block' for more exotic portfolio-based structures). Therefore, the credit default protection seller is similar to an investor in the subordinated tranche of a CBO or a CLO. The default protection seller takes the risk of the first loss in the basket-structure. The remaining credit in the basket is similar to the senior tranche of a CBO or a CLO. The holder of these remaining credits, the senior position, has been protected from the first loss by the default protection seller, the holder of the subordinated position. In addition, such structures double the size of the counterparties.

Although this comparison is frequently made in the market, there are very important differences between first-to-default baskets and senior or subordinated CBOs and CLOs. The risk of the default protection seller is an off-balance sheet risk as opposed to an on-balance sheet risk for the buyer of a subordinated tranche of a CBO or a CLO. Furthermore, the default protection seller does not invest capital but earns a fee. It is also important to note that the assets that compose a first-to-default basket are generally small in numbers and not necessarily well diversified. The construction of a first-to-default basket does not meet the same rigorous tests that rated CBOs and CLOs must meet in determining the composition of assets, which make up the collateral for these instruments, see also Tavakoli (2001).

For some illustrative examples and the regulatory issues see Tavakoli (2001). See also Masters (1998) for regulatory treatment of credit derivatives and balance sheet management, respective risk-based capital allocation and regulatory-based capital allocation.

2.2.3 The Consequences to the Financial Market from Capital Structure Arbitrage Strategies

The increase in counterparties given by hedge funds offers the banks to be more active in selling and buying protection to improve their balance sheet management and therefore meet better capital allocation and even to implement requirements and risk management frameworks from Basle II accord, which is a major issue in the banking industry today. Basle II will not set to take effect prior to 2006, but the impact of the changes in regulations is already anticipated in the industry. The evolution of the new Basel II accord will have a significant impact of the credit derivatives market in the future[6]. Furthermore, the increase in counterparties in form of hedge funds is a factor for completing the financial market.

> According to Moore and Watts (2003): "*An IMF Global Financial Stability report has advised that particularly as the markets mature and grow over time, credit risk transfers have the potential to enhance the efficiency and stability of credit markets overall and improve the allocation of capital. By separating credit origination from credit risk bearing, these instruments can make credit markets more efficient. They can also help to reduce the overall concentration of credit risk in financial systems by making it easier for non-bank institutions to take on the credit risks that banks have traditionally held.*"
>
> According to Patel (2003):"*Slatter* [JPM Chase] *says there is an alternative viewpoint not lacking in irony: When the accounting inconsistencies at Dutch*

[6] As we mentioned earlier, the thesis will not focus on this important issue. However, the reader should note that there is a strong impact from the Basle II evolution to the financial markets in the future.

> *retailer Ahold became apparent in late February, hedge funds were the main institutions facilitating price discovery. Hedge Funds are becoming a stabilizing factor in the credit market"*

Once can say, that hedge fund industry and the banks meet together in the capital structure arbitrage market within a win-to-win situation. By enhancing liquidity, credit derivatives or, in particular, CDSs achieve the financial equivalent of a "free lunch" whereby both buyers and sellers of risk benefit from associated efficiency gains.

A negative side effect of the liquidity in the CDS market is that the degree of success of the CDS market will reduce liquidity in the cash market. That can clearly create difficulties for investors who are restricted to investments in the cash bond market and unable to participate in credit derivatives. If it is indeed the case that the CDS market is now more liquid than the cash bond market in a number of sectors or individual credits, this implies that pricing in the CDS market provides more reliable signs of credit quality than the cash market. By extension, the CDS market ought to emerge as a more reliable benchmark than the cash market for the pricing of bonds in the primary market.

Traditionally the trading departments in banks are strictly separated between equity and fixed income activities. This has some consequences for the overall trading risk of mixed strategies such as capital structure arbitrage strategies. One major issue are differences in methodologies used for market and credit risk measure and -even more important- a separated view to liquidity risk. This fact comes from the different nature of the fixed income and the equity market as well as the underlying risk pricing methodologies. The changes in the structured credit market during the last years and the exponential development in the credit derivative market especially the outstanding rule of the CDS, in overcoming the classical fixed income cash market in terms of liquidity makes it more and more important to have an overall view in terms of a global market liquidity model. The increasing trading activities with credit derivatives combines more efficient the equity and the fixed income market and is therefore completing the financial markets as described before in subsection 1.3.6. This has a consequence to manage the overall risk of capital structure arbitrage strategies to be aware of liquidity holes in the single markets and segments.

In comparison to the traditional set up in banks the hedge fund industry is following mixed strategies from one desk and considers per se an over all risk position to manage the positions. This view is particular new in the proprietary trading business.

According to Patel (2003):

> *"In addition to bolstering overall market liquidity hedge funds voracious appetite for credit derivatives has caused dealers to radically rethink the way they organize their business" and "Our clients actually prefer to have CBs and CDSs covered by the same desk because they can get more efficient funding, UBS Warburgs Naro says"*

The fact that hedge funds activities take more parts in the CDS market bring another issue into the light: Hedge funds can influence market asset prices.

Shleifer and Vishny (1997) find out that professional arbitrageurs which are specialized and getting performance-based fees have a number of interesting implications for security pricing, including the possibility that arbitrage becomes ineffective in extreme circumstances, when prices diverge far from fundamental values. The model also suggests where anomalies in financial markets are likely to appear, and while arbitrage fails to eliminate them. Causal empiricism suggests that a great deal of professional arbitrage activity, such as that of hedge funds, is concentrated in a few markets. These tend to be the markets where extreme leverage, short selling, and performance-based fees are common.

Emmons and Schmidt (2002) analyze a three-date (two period) model of an professional arbitrageur (or "convergence trader" in the language of Kyle and Xiong (2001)) who must obtain financing from investors less informed than he is about the intrinsic value of a financial asset, that is its liquidation value at the end of the second period (asymmetry of information). In addition to these two types of individuals, there are noise traders who have wealth to invest but who misperceive the asset's intrinsic value. It is the noise trader who drives the asset's price away from the intrinsic value and is reducing volatility. They showed that arbitrageurs trading significantly affects the asset's price. While Shleifer and Vishny (1997) assume that the arbitrageur maximizes assets under management, Emmons and Schmidt (2002) assume that he maximizes his income. The arbitrageurs income is determined by an incentive scheme that resembles real-world contracts of hedge fund managers and therefore seems to be a more realistic set-up than Shleifer and Vishny (1997).

Like Shleifer and Vishny (1997), Emmons and Schmidt (2002) assume that hedge fund can influence the market price of assets. Hedge funds do in fact sometimes move market prices, because they operate in specialized market segments that have limited liquidity. (see, for example, Deutsche Telekom 2001/2002). It is also true, however, that hedge funds alone cannot prevent asset-price volatility or occasional mispricing, which might deepen before it eventually corrects.

As the market becomes more established in using credit derivatives (in particular CDSs) the margins will gradually decline and therefore participants in more complex derivatives will increase naturally, similar to the OTC derivatives market in equities and fixed income during the last decade.

Scepticism has been articulated sometimes from regulators and others such as Warren Buffet that the credit derivatives market would be unable to withstand the pressures exerted by multiple defaults and an economic crisis. However, even against the backdrop of a very weak global macroeconomic climate in the past exacerbated by accounting scandals and characterised by plummeting credit quality, the CDS market appears to have proved its stability and efficiency. Many agree that the losses sustained as a result of defaults ranging from Argentina to Enron and WorldCom were all minimized as a direct consequence of the expansion of the credit derivatives market, which has helped to diversify the concentration of risk highly effectively. Additionally, increased transparency and regulations in the credit derivatives market took they part to approach to better financial market efficiency. As a conclusion arbitrage opportunities will start disappearing faster and the volatility of profit and loss positions in financial institutions will fall.

Furthermore, internet trading blossomed after the first credit trade was closed over the internet by CreditTrade in September 1999. Internet trading in credit derivatives is still in the very early stages. ISDA short form confirmations for credit derivatives helped greatly in this effort.

Part II

Empirical Analysis

Chapter 3

Cointegration Analysis

In this part of the thesis, the empirical relationship between implied option volatilities and credit default swaps will be analysed for one company, France Telecom (FT in the following). According to traders, in the past this company presented interesting arbitrage opportunities involving capital structure financial instruments, such as CDS and equity options. The analysis will try to verify the findings of Hull et al. (2003a) and Blanco et al. (2003). Hull et al. (2003a) suggested that only two implied volatility are sufficient to determine an option-implied creditworthiness of a company. Blanco et al. (2003) suggested, among other issues, that CDS prices are well integrated with firm-specific equity market variables in the short-run. In fact, the work of Blanco et al. (2003) gives another topic to be explored in the empirical analysis: price discovery mechanism.

Firstly, a brief theoretical introduction about stationary and nonstationary stochastic processes for time series, vector autoregressive (VAR) models, cointegration and error correction model is presented. In the following, a statistical description of CDS spreads and implied equity option volatility data will be made. Then a long-term relationship between CDS rates and option volatility that could lead to some arbitrage trading strategies will be identified. The analysis will be made by a vector autoregressive (VAR) description of the relationship and by cointegration tests. Next it is presented what motivates this empirical analysis and which topics will be covered thereafter.

3.1 The Rationale of empirical Capital Structure Arbitrage Analysis

As mentioned in previous chapters, the nature of capital structure arbitrage and hedging strategies lies in the rule that CDSs play in the financial market. Typical trading strategies such as CDS versus equity or CDS versus equity options are known as capital structure arbitrage strategies. Strategies with CDS versus convertibles have also capital structure arbitrage characteristics but they are more known as convertible arbitrage strategies. The underlying key in all these strategies is in fact the relation between the CDS rate and the implied equity volatility or simply the relation between the CDS rate and the equity price. When traders set up such a strategy described they have to look on these market parameters. In reality, there is not only one implied volatility traded in the market, but rather an entire volatility surface traded which gives different volatilities for different strike prices with the lowest volatility always being the at-the-money volatility.

Just to make clear the differences between *surface*, *smile* and *skew*, according to Daglish et al. (2003):

> *"Option traders and brokers in over-the-counter markets frequently quote option prices using implied volatilities calculated from Black and Scholes (1973) and other similar models. Put-call parity implies that, in the absence of arbitrage, the implied volatility for a European call option is the same as that for a European put option when the two options have the same strike price and time to maturity. This is convenient: when quoting an implied volatility for a European option with a particular strike price and maturity date, a trader does not need to specify whether a call or a put is being considered. The implied volatility of European options on a particular asset, as a function of strike price and time to maturity, is known as the volatility surface. Every day traders and brokers estimate volatility surfaces for a range of different underlying assets from the market prices of options. Some points on a volatility surface for a particular asset can be estimated directly because they correspond to actively traded options. The rest of the volatility surface is typically determined by interpolating between these points. If the assumptions underlying Black-Scholes held for an asset, its volatility surface would be flat and unchanging. In practice the volatility surfaces for most assets are not flat and change stochastically. Consider for example equities and foreign currencies. Rubinstein (1994) and Jackwerth and Rubinstein (1996), among others, show that the implied volatilities of stock and stock index options exhibit a pronounced "skew" (that is, the implied volatility is a decreasing function of strike price). For foreign currencies we observe a "smile" rather than a skew (that is, the implied volatility is a U-shaped function of strike price). For both types of assets, the implied volatility can be an increasing or decreasing function of the time to maturity. The volatility surface changes through time, but*

> *the general shape of the relationship between volatility and strike price tends to be preserved."*

Practitioners are using the implied volatility surface from available European option prices as a tool to value a European option in case its price is not directly observable in the market. Doing this the trader prices the option consistent with the market. The fact that in the volatility skew the volatility decreases as the strike price increases yields to significantly higher volatilities to price a low-strike-option[1] than such volatilities used to price a high-strike-price option[2].

The volatility surface for equity options, named volatility skew, corresponds to the implied probability distribution of the underlying asset. It can be shown that this implied distribution has a fatter left tail and a thinner right tail than the lognormal distribution assumed by the Black-Scholes-Merton model; this means different kurtosis and skewness (see Hull (2003)). Therefore, a deep-out-of-the-money call has a lower price when the implied distribution is used for pricing than when the lognormal distribution is used, because of the fact that the probability to exercise an option for the implied probability distribution is lower than for the lognormal distribution. Similarly, for a deep-out-of -the money put option the probability to exercise is higher for the implied probability distribution than for the lognormal distribution. Therefore the price will be higher for such options. For more about surfaces, smiles and skews see Dupire (1997), Daglish et al. (2003), and Hull (2003).

One of the reasons for the volatility skew in equity options concerns leverage. As the equity value of a firm declines, the leverage increases. As the consequence the equity becomes more risky and its volatility increases. This means that the credit spread will widen. Furthermore, this results in an increasing of the CDS rate, as shown by Blanco et al. (2003). Conversely, as the equity value of the firm increases, the leverage decreases, and in fact the CDS rates and the credit spreads will tighten as well. Furthermore, it can be expected that the equity volatility is a decreasing function of the equity price. Therefore, equity market can sign early some future changes in credit quality (increase or decrease default probability) and/or of the level of leverage of companies. As shown in section 1.3.8, this is not really a surprise because the default probability is positively related to the CDS rate and a logical consequence is that the information of the CDS rate should be reflected in the volatility skew.

As shown by Hull et al. (2003a), the credit spread implied by Black-Scholes-Merton model is an increasing function of the implied put volatility[3]. Based on the framework in this paper,

---

[1] A low-strike-option is an out-of-the money put option or a deep-in-the-money call option.

[2] A high-strike-price option is a deep-in-the-money put option or a out-of-the-money call option.

[3] For an expression of the implied credit spread see Appendix I equation A.27.

we developed a semi-analytical formula, where the implied put volatility depends, among other parameters, on the implied credit spread and the leverage of a firm. (See 3.1.1).

As mentioned before, it is exactly this inter-relationship between equity volatility skew, CDS rate (as a proxy to default probability) and credit spreads that motivates the appearance of capital structure arbitrage strategies. If some of these financial trading instruments, or its proxies, exhibit mispricing or some reliable predictable feature, there will exist arbitrage opportunities. Implementing such strategies needs very detailed information on the leverage cycles of the firm to do a basic convergence trading[4]. In fact, banks and hedge funds are highly active in such strategies and a considerable amount of technical papers are been presented since the beginning of this year. For example, Ali Hirsa from Morgan Stanley gave recently a presentation entitled "Estimating Credit Spreads from Option Price" (see Hirsa (2003)). Joe Zou from Goldman Sachs is about to publish a paper entitled "Default Probability of Firms Implied by Equity Options Volatility Surfaces " (Zou (2003)). Kirill Illinski from JP Morgan Chase has developed a model that is currently being used by the bank, see Illinski (2003). Arthur M. Berd from Lehman Brothers was presenting as first a skew-adjusted Put-CDS relationship (see Bert (2003)). Finally, the web-based tool "CreditGrades", described in section 1.1.1, is in fact an application of a closed-formula that assesses default probabilities from the equity markets.

Therefore, the motivation for the empirical analysis in this thesis is the engagement of banks and hegde funds in such strategies, the special function of the CDS in the financial market, the empirical evidences in bond and equity markets and the classical relationships based on Merton´s model.

The empirical analysis will try

to assess if there is some long-run equilibrium between the CDS market and the equity options market and also to analyse the mechanism of price discovery between these markets for a specific company.

Price discovery is defined by Lehmann (2002) to be the efficient and timely incorporation of the information implicit in trading activities into market prices. If there exists merely one location for trading a financial asset, by definition all price discovery occurs in that market. When closely-related assets are traded in different locations or markets, by definition price discovery is split between markets.

For instance, Blanco et al. (2003) assess the mechanism of credit risk price discovery between the CDS and corporate bond markets. Their results is that CDS market contributes

[4] As seen in section 2.1.2.

around 80% of credit risk price discovery process. Following this line of research, this part of the thesis will try to assess the mechanism of credit risk price discovery between the CDS and equity markets and which of these two markets contributes most to the credit risk price discovery. Concerning the long-run equilibrium, a cointegration analysis will be carried out to assess the existence of this property (more about this in section 3.2). The analysis will be done for one particular company, France Telecom (FT). This company is particularly attractive, because according to certain traders it presented interesting arbitrage opportunities in the past.

Keeping the relationship between volatility skew, default probability and credit spread in mind, and to proceed with the empirical analysis, we will define a measure for the slope of the volatility skew (shortly SKEW or skew measure) as the difference between deep-out-of-the-money volatility and at-the-money-volatility, formally

$$SKEW_t = \sigma_{E,t}^{otm} - \sigma_{E,t}^{atm}. \tag{3.1}$$

Indeed, this skew measure is an indication of the expected future slope of the volatility skew and can be seen as an indication of overall credit-worthiness of a firm, meaning its level of credit spread and CDS rates. Therefore, it is expected that a steepening in the slope of the volatility skew will trigger an increase in CDS rates, as will be shown in the empirical analysis in section 3.4. The choice of how deeply out-of-the-money an option should be for an appropriate empirical analysis depends on the available data or perhaps simply on liquidity issues. The deeper the more significant will be the changes of the skew over time and therefore a good choice for an empirical analysis. Hull et al. (2003a) choose a similar skew measure for their analysis. They used an option moneyness[5] of 0.9 for the out-of-the-money options and in our analysis of FT we will use a simliar option moneyness of 0.8.

To compute the long-run relationship between the CDS and equity markets for FT, a cointegration analysis between the market parameters volatility skew and CDS rate, $SKEW_t$ and CDS_t[6], will be carried on (for a brief explanation about the procedures of a cointegration analysis refer to section 3.2). Following the general idea of a cointegration relationship between those variables, first of all a specification of the relation of the variables has to be made. The first choice would be to express a simple linear relationship between the variables. In fact, there is no empirical analysis in the literature about the relationship between the volatility skew and the CDS rates. As a first approach, verifying the correlation over time between both variables, as decipted in later on, Figure 3.6, it is indeed reasonable to assume a linear

[5] Moneyness is represented as the ratio of equity spot price over option strike price, and defines if an option is in-the-money, out-of-the-money and at-the-money. More details see Hull (1998).

[6] In the following, CDS rate will be noted as CDS_t or simply CDS, instead of s^{cds}.

relationship.

If there really is information on CDS rates in the slope of the volatility skew, defined in 3.1, then the relationship

$$CDS_t = a + b \; SKEW_t + \varepsilon_t, \tag{3.2}$$

where ε_t is a white noise process and a, b, are numbers, will lead to an error correction model representation (ECM) of the process in case both variable are cointegrated. The ECM is defined by:

$$\begin{aligned} \Delta CDS_t &= \gamma_1 \left(CDS_{t-1} - \beta_0 - \beta_1 SKEW_{t-1} \right) && \text{(3.3a)} \\ &\quad + \sum_{j=1}^{l} \phi_{1j} CDS_{t-j} + \sum_{j=1}^{k} \varphi_{1j} SKEW_{t-j} + \varepsilon_{1t} \\ SKEW_t &= \gamma_2 \left(CDS_{t-1} - \beta_0 - \beta_1 SKEW_{t-1} \right) \\ &\quad + \sum_{j=1}^{l} \phi_{2j} CDS_{t-j} + \sum_{j=1}^{k} \varphi_{2j} SKEW_{t-j} + \varepsilon_{2t}, && \text{(3.3b)} \end{aligned}$$

where γ_1 and γ_2 are the speed-of-adjustment coefficients for the cointegrating vector $\vec{\beta} = (1, -\beta_0, -\beta_1)'$, the vectors $\vec{\phi}$ and $\vec{\varphi}$ are the short-run dynamics among markets, and ε_{1t} and ε_{2t} are *i.i.d.*[7] white noise processes (see later, section 3.2 . A more detailed explanation about this is given in the next section.

The choice of the regression form in equation 3.2 is consistent with the results of Collin-Dufresne et al. (2000). They have shown *for changes in credit spreads* that the residuals from the first-pass regression are highly cross-correlated and empirical components analysis strongly suggests that they are driven by a single common factor. Furthermore, as seen in Hull et al (2003), CDS rates are theoretically a good proxy for credit spreads. This is also consistent with the market trading capital structure arbitrage strategies, as seen in chapter 2.

Blanco et al. (2003) suggested that CDS rates are well integrated with firm-specific equity market variables in the short-run, such as equity returns and near the money option implied volatilities. This motivates also the second empirical analysis in the thesis: the relationship between CDS rates and equity prices. Similarly to the equation 3.2, the regression is of the form:

$$CDS_t = a^* + b^* \; EQUITY_t + \varepsilon_t^*, \tag{3.4}$$

[7] *i.i.d. means a independent identical distributed random variable.*

where EQUITY stands for equity prices, ε_t^* is a white noise process and a^*, b^*, are numbers. As for the first empirical analysis, it is expected that the representation in 3.4 leads to an error-correction model similar to equation 3.3.

Besides the long-run relationship, the cointegration specification is also the base for a measure of the contributions to price discovery in each market. According to Blanco et al. (2003):

> *"The appropriate method to investigate the mechanics of price discovery is not clear. The two popular common factor models due to Hasbrouck (1995) and Gonzalo and Granger (1995) both rely on vector error correction models of market prices. Hasbrouck's model of "information shares" assumes that price volatility reflects new information, and so the market that contributes most to the variance of the innovations to the common factor is presumed to also contribute most to price discovery. Gonzalo and Granger's approach decomposes the common factor itself and ignoring the correlation between the markets attributes superior price discovery to the market that adjusts least to price movements in the other market. When price change innovations are correlated, Hasbrouck's approach can only provide upper and lower bounds on the information shares of each market."*

Therefore, if the equity option market is contributing significantly to the discovery of the CDS rate, then $\gamma_1 < 0$ and statistically significant as the CDS market adjusts to reestablish the equilibrium. Similarly, if the CDS market is contributing significantly to the discovery of the volatility skew, then $\gamma_2 > 0$ and significantly different from zero. If both coefficients are significant, then both markets contribute to price discovery. In this study the Gonzalo and Granger measure, referred to as GG, will be reported. The contributions of the CDS market to price discovery is defined by

$$GG_{CDS} = \frac{\gamma_2}{\gamma_2 - \gamma_1}, \tag{3.5}$$

and the contribution of the equity options markets, measured by the volatility skew defined in 3.1, is

$$GG_{SKEW} = \frac{\gamma_1}{\gamma_1 - \gamma_2}. \tag{3.6}$$

3.1.1 Relationship between the Credit Spread and the Volatility Skew implied by the Merton Model

We follow here the illustrations made in Hull et al. (2003a) and serve with formulas to estimate implied out-of and at-the-money put volatilities. This semi-analtic depends mainly on the implied credit spread and the leverage of a firm if the option moneyness, asset volatility, time to repayment and the time expiry time of the put option are given.

Using the credit spread implied by the Merton's model (see A.4) we found the following general result for the implied out-of-the-money put volatility[8]

$$\begin{aligned}\nu_E^{otm} \simeq\; & \tfrac{-1}{\sqrt{\omega}} N^{-1}\left(\frac{1}{\kappa}\left[\tfrac{lM\left(-a_2,d_2;-\sqrt{\frac{\omega}{T}}\right)-M\left(-a_1,d_1;-\sqrt{\frac{\omega}{T}}\right)}{1-le^{-s^cT}}+\kappa N\left(-a_2\right)-\Delta_\kappa\right]\right) \\ & \pm\tfrac{1}{\sqrt{\omega}}\sqrt{\left[N^{-1}\left(\frac{1}{\kappa}\left[\tfrac{lM\left(-a_2,d_2;-\sqrt{\frac{\omega}{T}}\right)-M\left(-a_1,d_1;-\sqrt{\frac{\omega}{T}}\right)}{1-le^{-s^cT}}+\kappa N\left(-a_2\right)-\Delta_\kappa\right]\right)\right]^2-2\ln\left(\kappa\right)},\end{aligned} \tag{3.7}$$

for $\kappa \neq 1$ and for $\kappa = 1$ simply

$$\nu_E^{atm} = \frac{-2}{\sqrt{\omega}} N^{-1}\left(\frac{lM\left(-a_2,d_2;-\sqrt{\frac{\omega}{T}}\right)-M\left(-a_1,d_1;-\sqrt{\frac{\omega}{T}}\right)}{1-le^{-s^cT}}+N\left(-a_2\right)-\Delta_\kappa\right), \tag{3.8}$$

where

$$d_1 = -\frac{\ln(l)}{\sigma\sqrt{T}}+\frac{1}{2}\sigma\sqrt{T}, \tag{3.9}$$

$$\begin{aligned} d_2 &= d_1-\sigma\sqrt{T}, \\ a_1 &= -\frac{\ln(\alpha)}{\sigma\sqrt{\omega}}+\frac{1}{2}\sigma\sqrt{\omega}, \\ a_2 &= a_1-\sigma\sqrt{\omega}, \\ \kappa\left[N\left(d_1\right)-lN\left(d_2\right)\right] &= \alpha N\left(d_{1,\omega}\right)-lN(d_{2,\omega}) \end{aligned}$$

$$d_{1,\omega} = -\frac{\ln\left(\frac{l}{\alpha}\right)}{\sigma\sqrt{(T-\omega)}}+\frac{1}{2}\sigma\sqrt{(T-\omega)}, \tag{3.10}$$

$$d_{2,\omega} = d_{1,\omega}-\sigma\sqrt{(T-\omega)} \tag{3.11}$$

and

[8] Th proof of this result is in Appendix A 3.1.1

$$l_{T>0} > e^{\frac{1}{2}\sigma^2 T},\ l_{T=0} > 1.$$

For a proof of this result see A.5. From the expression above we can see that the implied put volatility depends besides asset volatility on the implied credit spread and the leverage of a firm and is consistent with the results of Hull et al. (2003a). For a no-arbitrage condition for the evolution of the volatility surface and the basic dynamics of implied volatilities we refer to Daglish et al. (2003). Campbell and Taskler (2002) explores the effect of equity volatility on corporate bond yields.

Skew-Adjusted CDS-Equity-Put-relationship

Berd (2003) uses a put-equivalent CDS rate by applying the cost equivalence to develop a skew-adjusted CDS-put relationship for a short protection position as follow

$$\frac{s^{cds}\,[\%]}{100\,(1-R)} = \kappa^{-1}\left[PO\left(\kappa, 1, \sigma_E^{otm}\right) - PO\left(\kappa, 1, \sigma_E^{atm}\right)\right], \tag{3.12}$$

where σ_E^{otm} is an adjusted out-of-the-money equity option volatility, σ_E^{atm} is the at-the-money option volatility and $s^{cds}\,[\%]$ the CDS rate in percentage. Note that the right-hand-side of 3.12 is equivalent to the intensity, λ, in equation 1.86. One interpretation of 3.12 is that in case of no volatility skew the CDS rate is zero and the intensity or hazard rate is zero. The relationship 3.12 can be convert using an approximative vega to

$$\sigma_E^{otm} \simeq \sigma_E^{atm} + \frac{s^{cds}\,[\%]\,\kappa}{100\,(1-R)\ \ Vega\left(\kappa, 1, \sigma_E^{atm}\right)},$$

which should be in case of $s^c = s^{cds}$ close to the value of equation 3.7, $\nu_E^{otm} \simeq \sigma_E^{otm}$.[9]

3.2 Cointegration Econometrics

In this section a short description of stationary and non-stationary stochastic processes, vector autoregressive (VAR) models, cointegration and error correction model is presented. More details about these issues can be found in textbooks as Mills (1990), Enders (1995), Gujarati (1995) and Greene (2003). More technical analysis is available in Granger (1983), Engle and Granger (1987), Lütkepohl (1991), Banerjee *et al.* (1993), Hamilton (1994) and Johansen (1995).

[9]Further extensions, proofs and analysis of the results presented in this subsection 3.1.1 are not part of this work.

3.2.1 Stationary and Nonstationary Stochastic Process

A **time series** of a variable, y_t, is a set of observations on the values that that variable takes at different times and is collected at regular time intervals, such as daily, weekly, quarterly, annually, quinquennially, etc. Any time series can be represented as being generated by a **stochastic** or **random process** and a set of data can be thought as a (particular) **realization** or a **sample** of that underlying stochastic process. A **univariate time-series** model describes the behaviour of a given variable in terms of its own past values, and a **multivariate time-series** model describes the behaviour of a given variable in terms of its own past values and in terms of other variables past values.

The most simple time series is the **white noise** time series,

$$\{\varepsilon_t\}, \ t = -\infty, \ +\infty, \tag{3.13}$$

where each element in the sequence has zero mean, $E[\varepsilon_t] = 0$, constant variance σ_ε^2, $E[\varepsilon_t^2] = \sigma_\varepsilon^2$, and is non-autocorrelated, $Cov[\varepsilon_t, \varepsilon_s] = 0$ for all $s \neq t$. The sequence $\{\varepsilon_t\}$ with the previous characteristics is also called **innovation**.

A time series or stochastic process y_t is (**weakly**) **stationary** or **covariance stationary**[10] if its mean and variance are constant over time and the value of covariance between observations in the series is a function only of their distance in time, not the time at which they occur. Formally,

$$\begin{aligned} Mean &: \quad E[y_t] = \mu, & (3.14)\\ Variance &: \quad Var[y_t] = \sigma^2, & (3.15)\\ Covariance &: \quad Cov[y_t, y_{t+k}] = Cov_k. & (3.16) \end{aligned}$$

Obviously, if a time series is not stationary in the sense just defined, it is called a **nonstationary time series**.

Autocorrelation Function

The **autocorrelation function** (**ACF**) is a useful device for describing a time-series process stationarity. The ACF at lag[11] k, denoted by ρ_k, is defined as

[10] According to Greene (2003), strong stationarity requires that the joint distribution of all sets of observations be invariante to when the observarions are made.

[11] Lag 1 of a time series $\{y_t\}$ is defined as $\{y_t\} - \{y_{t-1}\}$, lag 2 of $\{y_t\}$ is $\{y_t\} - \{y_{t-2}\}$, and so on. In general, lag k of $\{y_t\}$ is $\{y_t\} - \{y_{t-k}\}$.

$$\rho_k = \frac{Cov_k}{Cov_0} = \frac{covariance\ at\ lag\ k}{variance}, \quad -1 \leq \rho_k \leq 1. \tag{3.17}$$

In equation 3.17, ρ_k denotes the true or populational autocorrelation function. In practice, only the realization or sample of a stochastic process is available. Therefore, $\hat{\rho}_k$ indicates the **sample autocorrelation function** and the plot of $\hat{\rho}_k$ against k is known as the **sample correlogram**, just like the ones depicted in Figure 3.5. One of the characteristics of a stationary stochastic process is an autocorrelation function pattern that eventually tends to zero. Contrarily, an ACF that presents a pattern that tends to zero very gradually is an indication of a nonstationary time series. The statistical significance of any $\hat{\rho}_k$ can be measured by its means of standard error. The sample autocorrelation estimators $\hat{\rho}_k$ are approximately normally distributed with zero mean and variance $1/n$, where n is the sample size. The 95% confidence interval is represented by the dashed lines in for instance Figure 3.5. If $\hat{\rho}_k$ lies outside this interval, the null hypothesis that the true ρ_k value is zero can be rejected. However, if it is inside this interval, the null hypothesis cannot be rejected.

Difference and Lag Operator

The n-**difference operator** (Δ^n) applied to a time series y_t is defined as

$$\Delta^n y_t = \Delta^{n-1} y_t - \Delta^n y_{t-1} = \sum_{r=0}^{n} (-1)^r \binom{n}{r} y_{t-r}, \tag{3.18}$$

where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$. The **lag operator**, denoted as L of a time series y_t is defined as

$$L^n y_t = y_{t-n},\ n = 0, 1, 2, \cdots. \tag{3.19}$$

The lag operator can be written in a polynomial form, where

$$y_t + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \cdots + \beta_n y_{t-n} \tag{3.20}$$

can be written as

$$\left(1 + \beta_1 L + \beta_2^2 L^2 + \cdots + \beta_n^n L^n\right) y_t = \beta\left(L\right) y_t, \tag{3.21}$$

where

$$\beta\left(L\right) = \left(1 + \beta_1 L + \beta_2 L^2 + \cdots + \beta_n L^n\right) \tag{3.22}$$

is the L polynomial.

Autoregressive Moving-Average Processes (ARMA)

A time series y_t follows an **autoregressive model** of order 1, AR(1), when its stochastic process can be represented as

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \tag{3.23}$$

where μ is the mean of y_t, ε_t is a white noise process, as described in 3.13, and ϕ, the autoregressive coefficient, is a real number of absolute value strictly less than 1. A more general representation of a pth-order autoregressive model, **AR**(p), can be written as

$$y_t = \mu + \phi y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t = \sum_{j=1}^{p} \phi_j y_{t-j} + \varepsilon_t. \tag{3.24}$$

In equation 3.23, the process with $\phi = 1$ is said to be a **random walk with drift**, where μ is the drift parameter, and display the characteristics of a non-stationary process. The process with $\mu = 0$ and $\phi = 1$ is a pure **random walk** process, and is a non-stationary process as well.

A time series y_t follows a **moving average model** of order 1, MA(1), when its stochastic process can be represented as

$$y_t = \mu + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{3.25}$$

where ε_t is a white noise process, as described in equation 3.13, and θ is the moving-average coefficient. A more general representation of a qth-order moving average model, **MA**(q), can be written as

$$y_t = \mu - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q} = \mu + \sum_{j=0}^{q} \theta_j \varepsilon_{t-j},\, \theta_0 \equiv 1. \tag{3.26}$$

A general process that encompasses 3.24 and 3.26 is the **autoregressive moving average**, **ARMA**(p,q), **model**:

$$y_t = \mu + \phi y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}, \tag{3.27}$$

where $\phi_p \neq 0$, $\theta_q \neq 0$, and ε_t is a white noise process. The description of the dynamics of the process can be simplified if equation 3.27 can be written in terms of the lag polynomial, such that

$$\Phi(L)\, y_t = \mu + \Theta(L)\, \varepsilon_t, \tag{3.28}$$

where the autoregressive and moving average lag polynomials are

$$\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p, \qquad \Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q. \tag{3.29}$$

Integrated Processess

A process or time series y_t is said to be **integrated** of order 1, denoted I(1), if it is stationary after first differencing or after applying the difference operator, defined in equation 3.18. Therefore, it is assumed

$$\Delta y_t = (1 - L) y_t = y_t - y_{t-1} = \varepsilon_t. \tag{3.30}$$

In general, a series y_t is integrated of order n, denoted I(n), if Δy_t is non-stationary while $\Delta^n y_t$ is stationary. A white noise series and an AR(1) with $\phi \neq 1$ are examples of I(0), while a random walk process, such as a AR(1) with $\phi = 1$ is an example of an I(1) series. According to Verbeek (2000), the main differences between I(0) and I(1) processes can be summarized as follows: An I(0) series fluctuates around its mean with a finite time-independent variance, while I(1) series has significant different features. An I(0) series is said to be mean reverting, i.e. displaying a tendency in the long-run to return to its mean. Moreover, an I(0) series has a limited memory of its past behaviour, while an infinitely long memory is inherent to a I(1) process.

Autoregressive Integrated Moving-Average Processes (ARIMA)

If Δy_t is described by a stationary ARMA(p,q) model, such as in equation 3.27, it is said that y_t is described by an **autoregressive integrated moving average** (**ARIMA**) **model** of order $p, 1, q$ or ARIMA($p, 1, q$). In general, the model would be

$$\Delta y_t = \mu + \phi_1 \Delta^d y_{t-1} + \cdots + \phi_p \Delta^d y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_q \varepsilon_{t-q}, \tag{3.31}$$

or in polynomial form:

$$\phi(L) \left[(1 - L)^d y_t - \mu \right] = \theta(L) \varepsilon_t. \tag{3.32}$$

equation 3.32 differs from equation 3.27 by the presence of a unit root in the autoregressive polynomial.

Testing for Unit Roots

The presence of a unit root in a time series is an interesting question from an economic point of view. In models with unit roots, external shocks have persistent effects that last forever, while in the case of stationary models, shocks have only temporary effect. Furthermore, the use of data characterized by unit roots has the potential to lead to serious errors in inferences. As already stated, an AR(1) model, such the one in equation 3.23, that displays $\phi = 1$ is a nonstationary (random walk) process with a **unit root**. A test for a unit root is a test for $\phi = 1$, and consequently for process stationarity.

Several statistical tests were developed to detect the presence of unit roots in time series. Depending on the **data-generating process** (**DGP**) of the process y_t, the **Dickey-Fuller** (**DF**) **test**, the **augmented Dickey-Fuller** (**ADF**) **test**, the **PP test**, the **DF-GLS test**, and the **KPSS test** can be used (for a complete discussion of such tests refer to Maddala and Kim (1998)). The **KPSS test**, proposed by Kwiatskowski *et al.* (1992), is an alternative test that circumvents the problem that unit root tests often have low power to detect stationarity. Basically, a time series is decomposed into the sum of a deterministic time trend, a random walk and a stationary error term. The null hypothesis of trend stationarity specifies that the variance of the random walk component is zero. The test is actually a Lagrange multiplier test and the computation of the test statistic is fairly simple, for details see Verbeek (2000). The test statistics is given by

$$KPSS = \sum_{t=1}^{T} \frac{S_t^2}{\hat{\sigma}^2}, \tag{3.33}$$

where $\hat{\sigma}^2$ is an estimator for the error variance. The asymptotic distribution is non-standard and the 5% critical value is 0.146. The null hypothesis of the **KPSS** test is that the series is stationary, or $H_0 : \hat{\sigma}^2 = 0$, against $H_1 : \hat{\sigma}^2 > 0$. The rejection of the null hypothesis implies that the series is nonstationary.

3.2.2 Vector Autoregressive Models

The ARMA model described in equation 3.27 can be extended to a multivariate time-series model to form a multivariate **vector autoregressive moving average** (**VARMA**) process. The most common approach and the one that often provides empirically a satisfactory fit to multivariate time series is the **vector autoregressive** (**VAR**) model. A VAR describes the dynamic evolution of a number of variables based on their past values. In general, a VAR(p) model for a k-dimensional vector $\vec{y}_t$ of variables is given by

$$\vec{y}_t = \vec{\mu} + \Theta_1 \vec{y}_{t-1} + \cdots + \Theta_p \vec{y}_{t-p} + \vec{\varepsilon}_t, \tag{3.34}$$

where $\vec{\mu}$ is a constant k-dimensional vector, each Θ_j is a $k \times k$ matrix and $\vec{\varepsilon}_t$ is a k-dimensional vector of white noise (or innovation) terms with covariance matrix Σ. Using the lag operator, a matrix lag polynomial can be defined as

$$\Theta(L) = I_k - \Theta_1 L - \cdots - \Theta_p L^p, \tag{3.35}$$

where I_k is the k-dimensional identity matrix, so that equation 3.34 can be written as

$$\Theta(L)\, \vec{y}_t = \vec{\mu} + \vec{\varepsilon}_t. \tag{3.36}$$

In the form of equation 3.34, VARs are easy to estimate. Although the equation system can be extremely large, it is a seemingly constitute of unrelated regressions with identical regressors. As such, the equations should be estimated separately by ordinary least squares (OLS). The innovation covariance matrix Σ can be estimated with average sums of squares or cross products of the least squares residuals.

Testing for Granger Causality

In the sense defined by Granger (1969), causality is inferred when lagged values of a variable, x_t, have explanatory power in a regression of a variable y_t on lagged values of y_t and x_t. A VAR model can be used to test the hypothesis and the tests of restrictions can be based on standard F tests in the single equations of the VAR model. Consider an example with a VAR representation of two variables,

$$\vec{y}_t = \vec{\mu} + \Theta_1 \vec{y}_{t-1} + \cdots + \Theta_p \vec{y}_{t-p} + \vec{\varepsilon}_t \tag{3.37}$$

where $\vec{y}_t = (y_t, x_t)$ and $\vec{\varepsilon}_t$ are uncorrelated. If variable x_t (Granger) causes variable y_t, then changes in x_t should *precede* changes in y_t, or, in other words, the lagged values of x_t improve statistically the prediction of y_t. In a similar fashion, if variable y_t (Granger) causes variable x_t, then the lagged values of y_t improve statistically the prediction of x_t.

Innovation Accounting: Impulse Response Functions and Variance Decomposition

A disturbing in one innovation of the k-dimensional vector $\vec{\varepsilon}_t$ in VAR leads to a chain reaction over time in all variables of the system. An **impulse response function** measures the effect of a transitory shock, or more precisely, a one standard deviation shock to one of the innovations on current and future values of the endogenous variables. Using impulse responses it is possible to examine the *dynamic interactions between the variables in the model.* For further discussion we refer to Hamilton (1994). The impulse response is highly sensitive to the ordering of the variables, due to the Cholesky decomposition of the error covariance matrix that orthogonalize the innovations of the VAR. Therefore, changing the order of equations could dramatically change the impulse responses and thus a common sense approach has to be taken to choose this ordering.

Variance decomposition provides a different method of analysing the system dynamics. Acording to Huang *et al.* (2002), it decomposes variation in an endogenous variable into the component shocks to the endogenous variables in the VAR and gives information about the relative importance of each random innovation to the variables in the VAR. Like the impulse response functions, the numerical variance decompositions are often very sensitive to the order in which the original innovation covariance matrix Σ is orthogonalized.

3.2.3 Cointegration

Consider two time series, y_t and x_t, both integrated of order 1, I(1), that is both are non-stationary and stationary after applying the first differencing. Then y_t and x_t are said to **cointegrated** if there exists a β such that $y_t - \beta x_t$ is I(0), with β being called the cointegrated parameter, or $\vec{\beta} = (1, -\beta)^T$ being called the **cointegrating vector**. What this means is that the relationship between y_t and x_t, expressed in the regression equation

$$y_t = \beta x_t + \varepsilon_t, \tag{3.38}$$

appears to be valid because y_t and x_t drift upward together over time, which means that there is a long-run equilibrium relationship between them. Furthermore, a distinction between this long-run equilibrium and the short-run dynamics, that is, the relationship between deviations of y_t from its long-run trend and deviations of x_t from its long-run trend can be established. As a consequence, the presence of a cointegrating vector indicates the presence of a long-run equilibrium relationship.

If y_t and x_t are cointegrated, then the short-run dynamics between y_t and x_t can be described by a **error correction model** (**ECM**) or, in another words, there exists a valid **error correction representation** of the data that describes the short-run dynamics consistently with the long-run relationship. This is known as the *Granger representation theorem*, according to Granger (1983) and Engle and Granger (1987). Therefore, if y_t and x_t are both I(1) and possess a cointegrating vector $(1, -\beta)^T$, there exists an error correction model, with $z_t = y_t - \beta x_t$, represented as

$$\theta(L)\Delta y_t = \mu + \phi(L)\Delta x_{t-1} - \gamma z_{t-1} + \alpha(L)\varepsilon_t, \tag{3.39}$$

where ε_t is white noise, and $\theta(L)$, $\phi(L)$, γ is the speed-of-adjustment coefficient, and $\alpha(L)$ are polynomials in the lag operator L (with $\theta_0 \equiv 1$). The speed adjustment coefficient measures how the elements in y_t are adjusted to the equilibrium error in z_{t-1}. A special case of equation 3.39, where the error term has no moving average part and the systematic dynamics are kept as simple as possible, can be written as

$$\Delta y_t = \mu + \phi_1 \Delta x_{t-1} - \gamma(y_{t-1} - \beta x_{t-1}) + \varepsilon_t. \tag{3.40}$$

Testing for Cointegration

There exists more than one method of conducting cointegration tests. The one used in this thesis was proposed by Johansen (1988), who developed a maximum likelihood estimation procedure that allows to test for the number of cointegrating relations. The technical details can be found in Johansen (1995). This procedure provides more robust results when there are more than two variables, according to Gonzalo (1994), and when the number of observations is greater than 100, according to Hargreaves (1994). The Johansen approach is to set up a vector autoregressive model given in equation 3.34 and to estimate this system by maximum likelihood, while imposing the restriction for a given value of r. Some assumptions have to be made regarding the trends of the time-series and regarding the intercepts and trends of the cointegrating equations. Johansen's procedure considers five possibilities:

1. Series y_t have no deterministic trends and the cointegrating equations do not have intercepts.

2. Series y_t have no deterministic trends and the cointegrating equations have intercepts.

3. Series y_t have linear trends but the cointegrating equations have only intercepts.

4. Both series y_t and the cointegrating equations have linear trends.

5. Series y_t have quadratic trends and the cointegrating equations have linear trends.

These five cases are nested from the most restrictive to the least restrictive, given any particular cointegrating rank r. The choice between these cases has to be based in an economic interpretation about the long-run relations among the variables as well as in statistical criteria, such as the normalized cointegrating vector.

The likelihood ratio **trace test** tests for the hypothesis of at most r cointegration relationships and at least $k-r$ common trends[12] and is given by

$$\lambda^*_{trace} = -T \sum_{j=r+1}^{k} \log\left(1-\hat{\lambda}_j\right), \tag{3.41}$$

where T is the sample size and $\hat{\lambda}_1 > \hat{\lambda}_2 > \cdots > \hat{\lambda}_k$ are the eigenvalues of a squared correlation matrix between two residual vectors from level[13] and first-difference regressions. An alternative test is the likelihood ratio **maximum eigenvalue test**. It tests the null hypothesis that there are exactly r cointegration relationships against $r+1$, and is given by

$$\lambda^*_{\max} = -T \log\left(1-\hat{\lambda}_{r+1}\right). \tag{3.42}$$

3.3 Descriptive statistics

This section will describe statistically the data set used. The set consists of daily equity prices, 1, 2 and 5-year daily CDS spread and 6-months and 1-year maturities daily implied option volatilities of France Telecom (FT), the French government-owned telecommunications company. The implied volatilities are available in five different moneyness measures, 0.8, 0.9, 1.0, 1.1, and 1.2, characterizing the volatility smile. The data covers the time period from July 31, 2001 until July 28, 2003, counting 520 observations.

Due to liquidity considerations and following academic researches, as Blanco et al. (2003) and Hull et al. (2003a), the 5-year CDS spread was chosen to be analysed, and is referred to as **CDS**. The volatility smile is reduced to a volatility skew and, as described in section 3.1, was calculated as the difference between deep-out-of-the-money implied volatility (0.8 moneyness measure) and at-the-money implied volatility (1.0 moneyness measure). The skew measure is referred to as **SKEW**. The daily equity prices is referred to throughout the text as **EQUITY**.

Figure 3.1 shows the time-series plot for 5-year CDS rates, the volatility skew and equity prices. Note that CDS rates and the volatility skew were normalized by the same factor, so that they could be plotted together, the equity prices plot is the highest one in the picture, and the two dotted vertical lines correspond to a delimitation in the time period that will be explained in section 3.4.2. Figures 3.2, 3.3, and 3.4 show the histograms of the 5-year CDS rate, the skew measure, and equity prices, respectively.

[12] Where k is the number of variables to be regressed.

[13] The level defines the original time series without differencing.

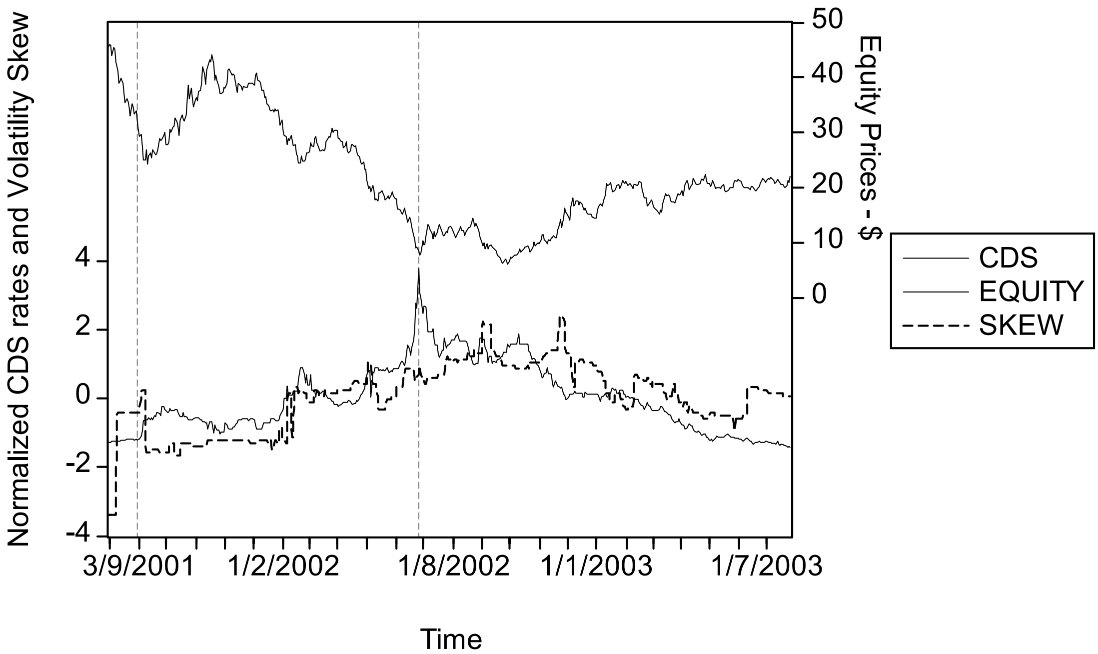

Figure 3.1: Plot of CDS rates, volatility skew and equity prices

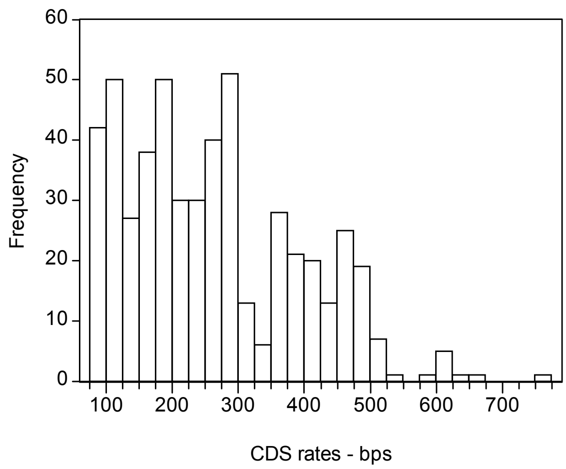

Figure 3.2: Histogram of CDS rates

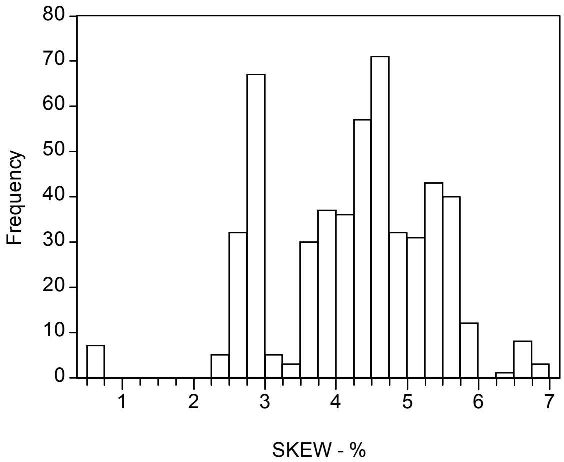

Figure 3.3: Histogram of volatility skew

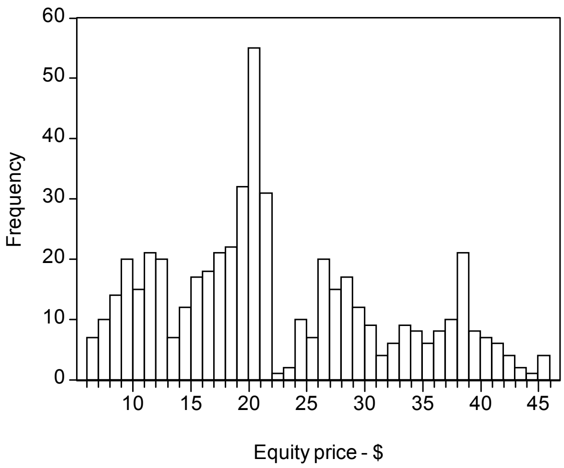

Figure 3.4: Histogram of equity prices

The descriptive statistics for the 5-year CDS rate, the skew measure and the equity price are shown in Table 3.1. The Jarque-Bera test for normality of each series is rejected at 1% level.

	CDS	**SKEW**	**EQUITY**
Mean	260.74	4.25	22.34
Median	244.00	4.40	20.46
Maximum	753.00	6.90	45.99
Minimum	76.00	0.50	6.01
Std. Dev.	130.15	1.10	9.75
Skewness	0.66	-0.48	0.48
Kurtosis	2.80	3.48	2.33
Jarque-Bera	38.30	24.99	29.32
Probability	0.00	0.00	0.00

Table 3.1: Descriptive statistics

The autocorrelation function (ACF), or correlogram, of the level (on the right column) and first difference (on the left column) of the volatility skew measure, the equity prices and the 5-year CDS rate are shown in Figure 3.5, from top to bottom, respectively. The dashed lines indicate the 95% confidence interval for each lag.

Table 3.2 shows a correlation matrix between CDS rates (CDS), equity prices (EQUITY) and skew measure (SKEW). Figure 3.6 shows scatter plots of the skew measure and equity prices against 5-year CDS spreads.

	CDS	**EQUITY**	**SKEW**
CDS	1.0000	-0.6627	0.6177
EQUITY	-0.6627	1.0000	-0.8322
SKEW	0.6177	-0.8322	1.0000

Table 3.2: Correlation Matrix

3.4 Cointegration Results

The empirical analysis will be conducted within the framework of a multivariate vector autoregressive (VAR) model. As stated before, VAR methodology enables to assess the evolution dynamics of a number of variables based on their past values. It estimates a multivariate ordinary least square (OLS) system where all the variables included are treated as endogenous. The advantage is that both the instantaneous and the lagged effects of each variable of one specific (or all) variable(s) can be captured. The section begins with a description of the cointegration tests and specifications used. In the following, the results are shown and analysed.

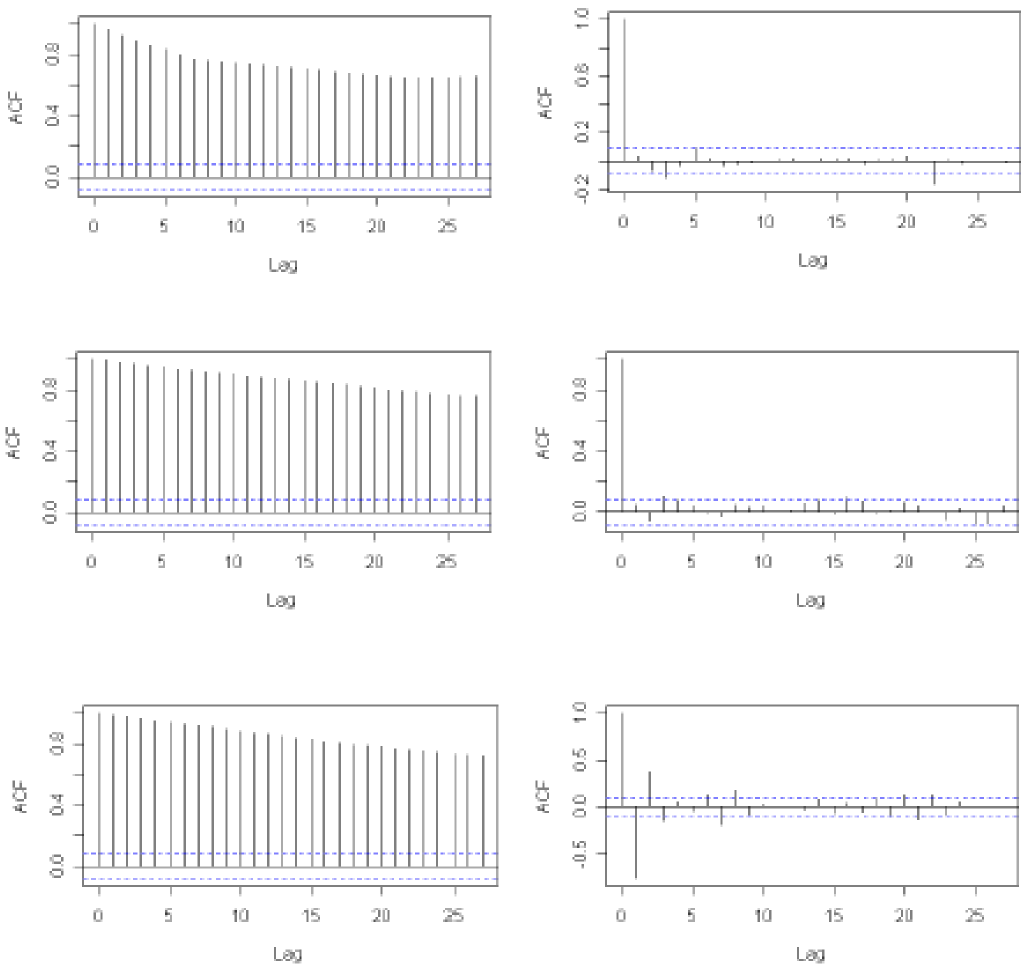

Figure 3.5: Correlogram of the volatility skew, equity prices and CDS rates (from top to bottom, respectively) for the level and first difference

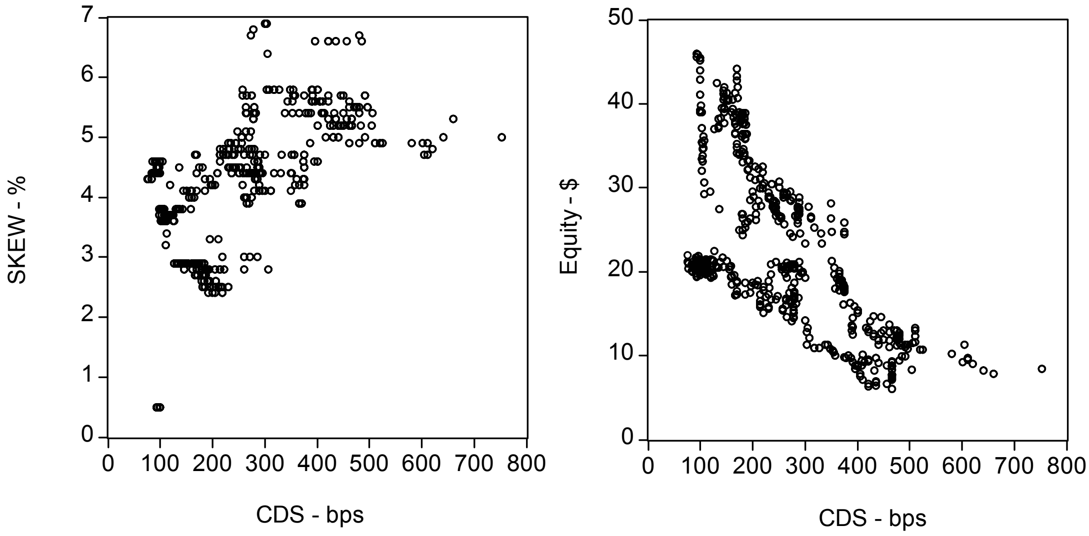

Figure 3.6: Scatter plot of the skew measure and the equity price against 5-year CDS rates

3.4.1 Description of the Tests

As already stated, the necessary condition for a vector $\vec{y}_t$ of variables to be cointegrated are twofold: First, each series must be non-stationary at level, and each series must have the same order of integration. Therefore, the test[14] for cointegration requires first an inspection in the **ACF** or correlogram for the level and for the first-difference and second testing formally for **unit roots** in each series. The unit roots test used in this thesis was the **KPSS** test, described in section 3.2.1.

In order to estimate the parameters of the VAR model that will be specified below, the **order of the maximum lag** p has to be fixed. One possible choice is to fix a sufficiently high order $p_{\max}$ and then to move to tests with smaller orders one by one, being a top-down procedure. A likelihood ratio (LR) statistic is calculated to test for the significance of imposing the restrictions. According to Gourieroux and Monfort (1997), the LR statistics is

$$\xi_T = T \ \log \frac{\det \hat{\Sigma}_{i-1}}{\det \hat{\Sigma}_i}, \tag{3.43}$$

where $\hat{\Sigma}_i$ is the estimated covariance matrix from the OLS residuals in a model with i lags for each variable, and T is the number of observations in the sample. Another alternative to choose the number of the lags is to use an information criterion, such as the Akaike information criterion (AIC), the Schwartz (or Bayesian) information criterion (SC or BIC), or the Hannan-

[14]All tests and model specifications were computed using the software EViews 4.1, from QMS Software.

Quinn information criterion (HQ). These criteria were proposed by Akaike (1969), Schwartz (1979) and Hannan and Quinn (1979), respectively. The BIC and HQ criteria lead to an asymptotically correct selection of the model, according to Hannan (1980), but the AIC is by far the most used criterion. For a more detailed explanation see Lütkepohl (1991). EViews gives the optimal lag order selection using these 4 different criteria: LR, AIC, BIC, and HQ. Some papers, like Neal et al. (2000) and Luo (2002), use the AIC criterion, while others, like Huang et al. (2002) use the LR statistics. In this work, the LR statistic will be used as outlined in Gourieroux and Monfort (1997) and Maddala (2001).

After selecting the appropriate lag length, Granger causality tests, VAR specifications, cointegration tests, and VEC models can be applied. The used **Granger causality tests** carried out tests whether an endogenous variable can be treated as exogenous. The procedure is the one described in section 3.2.2. The procedure implemented in EViews performs pairwise Granger causality tests, and for each equation in the VAR, the output displays χ^2 statistics for the joint significance of each of the other lagged endogenous variables in that equation. Furthermore, the procedure displays the statistic for joint significance of all other lagged endogenous variables in the equation.

The **VAR system** with its parameters are estimated using OLS and the t-test statistic of significance for each estimator is given in a standard form. The **impulse response functions** are shown in graphical form. In fact, the shock to each equation is equal to one positive standard deviation of the equation residual and the impulse responses of all the variables to the shock are traced out for a period of 300 days. The plus/minus two standard deviation bands are displayed in the graphs alongside the impulse responses. The ordering of variables are defined case by case. The **variance decomposition** results based on the VAR estimates are shown in a tabular format and the ordering of the equations are assumed to be the same as specified by the impulse response functions.

Furthermore, if the isolated variables underlying the various relationships are non-stationary and possess the same integration order, the **cointegration tests** can be applied. The cointegration test used in this thesis is based on the Johansen procedure described in section 3.2.3. The specification for the deterministic trend of the series,the specification of the intercept, and of the trends of the cointegrating equation have to be assumed among the five possibilities considered by Johansen. EViews provides tests for these five options and a summary showing for each test the number of cointegrating relations found. As it is impossible to know the error-correction model *a priori*, the tests will be applied assuming the third and the fourth possibilities in Johansen's procedure. That means, the series have linear trends but the cointegrating equations have only intercepts and, in the other case, both series and the cointegrating equations have linear trends. The one that gives better results will be presented.

3.4.2 Empirical Relationship between CDS Rates and the Volatility Skew

The first analysis is carried out between CDS rates (CDS) and the volatility skew (SKEW), as described in section 3.1. Before following the procedures presented in 3.4.1 to detect cointegrating relationships, it is interesting and useful to examine table 3.2 and figure 3.6. From the table and from the corresponding plot, it is particularly evident that **both variables are reasonably correlated** and may indicate indeed some form of relationship. This suggests that when CDS (SKEW) increases, SKEW (CDS) increases as well. The behaviour of the variables are in accordance with the common knowledge in the market.

Unit Roots Test

Continuing with the test procedures, the first item to analyse is the **ACF plots** of CDS and SKEW. From Figure 3.5 it is clear that **both series are non-stationary** on the level and stationary on the first-difference, the high degree of persistence on the level ACF is consistent with the presence of a unit root. Another indication of non-stationarity is given by a **unit root test**. Table 3.3 shows the result for the **KPSS** test. The test rejects the null hypothesis that the level of both series are stationary at the 1% critical value, but it fails to reject the null hypothesis that the first difference of both series are stationary. Therefore, **both series are nonstationary I(1) processes**. Therefore, it is assumed that the data generation process contains a constant and a linear time trend.

Variable	CDS	SKEW
Level		
KPSS statistic	0.6587	0.5871
First difference		
KPSS statistic	0.0435	0.0301
1% level critical value	0.2160	0.2160

Table 3.3: Results of KPSS test

Granger Causality Test

The **optimal lag length** is chosen according to the **LR** statistic at 5% level. According to the output of EViews:

The number of lagged terms to be included in the VAR representation is 8.

The results of the **Granger causality tests** are presented in Table 3.4 and Table 3.5. In Table 3.4, the columns reflect the marginal probability for the Granger causal impact of the column variables on the row variables. The last row represents the χ^2 statistics with 8 degrees of freedom. Examining a basic F test and the VAR χ^2 approach, it seems that t**he causality is running from the CDS rates to the skew measure, while the other direction is not statistically significant.**

χ^2 test	**CDS**	**SKEW**
CDS	0.0000	0.7741
SKEW	0.0120	0.0000
χ^2 statistic	19.6567	4.8845

Table 3.4: VAR pair-wise causality tests

F test	**F-statistic**	**p-value**
H_0 : SKEW does not Granger Cause CDS	0.6056	0.7735
H_0 : CDS does not Granger Cause SKEW	2.4573	0.0130

Table 3.5: Pairwise causality tests

VAR-Specification

The **VAR estimation** is shown in equations 3.44 and 3.45 merely with the statistically significant coefficients at 5% level[15]. The whole representation is given in Appendix B.

$$\begin{aligned} CDS &= 0.8864\, CDS_{t-1} + 0.2737\, CDS_{t-2} - 0.1983\, CDS_{t-3} + \\ &\quad +0.1267\, CDS_{t-5} - 0.2082\, CDS_{t-7} + 0.1266\, CDS_{t-8} \\ adjusted\ \bar{R}^2 &= 0.98 \end{aligned} \tag{3.44}$$

$$\begin{aligned} SKEW &= 0.1018 + 0.0016\, CDS_{t-3} - 0.002\, CDS_{t-4} + \\ &\quad +1.005\, SKEW_{t-1} - 0.1037\, SKEW_{t-2} + 0.1269\, SKEW_{t-5} \\ adjusted\ \bar{R}^2 &= 0.96 \end{aligned} \tag{3.45}$$

[15] The test for autocorrelation in the residuals at 1% level results that they are not auto-correlated.

It is worth nothing that the **VAR estimation gives the same results as Granger-causality tests**: in equation 3.44 there are no SKEW lagged terms, while in equation 3.45 the CDS lagged terms appear more than once. Note also the **high values** for the $\bar{R}^2$ measure for both equations, which is a **good indicator of goodness-of-fit** for the estimations. With the VAR regressions it is possible to examine the **impulse response functions**. As already said, the ordering of the variables significantly influences the results for the functions. Therefore, the analyses will be made in both directions: once considering CDS last and then first.

Innovation Accounting

The graphical output of the impulse responses when the SKEW enters first in the Cholesky decomposition is shown in Figure 3.7. Since it is interesting to discover the impact of the skew measure in CDS, it makes sense that CDS enters last. The output of the impulse responses when the CDS enters first in the Cholesky decomposition is shown in Figure 3.8. Because it is of interest to discover the impact of CDS in the skew measure, it makes sense that CDS enters first. The responses to the shock of one positive standard deviation in innovation for each equation are traced out for a period of 300 days. The standard deviations bands are displayed in dotted lines in the graphs.

Analysing the graphs, it is interesting to see that the response of CDS to SKEW is **slightly different in both representations**, due to the ordering. When CDS enters last, **volatility skew measure has a negative effect on CDS rates** and there is no apparent explanation for that. However, when CDS enters first, the **volatility skew measure** has a **slightly positive effect** on CDS rates (as seen in the upper-right graph of Figure 3.8). As the skew measure increases, with out-of-the-money options being more volatile than at-the-money options, or the skew becoming more steep, the CDS rate tends to rise. An increase in the slope of the skew generally means anticipation or reaction for *bad news* regarding the firm, which is reflected in the CDS market. The influence of the skew measure on CDS rates was not captured by the Granger causality tests made before.

The **response of SKEW to CDS is positive for both representations**, which is in accordance with the results of the Granger-causality tests presented (as seen in the lower-left graph of both representations). As CDS rates increase, the skew tends to increase also as a sign of *bad news*. It seems through the impulse response functions that there is a bidirectional effect or causality in both markets. This effect should be clarified in the cointegration analysis.

Following the same systematic of the impulse response functions, Panel A of Table 3.6 shows the variance decomposition of CDS, when CDS enters last and first in the ordering. Panel B shows the variance decomposition of SKEW, when SKEW enters first and last in the ordering.

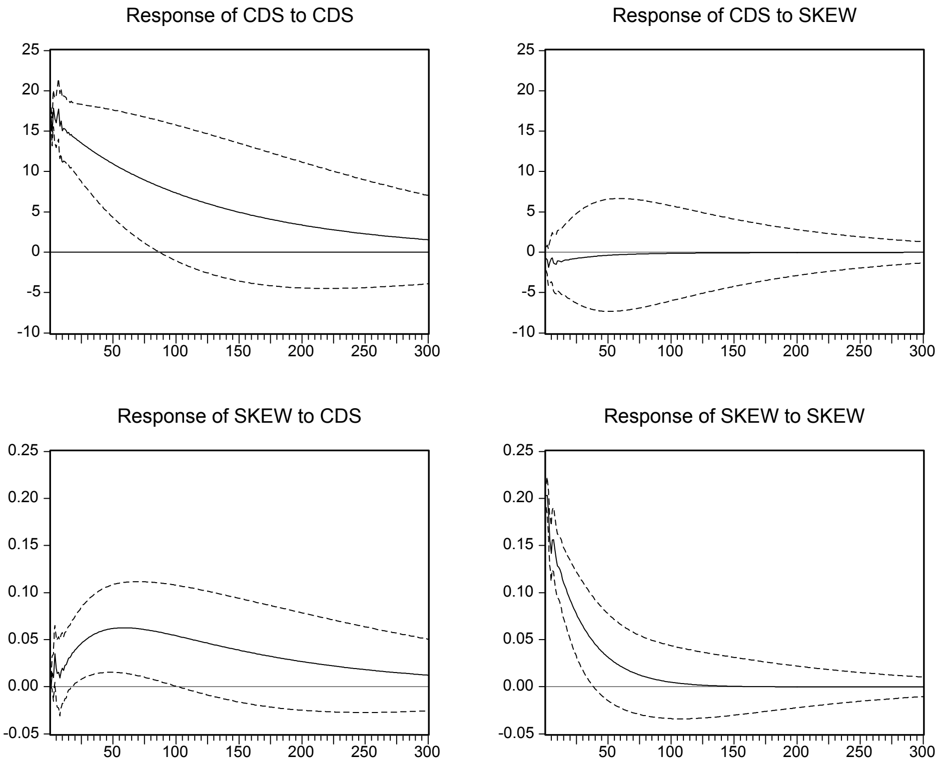

Figure 3.7: Impulse responses for CDS and SKEW with CDS entering last

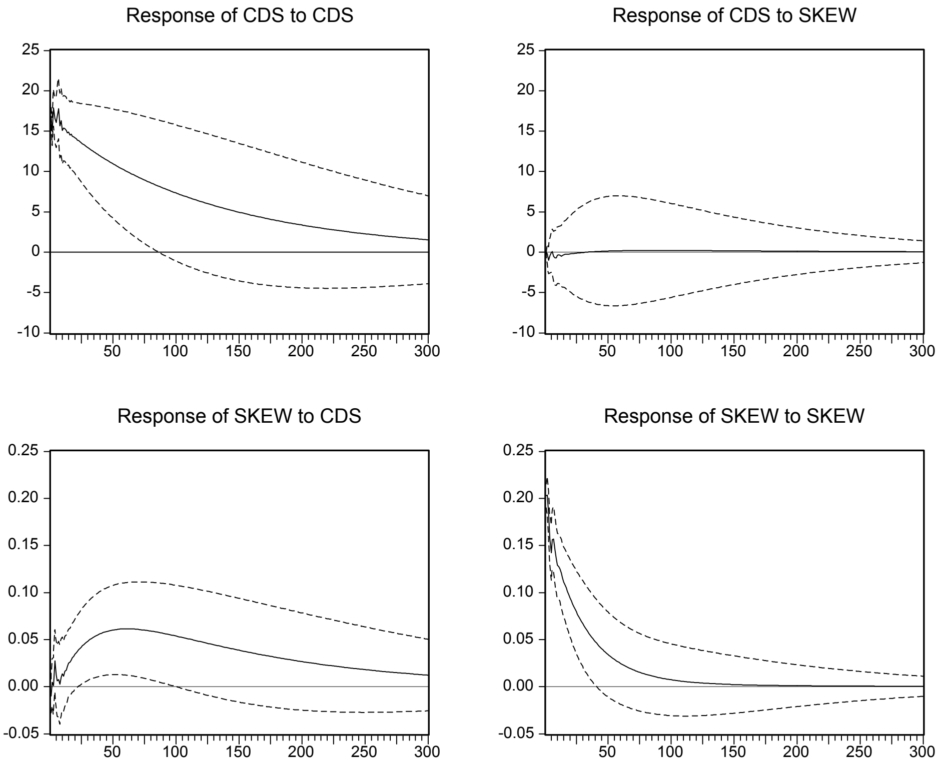

Figure 3.8: Impulse responses for CDS and SKEW with SKEW entering last

Panel A				
CDS variance decomposition				
Horizon (days ahead)	**CDS**	**SKEW**	**CDS**	**SKEW**
10	99.44	0.56	99.90	0.10
50	99.61	0.39	99.96	0.04
100	99.71	0.29	99.96	0.04
200	99.75	0.25	99.95	0.05
300	99.76	0.24	99.94	0.06
Panel B				
SKEW variance decomposition				
Horizon (days ahead)	**CDS**	**SKEW**	**CDS**	**SKEW**
10	1.15	98.85	0.53	99.47
50	17.25	82.75	15.08	84.92
100	35.98	64.02	33.76	66.24
200	46.89	53.11	44.96	55.04
300	48.85	51.15	46.98	53.02

Table 3.6: Variance decomposition

A first evidence drawn from the variance decomposition is that even with different ordering, CDS explains almost all of the forecast error variance of the CDS rates for the entire period.

In fact, the skew measure has little impact on the variance decomposition of the CDS rates. By contrast, there is a more balanced contribution of both the CDS rates and the skew measure to the SKEW variance decomposition, even changing the ordering of the variables. As can be seen, in the initial 100 days, about 35% of the variance is explained by the CDS rates and over the entire time horizon both variables contribute roughly equally to explain the variance in the skew measure.

Cointegration Tests

Since there is a **unit root in either variables** and they have the **same order of integration**, I(1), cointegration tests may be applied. Using the assumption that the data-generation process contains a constant and a linear time trend, the results of **Johansen's procedure** are displayed in Panel A of Table 3.7. Panel B shows the **normalized cointegrating vector**.

The trace tests in Panel A fail to reject either the null hypothesis of no cointegration, $r = 0$, and the null hypothesis of at most one cointegrating vector, $r \leq 1$. By contrast, the maximum eigenvalue test rejects the null hypothesis of no cointegration at 5% level but fails to reject the other null hypothesis of at most one cointegrating vector. Therefore,

CDS and SKEW are cointegrated with one cointegrating vector.

Panel A: Tests							
		Critical values				**Critical values**	
H_o	Trace	5%	1%	H_o	Max. Eigenvalue	5%	1%
$r = 0$	23.71	25.32	30.45	$r = 0$	19.44	18.96	23.65
$r \leq 1$	4.27	12.25	16.26	$r = 1$	4.27	12.25	16.26

Panel B: Normalised cointegrating vector				
	CDS	**SKEW**	**Trend**	**Constant**
Coefficient	1.00	-155.37	0.61	244.32

Table 3.7: Results of the Johansen cointegration tests

In Panel B, all coefficients of the **normalized cointegrating vector** are significant at 1% level. Thus the vector is

$$\vec{\beta} = (1.00, -155.37, 0.61)^T .$$

Since the cointegration relationship can be found between CDS rates and the skew measure, there must exist a **representation of an error correction model (ECM)** which shows long and short run dynamics among the cointegrated variables, as described in equation 3.40. In equation 3.46, the estimated VECM is presented only with coefficients significant at 10% level. The complete representation is shown in Appendix B.

$$\begin{aligned}
\Delta CDS_t &= -0.0187\left(CDS_{t-1} - 155.3719 SKEW_{t-1} + 0.6061t + 244.32\right) \\
&\quad -0.0935\Delta CDS_{t-1} + 0.1702\Delta CDS_{t-2} + 0.0796\Delta CDS_{t-6} \qquad (3.46a) \\
&\quad -0.1156\Delta CDS_{t-7} - 5.5063\Delta SKEW_{t-2} \\
adjusted\ \bar{R}^2 &= 0.09 \\
\Delta SKEW_t &= 0.0004\left(CDS_{t-1} - 155.3719 SKEW_{t-1} + 0.6061t + 244.32\right) \qquad (3.46b) \\
&\quad +0.0012\Delta CDS_{t-3} - 0.073\Delta SKEW_{t-3} + 0.1044\Delta SKEW_{t-5} \\
adjusted\ \bar{R}^2 &= 0.10
\end{aligned}$$

As expected, the coefficient of adjustment (-0.0187) in the first equation of 3.46 is **negative** and **statistically significant**. That is, the skew measure of FT options contributes significantly to the daily price levels of FT's CDS rates, and consequently the market for FT CDS adjusts to incorporate this information. Similarly, the coefficient of adjustment (0.0004) in the second equation of 3.46 has the expected **positive signal** which is also **significantly different from zero**. This means that FT's CDS rates contribute significantly to the daily levels of the skew measure. As both coefficients are significant,

both variables, the CDS rate and the skew measure contribute to daily levels of information but with different speeds.

Comparing the speed of adjustment in both equation using the half-life parameter[16], the skew measure adjusts faster (within 36 days) to the long-run equilibrium than CDS rates (more than 200 days). Furthermore, having analysed the short-run terms of both equations it is clear that there **exists a relationship between the changes in levels of CDS and SKEW** (ΔCDS_t and $\Delta SKEW_t$), their own lagged terms, and the cross-market lagged terms. Therefore,

the source of causality is running in both directions (CDS rates to skew measure and vice-versa) with the skew measure contributing more to the long-run equilibrium adjustment than the CDS rates.

It is interesting to compare the cointegrating vector $\vec{\beta}$ with the time series of CDS rates and of deep-out-of-the-money implied volatilities, as seen in Figure 3.9.The vector exhibits **strikingly similar movements patterns** both series and which is a signal that it could capture both long-run and short-run dynamics of the interaction between CDS rates and implied volatilities. In Figure 3.9, the graph for CDS rates corresponds to the lowest plot and VOL6M08 represents an option with moneyness option of 0.8 and maturity 6-month.

Price discovery

In terms of price discovery measures, as explained in section 3.1 and calculated by equations 3.5 and 3.6, we get:

$$GG_{CDS} = 0.02 \tag{3.47}$$

$$GG_{SKEW} = 0.98. \tag{3.48}$$

These results suggest that 98% of the credit risk price discovery occurs in the option markets and only 2% in CDS market, as shown for the case of FT.

Discussion of results

Why does exist such a strong evidence that the skew measure leads or even drives CDS rates? From a market point of view, it is evident that price discovery occurs in the market where (informed) traders trade the most. Although not knowing which market has more liquidity, be it the market for FT's options or for FT CDS, it could be argued from the cointegration analysis that the FT's options market is more liquid than FT's CDS market. Even if the CDS market is the easiest place for trading credit risk, this seems not to be the case for FT.

[16]The half-life parameter is defined by $-\frac{\log(2)}{\log(1+\alpha)}$. According to Madhavan and Smidt (1993), it represents the expected number of days required for a deviation return to the long-term equilibrium by 50 percent.

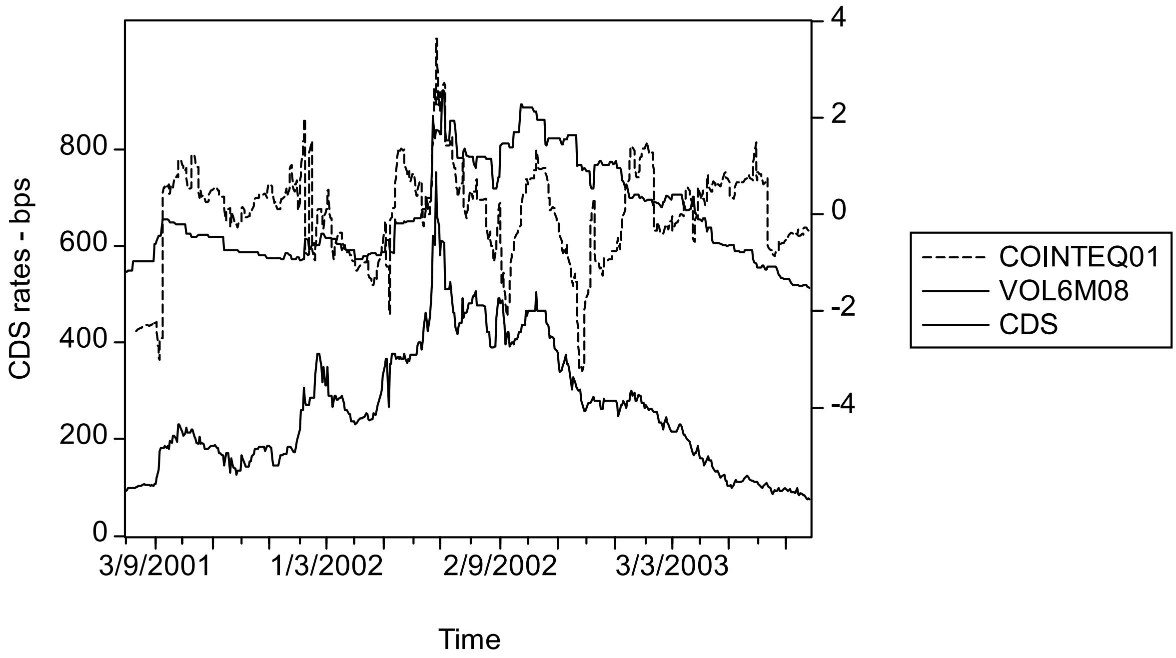

Figure 3.9: Cointegrating vector

Some explanations arise for that: First, it could be explained by the market entering level of both securities. It is known that options contracts have a lower contract size than CDS contracts, which could lead to more trading activities in the option market. This is verified in practice as deep-out-of-the-money put options are commonly used as a proxy for hedging credit risk instead of CDS securities, as seen in section 2.1.1 on equity and debt market trading strategies. Another point is a type of risk inherent to the CDS markets that does not exist in option markets. As already stated, CDS is an OTC security, while options are traded mainly in large exchanges. It could be said that, keeping all other risks equal for both markets, a CDS security bears in terms of counterparty risk more risks than a listed option security. But nonetheless, even with this plausible explanations it is reasonable not to accept that the market for FT's credit risk is almost entirely traded in the option market. In case of considering the bubble burst cycle only, see next section, the analysis will reduce the GG-result.

Bubble Burst Cycle

Given that CDS rates and volatility skew are linked by an arbitrage relationship, how can one take advantage of this? First of all, knowing that there is a direct link between the leverage of a company and its probability of default, equity markets respond according for the current (or future) debt payment capacity of the company. This response is reflected in the implied volatility for the company's traded equity options. Therefore, traders may have some indicators to anticipate future problems related to the current financial status of a company that could trigger a default or another credit event situation. It seems that this indicator might be based on the corporate business cycle, or specifically in the leverage cycle (as seen in section 2.1.2). If in reality this is the case, fundamental economic analysis and deep knowledge about the company could anticipate financial problems and lead to arbitrage opportunities.

In fact, a good indicator for cycles in the leverage life of FT, for instance, could lead the analysis presented to a slightly different approach. Instead of analysing the entire time-series of the CDS rate and the volatility skew, the series could be split according to the different leverage cycles. The analysis being carried out in sub-sets, which is more related to market practices. Even without any indicator for a leverage cycle, one can identify at least two distinct cycles for FT that matches the description in Figure 2.4: a **bubble burst** cycle from **August 30, 2001 to June 26, 2002** and a **deleveraging** cycle from **June 26, 2002 to July 28, 2003**. The period of the bubble burst cycle corresponds to an a strong increase in both CDS rates and implied volatilities happening after a short period of stability. On August 30, 2001 the CDS rate was 103 bps and until June 26, 2002 it surged to 753 bps, an increase of about 630%!

By contrast, the deleveraging period corresponds to a downturn of both CDS rates and implied volatilities. For instance, the CDS rate sunk by approximately 90% in the period. For a graphical representation of these periods, note the two dotted vertical lines in Figure 3.1, shown previously

To show that it makes indeed sense to split the analysis by cycles, a cointegration analysis for the bubble burst cycle will be briefly represented. It is expected that in terms of cointegrating analysis the results are similar to the previous ones, rflecting, however, higher *adjusted* $\bar{R}^2$ measures and higher adjustment coefficients. In terms of the price discovery mechanism it is expected that the GG measure of the CDS market increases and consequently contributes more to the price discovery of credit risk for FT. The test procedures are the same as before and the complete results are presented in Appendix C.

To start with, the **correlation between CDS and SKEW has risen to 0.72**, suggesting a stronger interaction than before. **CDS and SKEW are nonstationary** time-series and using the LR ratio a **optimal lag length is found to be 7**, assuming the data generation process contains a constant and a linear time trend. Results from the Granger-causality test for both the F test and the χ^2 approaches show that there **exists a bidirectional Granger**

causality between the variables. A VAR representation is estimated and impulse response functions as well as a variance decomposition are derived accordingly. It is worth nothing to consider the impulse response of CDS to SKEW. In the previous analysis, when the CDS entered last in the ordering there could be seen a slightly positive relationship between both variables. In the present analysis with both ordering, the **relationship between CDS and SKEW is reasonably positive** for the entire period. Therefore, the relations drawn from the impulse response functions are in agreement with Granger-causality tests.

The **tests for cointegration show the existence of one cointegrating relation** at 1% level between CDS and SKEW with the normalized cointegrating vector

$$\vec{\beta} = (1.00, -125.96, 1.11)^T .$$

significant at 10% level. The VECM with merely the significant terms at 10% level is given by:

$$\begin{aligned}
\Delta CDS_t &= -0.0023\,(CDS_{t-1} - 125.9671 SKEW_{t-1} + 1.10941t + 55.19) && (3.49a)\\
&\quad -0.0079\Delta CDS_{t-1} + 0.2817\Delta CDS_{t-2} - 0.1662\Delta CDS_{t-4}\\
&\quad -3.7694\Delta SKEW_{t-1} + 2.6372\\
adjusted\ \bar{R}^2 &= 0.22\\
\Delta SKEW_t &= 0.0063\,(CDS_{t-1} - 125.9671 SKEW_{t-1} + 1.10941t + 55.19) && (3.49b)\\
&\quad +0.0234\Delta CDS_{t-2} + 0.0247\Delta CDS_{t-3} + 0.0154\Delta CDS_{t-6}\\
&\quad +0.0201\Delta CDS_{t-7} + 0.5119\Delta SKEW_{t-1} - 0.2757\Delta SKEW_{t-2} -\\
&\quad -0.2067\Delta SKEW_{t-4} - 0.2465\Delta SKEW_{t-6} - 0.2325\\
adjusted\ \bar{R}^2 &= 0.56
\end{aligned}$$

As expected, the coefficient of adjustment (-0.0023) in the first equation of 3.49 **is negative and statistically significant**. Similarly, the coefficient of adjustment (0.0063) in the second equation of 3.49 has the expected **positive signal and is significantly different from zero** as well. As two coefficients are significant, both variables contribute to daily levels of information but with different speeds. Comparing the speed of adjustment of both equations using the half-life parameter, the skew measure adjusts faster (within 30 days) to the long-run equilibrium than the CDS rates (about 106 days). Comparing these half-life parameters with those computed before shows clearly, that the speed of adjustment is faster in the bubble burst cycle than before. It is worth nothing to verify the **expected rise in**

the *adjusted* $\bar{R}^2$ measure in equation 3.49. Comparing with those values in equation 3.46, the average adjusted $\bar{R}^2 s$ for the bubble burst cycle are much higher than those for the entire period, suggesting that the stronger interaction in the correlation coefficient (0.72) observed in this cycle is verified in the regression as well.

In terms of price discovery measures, this means as explained in section 3.1 and calculated by equations 3.5 and 3.6:

$$GG_{CDS} = 0.22, \tag{3.50}$$

$$GG_{SKEW} = 0.78. \tag{3.51}$$

These results suggest that

78% of credit risk price discovery is performed in the options market and 22% in the CDS market.

The difference between these results to the previous ones is impressive. In fact, it is more reasonable to state that even while not being the most contributor to the credit risk price discovery for FT, the CDS market has a considerable participation in the price discovery. As a conclusion, it does make a difference when splitting the analysis based in on indicator for each leverage cycle. Indeed, the analysis for the bubble burst shows that the inter-relationship between CDS rates and the volatility skew has a different dynamic process that depends strongly on the period analysed.

Deleveraging Cycle

The cointegration tests for the deleveraging cycle was not done in this work. Nonetheless the following extract illustrates, from a market perspective, the existence of the deleveraging cycle in France Telecom. According to Moore and Watts (2003):

> *Anticipating good news:*
>
> *"On a more positive note, the CDS market can also function as an accurate litmus test of the market's perception of changes in corporate management or strategy leading to an improvement in credit quality. A good example here is provided by the heavily indebted France Télécom, which at the start of October 2002 appointed a new CEO in the form of Thierry Breton, who was well-respected in the market as a specialist in corporate turnarounds. CDS traders clearly responded positively to Breton's appointment and to the deleveraging strategy he announced soon after his arrival. Between October 10 and November 1, the offered price of the CDSs on France Télécom's five-year euro bonds*

almost halved, from 500bp to 260bp. Again, this trend foreshadowed other key market developments: in December, Moody's confirmed its Baa3 rating on France Télécom with a stable rating, while between mid-November and the middle of January, the spread on the company's benchmark 2011 bond fell from 360bp to 264bp over swaps. In other words, the CDS market anticipated both the action of the rating agencies and the performance of France Télécom's benchmark bonds in the secondary market."

Summary of Results

The results from the empirical analysis between CDS rates and the volatility skew can be summarized as follows, when considering the entire period:

- The variables have a highly negative correlation: -0,61.
- The Granger causality tests indicate that CDS rates Granger-cause volatility skew.
- A sinle cointegrating relationship is found between the variables.
- Both coefficient of adjustments are statistically significant and show the expected sign.
- The influence of the skew volatility on the CDS rates is more pronounced than vice versa.
- The cointegrating relation plot shows strikingly similarity patterns with CDS rates and deep-out-of-the-money implied option volatilities.
- It is found that the credit risk price discovery is almost entirely done in the option markets via the volatility skew.

In a similar way, the results from the empirical analysis between CDS rates and the volatility skew can be summarized as follows, when considering only the bubble burst cycle:

- The variables have a higher than before negative correlation: -0,72.
- The Granger causality tests suggest a bidirectional Granger causality between the variables.
- A single cointegrating relationship is found between the variables.
- Both coefficient of adjustments are statistically significant and show the expected sign.

- The influence of the skew volatility on the CDS rates is more pronounced than vice versa, and is even more pronounced when compared to the analysis of the entire period..
- It is found that the credit risk price discovery is done in a more reasonable way than before, with the volatility skew contributing about 78% to the discovery.

3.4.3 Empirical Relationship between CDS and Equity Prices

The second analysis is carried out between CDS rates and equity prices, as described in section 3.1. Before following the procedures presented in 3.4.1 to detect cointegrating relationships, it is interesting and useful to examine Table 3.2 and Figure 3.6. From the table and the plot, it is strikingly evident that **both variables are highly correlated** and may possess a certain relationship. This suggests that when CDS (EQUITY) falls, EQUITY (CDS) will rise. The behaviour of those variables are in accordance with common knowledge in the market.

Unit Root Tests

Continuing with the test procedures, the first thing to analyse is the **ACF plots** of CDS and SKEW. From Figure 3.5 it is clear that **both series are non-stationary** on the level and **stationary** on the first-difference, where the high degree of persistence on the level ACF is consistent with the presence of a unit root. Another indication of non-stationarity is given by a **unit root test**. Table 3.8 shows the result for **KPSS** test. The test rejects the null hypothesis that the level of both series are stationary at the 1% critical value, but it fails to reject the null hypothesis that the first difference of both series are stationary. Therefore, **both series are nonstationary I(1) processes** and as a consequence it is assumed that the data generation process contains a constant and a linear time trend.

Variable	**CDS**	**EQUITY**
Level		
KPSS statistic	0.6587	0.5378
First difference		
KPSS statistic	0.0435	0.0640
1% level critical value	0.2160	0.2160

Table 3.8: Results of the KPSS test

Granger Causality Test

The **optimal lag length** is chosen according to an **LR** statistic at 5% level. The number of lagged terms to be included in the **VAR** representation is **18**, according to the output of

EViews. The results for the **Granger causality tests** are presented in Table 3.9 and Table 3.10. In Table 3.9, the columns reflect the marginal probability for the Granger-causal impact of the column-variables on the row-variables. The last row represents the χ^2 statistics with 18 degrees of freedom. Examining the basic F test and the VAR χ^2 approach, it seems that

the causality is moving from the equity prices to the CDS rates, while the other direction is statistically not significant.

χ^2 test	**CDS**	**EQUITY**
CDS	0.0000	0.0002
EQUITY	0.1500	0.0000
χ^2 statistic	24.1543	47.5934

Table 3.9: VAR pair-wise causality test

F test	F**-statistic**	**p-value**
H_0 : EQUITY does not Granger Cause CDS	2.6441	0.0003
H_0 : CDS does not Granger Cause EQUITY	1.3419	0.1567

Table 3.10: Pair-wise causality test

VAR-Specification

The **VAR estimation** is shown in equation 3.52 and 3.53 with the statistically significant coefficients at 5% level. The whole representation is given in Appendix D.

$$\begin{aligned} CDS &= 0.8052\, CDS_{t-1} + 0.2990\, CDS_{t-2} - 0.1966\, CDS_{t-3} + \\ &\quad +0.1344\, CDS_{t-5} - 0.2039\, CDS_{t-7} + 0.1493\, CDS_{t-8} - \\ &\quad -3.7721 EQUITY_{t-1} + 3.1202 EQUITY_{t-2} \\ adjusted\ \bar{R}^2 &= 0.98 \end{aligned} \tag{3.52}$$

$$\begin{aligned} EQUITY &= 0.0086\, CDS_{t-3} - 0.0088\, CDS_{t-13} + 1.015\, EQUITY_{t-1} - \\ &\quad -0.2270\, EQUITY_{t-3} + 0.1269\, EQUITY_{t-16} \\ adjusted\ \bar{R}^2 &= 0.99 \end{aligned} \tag{3.53}$$

It is worth nothing that the **VAR estimation does not give the same results as Granger-causality tests**: there seems to exist a bidiretional influence on the variables. Note further the **high values for the** $\bar{R}^2$ measure for both equations, which is a **good indicator for the goodness-of-fit for the estimations**. With the VAR regressions it is possible to examine the **impulse responses functions**. As already said, the ordering of the variables influences highly the results of the functions. Therefore, the analyses will be made in both directions: first CDS entering last and then entering first.

Innovation Accounting

The graphical output of the impulse responses when EQUITY enters first in the Cholesky decomposition are shown in Figure 3.10. Since it is interesting to discover the impact of equity prices in CDS, it makes sense that CDS enters last. The output of the impulse responses when the CDS enters first in the Cholesky decomposition are shown in Figure 3.11. Since it is interesting to discover the impact of CDS in equity prices, it makes sense that CDS enters first. The responses to the shock of one positive standard deviation in innovations for each equation are traced out for a period of 300 days. The standard deviations bands are displayed as dotted lines in the graphs.

Analysing the graphs, it is interesting to see that the response of CDS to EQUITY (as seen in the upper-right plot of Figure 3.10) is **quite ambiguous in both representations**. It would be expected that as soon as equity prices fall, CDS rates would rise. According to the graphs, during about the first 120 days, this is exactly what happens. Later on, however, the relationship between equity prices and CDS rates becomes positive and there is not a logical explanation for that feature. Although the influence of equity prices on CDS rates was captured by the Granger-causality tests, the impulse responses **give no clear signal of the correct relationship between them**.

However, the response of EQUITY to CDS is as expected (as seen in the lower-left plot of Figure 3.11). Despite of a few days of positive relationship (in particular in the first representation), the relationship between EQUITY and CDS is strongly negative. In this case, as CDS spread increases equity prices fall. The influence of CDS rates on equity prices was not captured by the Granger-causality tests made before. It seems through the impulse response functions that there is **a bidirectional effect or causality presenting in both markets**.

Following the same systematics of the impulse response functions, Panel A of Table 3.11 shows the variance decomposition of CDS, when CDS enters last and first in the ordering, respectively. Panel B shows the variance decomposition of EQUITY, when EQUITY enters

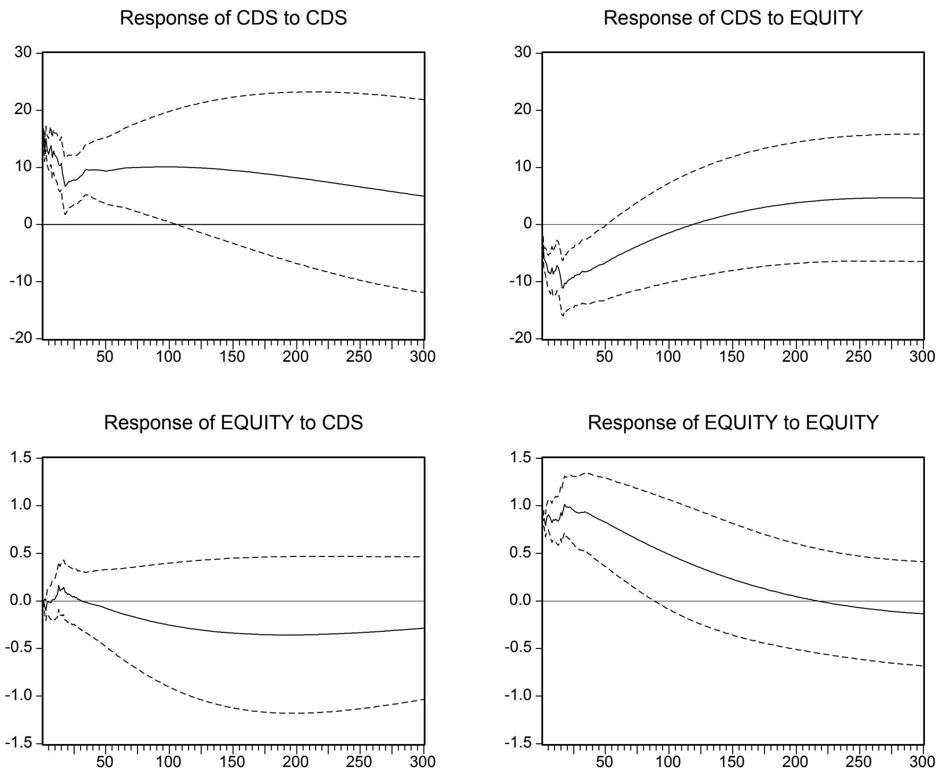

Figure 3.10: Impulse responses for CDS and EQUITY with CDS entering last

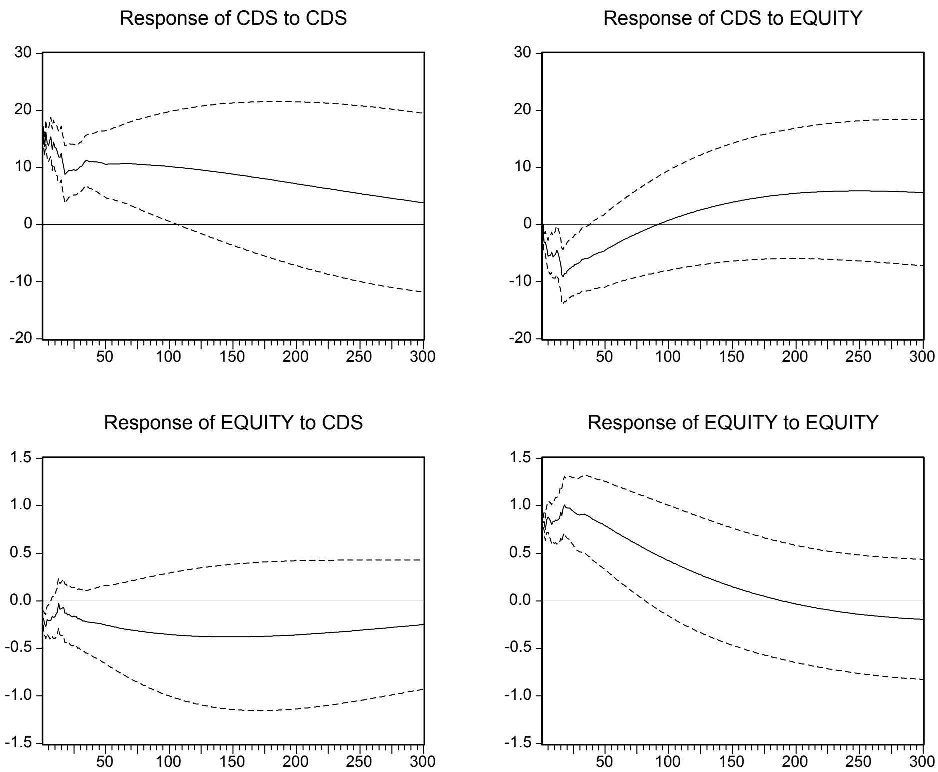

Figure 3.11: Impulse responses for CDS and EQUITY with EQUITY entering last

first and last in the ordering, respectively.

Panel A				
CDS variance decomposition				
Horizon (days ahead)	**CDS**	**EQUITY**	**CDS**	**EQUITY**
10	75.92	24.07	90.96	9.04
50	59.18	40.82	78.09	21.91
100	69.63	30.37	85.23	14.77
200	79.44	20.56	84.58	15.42
300	77.17	22.83	76.73	23.27
Panel B				
EQUITY variance decomposition				
Horizon (days ahead)	**CDS**	**EQUITY**	**CDS**	**EQUITY**
10	0.20	99.80	5.75	94.25
50	0.45	99.55	4.67	95.33
100	2.94	97.06	10.75	89.25
200	15.52	84.48	25.26	74.74
300	25.25	74.75	32.36	67.65

Table 3.11: Variance decomposition

A first evidence drawn from the variance decomposition is that even with different ordering, **CDS explains almost 70% of the forecast error variance of the CDS rates** for the entire period. Similarly, even with different ordering, EQUITY explain almost 70% of the forecast error variance of the equity prices for the whole period as well.

Cointegration Tests

Since there is **a unit root in either variable** and as they have the **same order of integration**, I(1), cointegration tests may be carried out. Using the assumption that the data-generation process contains a constant and a linear time trend, the results of **Johansen's procedure** are displayed in Table 3.12.

		Critical Values				**Critical Values**	
H_o	Trace	5%	1%	H_o	Max. Eigenvalue	5%	1%
$r = 0$	15.79	25.32	30.45	$r = 0$	11.53	18.96	23.65
$r \leq 1$	4.27	12.25	16.26	$r = 1$	4.27	12.25	16.26

Table 3.12: Results of the Johansen cointegration tests

The trace tests fail to reject either the null hypothesis of no cointegration, $r = 0$, and the null hypothesis of at most one cointegrating vector, $r \leq 1$. Similarly, the maximum eigenvalue tests fail to reject the null hypothesis of no cointegration and fails to reject the other null hypothesis of at most one cointegrating vector.

Therefore, CDS and EQUITY are not cointegrated

and hence a VECM representation is not valid. It could be argued that rejection is due to the fact that

credit risk for this specific company is not revealed or priced in its cash equity market,

conversely to the evidence of cointegrating relation between CDS rates and the volatility skew.

Since there is no cointegration among CDS and EQUITY, the VAR system estimated in equation 3.52 and equation 3.53 has to be re-estimated in first difference. That is, the first difference will be eliminated the non-stationarity feature of the variables, as could be seen in Table 3.8, and **Granger-causality tests can be an indication**, even a simplified one, of **causality**. The results are given in Table 3.13 and 3.14. As before, it seems that

the causality is moving from the equity prices to the CDS rates, while the other direction is statistically not significant.

χ^2 test	**ΔCDS**	**ΔEQUITY**
ΔCDS	0.0000	0.0001
ΔEQUITY	0.1413	0.0000
χ^2 statistic	23.2500	47.0621

Table 3.13: VAR pair-wise causality test

F test	**F-statistic**	**p-value**
H_0 : ΔEQUITY does not Granger Cause ΔCDS	2.7684	0.0003
H_0 : ΔCDS does not Granger Cause ΔEQUITY	1.3676	0.1475

Table 3.14: Pair-wise causality test

As a conclusion, even if there is **no cointegration** between CDS and EQUITY, it was shown that exists a **strong correlation** between these markets and that indeed **equity prices Granger-cause CDS rates**. The same reasoning for using an indicator for leverage applies here: the results could serve to improve the understanding of the interaction between CDS rates and equity prices. In fact, splitting the period as before, the correlation between CDS and EQUITY increases in absolut terms to 0.87, suggesting an even stronger relationship between them. Further analyses on this subject was not part of the thesis.

Summary of Results

The results from the empirical analysis between CDS rates and equity prices can be summarized as follows, when considering the whole period:

- The variables have a highly negative correlation: -0,66.
- The Granger causality tests indicate that equity prices Granger-cause CDS rates.
- No cointegrating relationship is found between the variables.
- VAR in first-difference still suggests that equity prices Granger-cause CDS rates.
- There is a stronger negative correlation between the variables of -0,87 than before, when considering the bubble burst cycle.

Chapter 4

Conclusion

The objective of this thesis was to present the principles of capital structure arbitrage: the theoretical background that supports the existence of such arbitrage techniques, the main trading strategies involved in this techniques, the existing market for capital structure arbitrage, and empirical evidence for the relationship between CDS prices, implied equity option volatility skew and equity prices.

Apparently, capital structure arbitrage trading strategies in recent years became of paramount importance in financial markets while not beeing a new area in academic research. It might be said that the theoretical foundation of capital structure arbitrage is mainly based on the contingent claim analysis (CCA) of the so-called Merton model, see Merton (1974). CCA states that there exists a direct link between a company's bond credit spread, and consequently its probability of default, and its asset value volatility, or its observable equity price and corresponding volatility. After the seminal work of Merton, various extensions and developments were proposed, in particular the KMV model and the web-based tool CreditGrades. All the CCA modelling proposed so far, however, has to make some kind of assumptions to derive a closed-formula linking credit spreads, the probability of default, and equity volatilities.

In fact, it is exactly the modelling of this relationship that enables the existence of capital structure arbitrage: the possibility of extracting the probability of default, or as its proxy the CDS rates, from observable equity option prices, or in particular the deduced volatility skew, via a mathematical model. At present, trading CDS versus put options is the most common trading strategy in capital structure arbitrage. Hence, the more accurate the theoretical model, the more arbitrage opportunities might be identified. However, instead of using such mathematical modelling one could verify empirically the relationship between CDS rates and the equity option volatility skew.

It is known that cointegration analysis is a suitable econometric tool to model dynamically long-run equilibrium with short-run divergence. Knowing that CDS rates and the volatility skew present precisely this feature, cointegration analysis was chosen in this thesis to try to describe this relationship. Besides the cointegration analysis, the empirical analysis focused on the mechanism of price discovery for credit risk, assessed by the Gonzales-Granger measure. This measure reveals in which of the two markets, equity options or credit instruments, credit risk is better assessed.

An empirical analysis was carried out to study the relationship between CDS prices and a volatility skew measure defined as diference between deep-out-of-the-money and at-the-money put options. The relationship between CDS prices and cash equity prices was studied as well. The entire analysis was done for one company: France Telecom.

First of all, when considering the whole time period available in the data set, the highly negative correlation of -0,61 between CDS rates and the volatility skew should be stressed. Granger causality tests indicate that CDS rates Granger-cause the volatility skew. The results for the cointegration analysis showed that there exists one cointegrating vector between CDS rates and the volatility skew measure. The existence of one cointegrating vector means that CDS rates and the volatility skew have in indeed a long-run equilibrium despite the short-run deviations.

In fact, the coefficients of adjustment of the error correction model indicate that the volatility skew drives CDS rates for this time period. Another interesting result is that the plot of the cointegrating vector presents a strikingly similar movement pattern with CDS rates and deep-out-of-the-money implied equity volatilities for the period considered.

The GG measure of price discovery indicates that 98% of credit risk price discovery occurs in options market via the volatility skew. This astonishing result could lead to the wrong intepration that credit risk is almost entirely traded in France Telecom's equity options. However, when taking into account the leverage indicator, the above result changes significantly. In fact, it is reasonable to adopt such indicator as companies indeed present leverage or debt cycles throughtout its corporate life. For the present case, without a specific leverage indicator, such as debt per share ratio, CDS rates were used to split the time period in two sub-sets. The first cycle, namely the bubble burst cycle, was empirically analysed in this study.

Secondly, considering the relationship between CDS rates and the volatility skew in the bubble cycle the correlation between the variables is even more negative: -0,72. Granger causality tests suggest a bi-directional Granger causality between the variables. The results for the cointegration analysis showed that there exists one cointegrating vector between CDS rate and the volatility skew measure. In fact, the coefficients of adjustment are greater than before; meaning that in the bubble cycle the relationship reverts faster to its long-run equilibrium.

Furthermore, the influence of the skew volatility on the CDS rates is more pronounced than the vice versa and is even stronger than before. These facts indicate that in the bubble cycle skew volatility drives CDS rates, as already expected. The fact that the skew volatility drives the CDS rates during the bubble cycle is also confirmed by the price discovery measure. It is found that the credit risk price discovery is revealed in a more reasonable way than before, with volatility skew contributing to 78% of the discovery mechanism against 98% before.

Thirdly, considering the relationship between CDS rates and equity prices for the whole period, the variables present a highly negative correlation: -0,66. Granger causality tests indicate that equity prices Granger-cause CDS rates. Contrarily to the previous analysis, cointegrating relationship between the variables was not found. However, VAR in first-difference still suggests that equity prices Granger-cause CDS rates.

The present results for France Telecom empirically suggest what was predicted by Merton's model: there is a direct link between equity volatilities (via the volatility skew), credit spreads (via CDS rates) and leverage. Furthermore, the results empirically confirm that CDS rates and the volatility skew for France Telecom can give early signs of corporate distressed, and consequently indicate a corrsponding trading strategy. Indeed the results show that there exists some kind of mis-pricing between these two asste classes. In fact, the results for France Telecom indicate what banks and hegde funds have already discovered for a variety of other companies: capital structure arbitrage is an effective trading strategy.

Recommendations

This thesis introduces a novel and innovative empirical study concerning a relatively new trading strategy in that it presents a cointegration analysis of CDS prices and the volatility skew as the relevant variables to be analysed. Actually, the authors believe this is a research niche that was not explored in detailed so far, at least until the completion of this work. As a cosequence recommendation for future works is thus to extend the cointegration analysis to a much broader range of companies. In that sense, Merton's model could be tested empirically with market variables and market prices. However, a good leverage indicator is suggested to capture the appropriate leverage or debt cycles.

Furthermore, it is recommended that the analysis of the price discovery mechanism of credit risk should be extended. This could be implemented calculating another price discovery measure, such as the Hasbrouck measure. This analysis is important in that its complements and even validates the cointegration results, as demonstrated in this thesis. Another suggestion is to perform trading-rule simulations using the cointegrating vector results as an indicator for possible trading actions.

Difficulties

The most critical points in the thesis were to have access to proprietary capital structure arbitrage trading strategies and to obtain data. It is reasonable to expect some resistance of banks and hedge funds for revealing their trading strategies. In fact these strategies contribute significantly to their trading profit. However, the authors encountered faced problems to obtain the necessary data, in particular CDS rates and implied equity option volatilities.

Even the considerable amount of contents presented in this study does not exhaust the capital structure arbitrage theme. The subject is indeed vast and wide-ranging with ramifications into numerous and different areas of finance. The important and relevant empirical results obtained in this thesis show that capital structure arbitrage can provides an interesting research topic and will attract the attention of more researchers.

Appendix A

Detailed Proofs

In this appendix, following the work of Hull et al. (2003a) detailed proofs of the formulas (8) and (9) in Hull et al. (2003a) are presented mainly to show the correct formula (8). The formula A.23 below is the correct formula (8) in Hull et al. (2003a). Based on the knowledge of the formulas (8) - here A.23 - and (9) - here A.18 - and the credit spread implied by the Merton model -here A.27 we developed a formula to calculate the implied put volatilities (see Section 3.1.1 formula 3.7) where the proof is presented within the Appendix.

A.1 Notations:

- E is the value of the firms equity
- A is the value of the firms asset
- B is the value of the firms bond
- T is the time to maturity of the debt, the time to repayment to the shareholders
- E_0, A_0 and B_0 are the values of E, A and B today and E_T, A_T are the values of E and A at time T
- CO_t is the value of the call option on E today with strike price K, maturity $T - t$, where t is the time to start the option usually today ($t = 0$), the risk free interest rate is denoted as r, and the equity volatility as σ_E
- PO_t is the value of the put option on E today with strike price K, maturity $T - t$, risk free interest rate r, an the equity volatility σ_E
- F is the value of the promised debt payment (face value of the debt)

- σ is the asset volatility
- σ_E is the equity volatility
- $N(\,)$ is the normal distribution function
- $M(,;)$ is the cumulative bivariate normal distribution function
- A^*_ω is the critical asset value at time ω, $\omega < T$, the asset value for which the equity value at ω is equal K
- Debt is a pure discount bond where a payment, F, is promised at time T. The present value of the promised debt payment is

$$F^* = Fe^{-rT} = FB(0,T) \tag{A.1}$$

- The measure of Leverage is defined by

$$l = F^*/A_0 \tag{A.2}$$

- α is the scalar multiple of the forward asset value at which the option is at the money (implied strike level) defined as

$$A^*_\omega = \alpha A_0 e^{r\omega} \tag{A.3}$$

- κ is the ratio of the strike price to the forward equity price (option moneyness) defined as

$$K = \kappa E_0 e^{r\omega} \tag{A.4}$$

- ν is the implied volatility of the put option on . The implied volatilities are the ones which, when substituted into Black-Schloles formula for given market prices
- PD is the probability of default
- The credit spread is defined as $s^c = y - r$, where r is denoted as the risk-free interest rate and y will be the yield to maturity of the risky bond
- Δ is the equity option delta

A.2 Proof of formula (9) in Hull et al. (2003a)

To start with the proof of A.18 we have to look on the payment to the shareholders at time T , which is $E_T = \max[A_T - F, 0]$. As we know from Merton (1974) the equity of a firm is a call option on the assets of the firm with strike price equal to the promised debt payment, F. The current equity price is given by

$$E_0 = A_0 N(d_1) - Fe^{-rT} N(d_2), \tag{A.5}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{A_0 e^{rT}}{F}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ d_2 &= d_1 - \sigma (T)^{\frac{1}{2}}. \end{aligned}$$

using A.1 and A.2 we get

$$\begin{aligned} E_0 &= A_0 N(d_1) - F^* N(d_2) \\ &= A_0 N(d_1) - l A_0 N(d_2) \\ &= A_0 [N(d_1) - l N(d_2)] \end{aligned} \tag{A.6}$$

and

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{A_0}{Fe^{-rT}}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ &= \frac{\ln\left(\frac{A_0}{F^*}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ &= \frac{\ln\left(l^{-1}\right)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ &= -\frac{\ln(l)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \\ d_2 &= d_1 - \sigma\sqrt{T}. \end{aligned}$$

Using Itô's lemma the instantaneous volatility of the equity is given by

$$\sigma_E = \frac{\partial E}{\partial A}\frac{A_0}{E_0}\sigma = \Delta\frac{A_0}{E_0}\sigma = N(d_1)\frac{A_0}{E_0}\sigma. \tag{A.7}$$

Jones, Mason and Rosenfeld (1984) have shown the empirical evidence of this relation.

Equations A.6 and A.7 allow A_0 and σ to be obtained from E_0, σ_E, l and T. Remember that PD is the risk neutral probability that the company will default by time T, which is the probability that share holders will not exercise their call option to buy the assets of the company for the value of F at time T, formally

$$PD = N(-d_2). \tag{A.8}$$

The probability of default depends on the leverage, l, the asset volatility, σ, and the time until repayment, T .

Geske (1979) developed a formula for a put option on the value of the firms equity with strike price K and expiry time $\omega < T$. Within the framework of Merton (1974), an option on the firms equity that expires before the debt matures is a compound option, i.e. an option on a European call.

$$PO_0 = Fe^{-rT} M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + Ke^{-r\omega} N\left(-a_2\right), \tag{A.9}$$

where

$$\begin{aligned} a_1 &= \frac{\ln\left(\frac{A_0}{A^*_\omega e^{-r\omega}}\right)}{\sigma\sqrt{\omega}} + \frac{1}{2}\sigma\sqrt{\omega} \\ a_2 &= a_1 - \sigma\sqrt{\omega}. \end{aligned}$$

and A^*_ω is the critical asset value at time ω, which represents the asset value such that the equity value at ω is equal K.

The Put-Call-Parity gives the following relation

$$\begin{aligned} CO^*_0 - PO^*_0 &= E_0 - Ke^{-r\tilde{T}} \\ PO^*_0 &= Ke^{-r(\tilde{T})} N\left(-\tilde{d}_2\right) - E_0 N\left(-\tilde{d}_1\right), \end{aligned} \tag{A.10}$$

where now $T \neq \tilde{T}$ and

$$\begin{aligned} \tilde{d}_1 &= \frac{\ln\left(\frac{Ke^{-r\tilde{T}}}{E_0}\right)}{\nu\sqrt{\tilde{T}}} + \frac{1}{2}\nu\sqrt{\tilde{T}} \\ \tilde{d}_2 &= \tilde{d}_1 - \nu\sqrt{\tilde{T}}. \end{aligned}$$

The Idea is that at time ω, $\omega < T$, where the asset value is such that the equity value is equal the strike price K, the value of the put option in equation A.10, PO^*_0,at time $\tilde{T} = \omega$ and equation A.9, PO_0, at time $T = \omega$ must be equal.

Let's first modify equation A.10 using equation A.4, which yields the following Black-Scholes put option price

$$PO_0^* = \kappa E_0 N(-d_2^*) - E_0 N(-d_1^*),$$

where

$$\begin{aligned} d_1^* &= \frac{\ln\left(\frac{E_0}{\kappa E_0}\right)}{\nu\sqrt{\omega}} + \frac{1}{2}\nu\sqrt{\omega} = \frac{\ln(\kappa^{-1})}{\nu\sqrt{\omega}} + \frac{1}{2}\nu\sqrt{\tau} = -\frac{\ln(\kappa)}{\nu\sqrt{\omega}} + \frac{1}{2}\nu\sqrt{\omega} \\ d_2^* &= d_1^* - \nu\sqrt{\omega}. \end{aligned} \tag{A.11}$$

Modifying equation A.9 using equation A.1 and equation A.4 yields

$$PO_0 = F^* M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa E_0 N(-a_2),$$

and by using equation A.3 yields

$$\begin{aligned} a_1 &= \frac{\ln\left(\frac{A_0}{\alpha A_0}\right)}{\sigma\sqrt{\omega}} + \frac{1}{2}\sigma\sqrt{\omega} = -\frac{\ln(\alpha)}{\sigma(\omega)^{\frac{1}{2}}} + \frac{1}{2}\sigma\sqrt{\omega} \\ a_2 &= a_1 - \sigma\sqrt{\omega}. \end{aligned}$$

Now we set $PO_0 = PO_0^*$ and get

$$F^* M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa E_0 N(-a_2) = \kappa E_0 N(-d_2^*) - E_0 N(-d_1^*). \tag{A.12}$$

Formula A.12 is identical with formula (7) in Hull et al. (2003a)

A variation of equation A.5 can be used to determine the implied strike level, α, using the fact that A_ω^* is the critical asset value at time ω, the asset value for which the equity value at ω is equal K. This means that we are looking for the payment to shareholders at time ω is $E_\tau = \max[A_\omega^* - F, 0]$ and get

$$K = A_\tau^* N(d_{1,\tau}) - F e^{-r(T-\tau)} N(d_{2,\tau}), \tag{A.13}$$

where

$$
\begin{aligned}
d_{1,\omega} &= \frac{\ln\left(\frac{A^*_\omega e^{r(T-\omega)}}{F}\right)}{\sigma\left(T-\omega\right)^{\frac{1}{2}}} + \frac{1}{2}\sigma\sqrt{(T-\omega)},\\
d_{2,\omega} &= d_1 - \sigma\sqrt{(T-\omega)}.
\end{aligned}
$$

Using equations A.1, A.2, A.3 and A.4 we can write

$$
A_0 = \frac{F^*}{l} = \frac{Fe^{-rT}}{l} = \frac{A^*_\omega}{\alpha}e^{-r\omega}, \tag{A.14}
$$

which yields to

$$
\frac{A^*_\omega}{\alpha} = \frac{F}{l}e^{-r(T-\omega)}
$$

after rearranging being equivalent to

$$
A^*_\omega\frac{l}{\alpha} = Fe^{-r(T-\omega)}
$$

or

$$
F = A^*_\omega\frac{l}{\alpha}e^{r(T-\omega)}.
$$

Using equation A.13 we get

$$
\kappa E_0 e^{r\omega} = K = A^*_\omega N\left(d_1,\omega\right) - A^*_\omega\frac{l}{\alpha}N\left(d_2,\omega\right) = A^*_\omega\left[N\left(d_1,\omega\right) - \frac{l}{\alpha}N\left(d_2,\omega\right)\right], \tag{A.15}
$$

where now

$$
\begin{aligned}
d_{1,\omega} &= \frac{\ln\left(\frac{A^*_\omega e^{r(T-\omega)}}{A^*_\omega\frac{l}{\alpha}e^{r(T-\omega)}}\right)}{\sigma\sqrt{(T-\omega)}} + \frac{1}{2}\sigma\sqrt{(T-\omega)} = \frac{\ln\left(\frac{1}{\frac{l}{\alpha}}\right)}{\sigma\sqrt{(T-\omega)}} + \frac{1}{2}\sigma\sqrt{(T-\omega)} && \text{(A.16)}\\
&= -\frac{\ln\left(\frac{l}{\alpha}\right)}{\sigma\sqrt{(T-\omega)}} + \frac{1}{2}\sigma\sqrt{(T-\omega)} && \text{(A.17)}\\
d_{2,\omega} &= d_{1,\omega} - \sigma\sqrt{(T-\omega)}.
\end{aligned}
$$

From equation A.15 follows

$$\begin{aligned} E_0 &= \frac{e^{-r\omega}}{\kappa} A^*_\omega \left[N(d_1,\omega) - \frac{l}{\alpha} N(d_2,\omega) \right] \\ &= \frac{e^{-r\omega}}{\kappa} \alpha A_0 e^{r\omega} \left[N(d_1,\omega) - \frac{l}{\alpha} N(d_2,\omega) \right] \\ &= \frac{\alpha}{\kappa} A_0 \left[N(d_1,\omega) - \frac{l}{\alpha} N(d_2,\omega) \right] \\ &= \frac{A_0}{\kappa} \left[\alpha N(d_1,\omega) - l N(d_2,\omega) \right]. \end{aligned}$$

Using again equation A.5 yields

$$\kappa = \frac{\alpha N(d_{1,\omega}) - l N(d_{2,\omega})}{N(d_1) - l N(d_2)}, \tag{A.18}$$

which is exactly formula (9) in Hull et al. (2003a). q.e.d.

A.3 Proof of formula (8) in Hull et al. (2003a)

Using equations A.2, A.6, and A.12 we get the following equation:

$$\begin{aligned} & l A_0 M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - A_0 M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) && \text{(A.19)} \\ & + \kappa A_0 \left[N(d_1) - l N(d_2)\right] N(-a_2) && \text{(A.20)} \\ = \; & \kappa A_0 \left[N(d_1) - l N(d_2)\right] N(-d_2^*) - A_0 \left[N(d_1) - l N(d_2)\right] N(-d_1^*). \end{aligned}$$

Dividing this equation by A_0 yields to

$$\begin{aligned} & l M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) && \text{(A.21)} \\ & + \kappa \left[N(d_1) - l N(d_2)\right] N(-a_2) && \text{(A.22)} \\ = \; & \kappa \left[N(d_1) - l N(d_2)\right] N(-d_2^*) - \left[N(d_1) - l N(d_2)\right] N(-d_1^*). \end{aligned}$$

Rearranging A.21 yields

$$\begin{aligned} & l M\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) && \text{(A.23)} \\ & + \kappa \left[N(d_1) - l N(d_2)\right] N(-a_2) && \text{(A.24)} \\ = \; & \left[N(d_1) - l N(d_2)\right] \left[\kappa N(-d_2^*) - N(-d_1^*)\right]. \end{aligned}$$

Which seems to be is the corrected equation (8) in Hull et al. (2003a) q.e.d.

A.4 The credit spread of a risky bond implied by the Merton model

To explain risky bond yields the Merton model can be used. The value of the assets at any time equals the total value of the value firms equity and bonds so that

$$A_0 = E_0 + B_0.$$

Using equation A.6 this yields

$$\begin{aligned} B_0 &= A_0 - E_0 \\ &= A_0 - A_0 \left[N(d_1) - lN(d_2)\right] \\ &= A_0 \left[N(-d_1) - lN(d_2)\right]. \end{aligned} \tag{A.25}$$

The implied yield to maturity is defined by

$$B_0 = Fe^{-yT} = F^* e^{(r-y)T}. \tag{A.26}$$

Substituting A.26 into A.25 using equation A.14 and the definition of the credit spread gives

$$\begin{aligned} F^* e^{(r-y)T} &= A_0 \left[N(-d_1) - lN(d_2)\right] \\ \frac{F^*}{A_0} e^{(r-y)T} &= \left[N(-d_1) - lN(d_2)\right] \\ Le^{(r-y)T} &= \left[N(-d_1) - lN(d_2)\right] \\ e^{(r-y)T} &= \left[\frac{N(-d_1)}{l} - N(d_2)\right] \\ e^{-s^c T} &= \left[\frac{N(-d_1)}{l} - N(d_2)\right] \\ s^c &= -\ln\left[\frac{N(-d_1)}{l} - N(d_2)\right]\frac{1}{T}. \end{aligned} \tag{A.27}$$

According to the expression for the risk-neutral probability of default in equation A.8 the credit spread implied by the Merton model, equation A.27 and therefore so-called *implied credit spread* depends on the leverage l, the asset volatility, σ, and the time until repayment T.

Using the risk-neutral probability of default in equation A.8 the expression

$$s^c = -\ln\left[\frac{N(-d_1)}{l} - 1 - PD\right]\frac{1}{T}$$

shows the dependence on the risk-neutral probability of default, which by it self depends on leverage the , L, the asset volatility, σ, and the time until repayment, T .

A.5 A formula for the implied put option volatilities

To develop a formula for the implied put option as presented in 3.1.1. We solve the two equations A.23 and A.18 subject to the implied put volatility, ν. For this, first of all, the α, the scalar multiple of the forward asset value at which the option is at the money (implied strike level) has to be estimated. Because within formula A.23 the variables a_2 and a_1 depending on α as well the variables $d_{1,\omega}$ and $d_{2,\omega}$ in formula A.18. Ones we have estimate the $\alpha's$, then the implied put volatilities can be estimated using the variables d_1^* and d_2^*. All calculations depending on the leverage parameter, l.

To estimate $\alpha's$ for different leverage ratios we use A.18 to get

$$\kappa\left[N(d_1) - lN(d_2)\right] = \alpha N(d_{1,\tau}) - lN(d_{2,\tau}). \tag{A.28}$$

Knowing that the left hand side of this equation can be estimated directly and that $d_{1,\tau}$, $d_{2,\tau}$ depends on α a simple Newton method can be used to solve A.28 numerically with subject to α. Based on these estimated $\alpha's$ the right hand side of the rearrange formula A.23

$$\kappa N(-d_2^*) - N(-d_1^*) \simeq \frac{lM\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa[N(d_1) - lN(d_2)]N(-a_2)}{[N(d_1) - lN(d_2)]}, \tag{A.29}$$

can be calculated in approximation. The quality of the approximation depends on the number of iterations used within the used Newton method. Using now the fact that for a given moneyness κ the option delta for the put option, $\Delta_\kappa^p = -N(-d_1^*)$, is known in approximation. Then follows from A.29

$$d_2^* \simeq -N^{-1}\left(\frac{1}{\kappa}\left[\frac{lM\left(-a_2, d_2; -\sqrt{\frac{\omega}{T}}\right) - M\left(-a_1, d_1; -\sqrt{\frac{\omega}{T}}\right) + \kappa[N(d_1) - lN(d_2)]N(-a_2)}{[N(d_1) - lN(d_2)]} - \Delta_\kappa^p\right]\right). \tag{A.30}$$

Using the representations from A.11 to estimate the implied put volatilities using the following quardratic equation

$$\begin{aligned} d_2^* &= -\frac{\ln(\kappa)}{\nu\sqrt{\omega}} + \frac{1}{2}\nu\sqrt{\omega} - \nu\sqrt{\omega}, \\ \nu\sqrt{\omega}d_2^* &= -\ln(\kappa) - \frac{1}{2}\nu^2\sqrt{\omega}, \\ \nu^2 - \nu\frac{2d_2^*}{\sqrt{\omega}} + \frac{2\ln(\kappa)}{\omega} &= 0. \end{aligned} \tag{A.31}$$

The equation A.31 yields to the following solution for the implied put volatilities

$$\nu \simeq \frac{d_2^* \pm \sqrt{d_2^{*2} - 2\ln(\kappa)}}{\sqrt{\omega}}, \tag{A.32}$$

and for $\kappa = 1$ simply

$$\nu \simeq \frac{2d_2^*}{\sqrt{\omega}}.$$

Rearranging 3.1.1 yields to

$$1 - le^{-s^c T} = N(d_1) - lN(d_2). \tag{A.33}$$

This expression can be substitute in A.30 and we get our general result as presented in 3.7.

Because of the fact that $-d_1 \geq 0$ we get

$$\begin{aligned} \frac{\ln(l)}{\sigma\sqrt{T}} &\geq \frac{1}{2}\sigma\sqrt{T} \\ \ln(l) &\geq \frac{1}{2}\sigma^2 T, \end{aligned}$$

which yields to

$$l > e^{\frac{1}{2}\sigma^2 T}$$

q.e.d.

Appendix B

VAR and VECM representations of section 3.4.2

The complete VAR and VECM representations with all the coefficients and its t-statistics of the analysis of CDS rates and volatility skew, SKEW, (section 3.4.2) are reported here. Table B.1 shows the 8-lags VAR estimation, where the first number in brackets are the standard errors and second the t-statistic. Table B.2 shows the 8-lag VECM representation, where the first number in brackets are the standard errors and second the t-statistic. In Table B.2, adjustment stands for the coefficient of adjustment of the cointegrating equation. The 5% critical level is 1.96. For the VAR estimation th e adjusted sample is from 10/08/2001 to 28/07/2003, and the number of observations is 512, after adjusting endpoints. For the VECM representation the adjusted sample is from 13/08/2001 to 28/07/2003, and the number of observations is 511, after adjusting endpoints.

	CDS	SKEW
CDS(-1)	0.8864	0.009
	(0.0446)	(0.0005)
	(19.8652)	(1.5981)
CDS(-2)	0.2737	-0.0010
	(0.0592)	(0.0007)
	(4.6257)	(1.4775)
CDS(-3)	-0.1983	0.0016
	(0.0604)	(0.0007)
	(-3.2810)	(2.1836)
CDS(-4)	-0.0302	-0.0020
	(0.0610)	(0.0007)
	(-0.4950)	(-2.7219)
CDS(-5)	0.1267	0.0001
	(0.0613)	(0.0007)
	(2.0670)	(0.1703)
CDS(-6)	0.0148	(0.0007)
	(0.0607)	(0.0007)
	(0.2440)	(0.9479)
CDS(-7)	-0.2082	-0.0005
	(0.0596)	(0.0007)
	(-3.4928)	(0.7083)
CDS(-8)	0.1267	0.0005
	(0.0451)	(0.0005)
	(2.8107)	(0.9111)
SKEW(-1)	-1.0951	1.0049
	(3.0783)	(0.0371)
	(-0.3558)	(27.1147)
SKEW(-2)	-2.9033	-0.1038
	(4.3148)	(0.0520)
	(-0.6729)	(-1.9972)
SKEW(-3)	6.4267	-0.0301
	(4.3160)	(0.0520)
	(1.4891)	(-0.5783)
SKEW(-4)	-0.0394	0.0490
	(4.3058)	(0.0518)
	(0.0091)	(0.9452)

	CDS	SKEW
SKEW(-5)	-2.5973	0.1269
	(4.2943)	(0.0517)
	(-0.6048)	(2.4550)
SKEW(-6)	-2.6919	-0.0834
	(4.2979)	(0.0518)
	(-0.6263)	(1.6115)
SKEW(-7)	2.8199	-0.0299
	(4.2887)	(0.0516)
	(0.6575)	(-0.5785)
SKEW(-8)	0.1314	0.0295
	(3.0111)	(0.0363)
	(0.0437)	(0.8134)
Constant	1.9882	0.1018
	(3.4121)	(0.0411)
	(0.5827)	(2.4775)
R-squared	0.9830	0.9603
Adj. R- squared	0.9825	0.9590
Sum sq. resids	145370.20	21.0729
S.E. equation	17.1370	0.2063
F-statistic	1793.7450	748.8519
Log likelihood	-2172.5670	90.2293
Akaike AIC	8.5530	-0.2861
Schwarz SC	8.6937	-0.1453
Mean dependent	263.2949	4.2994
S.D. dependet	129.5324	1.0195

Determinant Residual Covariance	12.4732
Log Likelihood (d.f. adjusted)	-2099.0300
Akaike Information Criteria	8.3321
Schwarz Criteria	8.6136

Table B.1: VAR Estimation

	ΔCDS	**ΔSKEW**
Adjustment[a]	-0.0188	0.0004
	(0.0088)	(0.0001)
	(-2.1419)	(3.8758)
ΔCDS(-1)	-0.0935	0.0006
	(0.0451)	(0.0005)
	(-2.0760)	(1.0341)
ΔCDS(-2)	0.1702	-0.0004
	(0.0449)	(0.0005)
	(3.7929)	(-0.6510)
ΔCDS(-3)	-0.0275	0.0012
	(0.0454)	(0.0006)
	(-0.6063)	(2.2182)
ΔCDS(-4)	-0.0544	-0.0009
	(0.0455)	(0.0006)
	(-1.1935)	(-1.5703)
ΔCDS(-5)	0.0698	-0.0007
	(0.0455)	(0.0006)
	(1.5360)	(-1.2247)
ΔCDS(-6)	0.0796	(0.0001)
	(0.0455)	(0.0006)
	(1.7499)	(0.2366)
ΔCDS(-7)	-0.1157	-0.0006
	(0.0451)	(0.0005)
	(-2.5660)	(-1.0834)
ΔCDS(-8)	0.0534	-0.0007
	(0.0452)	(0.0005)
	(1.1809)	(-1.2232)
ΔSKEW(-1)	-2.8387	0.0740
	(3.7684)	(0.0453)
	(-0.7533)	(1.6358)
ΔSKEW(-2)	-5.5063	-0.0428
	(3.0934)	(0.0372)
	(-1.7801)	(-1.1528)
ΔSKEW(-3)	0.9653	-0.0732
	(3.1008)	(0.0372)
	(0.3113)	(-1.9667)

	ΔCDS	**ΔSKEW**
ΔSKEW(-4)	1.2670	-0.0245
	(3.1056)	(0.0373)
	(0.4080)	(-6564)
ΔSKEW(-5)	-1.4260	0.1044
	(3.0853)	(0.0371)
	(-0.4622)	(2.8178)
ΔSKEW(-6)	-3.7470	0.0132
	(3.0638)	(0.0368)
	(-1.2230)	(0.3597)
ΔSKEW(-7)	-1.4448	-0.0106
	(3.0506)	(0.0366)
	(-0.4736)	(-0.2885)
SKEW(-8)	1.4717	0.0052
	(3.0384)	(0.0365)
	(0.4844)	(0.1421)
Constant	0.0254	0.0011
	(3.7599)	(0.0091)
	(0.0335)	(0.1220)

R-squared	0.0915	0.1030
Adj. R- squared	0.0601	0.0721
Sum sq. resids	144092.40	20.7829
S.E. equation	17.0961	0.2053
F-statistic	2.9194	3.3303
Log likelihood	-2166.5680	93.0951
Akaike AIC	8.5502	-0.2939
Schwarz SC	8.6994	-0.1447
Mean dependent	-0.0450	0.0010
S.D. dependet	17.6345	0.2131

Determinant Residual Covariance	12.3096
Log Likelihood	-2073.2330
Log Likelihood (d.f. adjusted)	-2073.5570
Akaike Information Criteria	8.3388
Schwarz Criteria	8.6621

a: The cointegrating vector is shown in equation 3.46

Table B.2: VECM Representation

Appendix C

VAR and VECM representations of section 3.4.2

Cointegrating analysis of CDS and SKEW in the bubble cycle follows the same steps of the analysis for the whole period, as presented is section 3.4.2. To begin with, unit roots test will follow.

C.1 Unit Roots Test

Following with the test procedures, the first thing to analyse is the **ACF plots** of CDS and SKEW. From the plots (not shown), it is clear that **both series are nonstationary** on the level and stationary on the first-difference, the high degree of persistence on the level ACF is consistent with the presence of a unit root. Another indication of nonstationarity is given by a **unit root test**. Table C.1 shows the result for **KPSS** test. The test rejects the null hypothesis that the level of both series are stationary at the 1% critical value, but it fails to reject the null hypothesis that the first difference of both series are stationary. Therefore, **both series are nonstationary I(1) processes**. It is assumed that the data generation process contains a constant and a linear time trend.

Variable	**CDS**	**SKEW**
Level		
KPSS statistic	0.2348	0.2395
First difference		
KPSS statistic	0.1806	0.1595
1% level critical value	0.2160	0.2160

Table C.1: KPSS test

C.2 Granger Causality Test

The **optimal lag length** is chosen according to **LR** statistic at 5% level. The number of lagged terms to be included in the **VAR** representation is **7**, according to the output of EViews. **Granger causality tests** are presented in Table C.2 and Table C.3. In Table C.2, the columns reflect the marginal probability for the Granger-causal impact of the column-variables on the row-variables. The last row informs the χ^2 statistics with 7 degrees of freedom. Examining the simple F test and the VAR χ^2 approach, it seems that **the causality is running from both directions**.

χ^2 test	**CDS**	**SKEW**
CDS	0.0000	0.0000
SKEW	0.0019	0.0000
χ^2 statistic	22.7691	43.1134

Table C.2: VAR pairwise causality test

F test	**F-statistic**	**p-value**
H_0 : SKEW does not Granger Cause CDS	3.2527	0.003
H_0 : CDS does not Granger Cause SKEW	6.1591	0.000

Table C.3: Pairwise causality test

C.3 VAR Especification

The **VAR estimation** is shown in equation C.1 and C.2 just with the statistically significant coefficients at 5% level[1]. The whole representation is the last section of this Appendix.

$$\begin{aligned} CDS &= -8.6738 + 0.8559\ CDS_{t-1} + 0.3678\ CDS_{t-2} \\ &\quad -0.2721\ CDS_{t-4} + 2.8665\ SKEW_{t-2} \\ adjusted\ \bar{R}^2 &= 0.96 \end{aligned} \tag{C.1}$$

[1] The test for autocorrelation in the residuals at 1% level results that they are not autocorrelated.

$$
\begin{aligned}
SKEW &= 0.0191\ CDS_{t-1} - 0.0214\ CDS_{t-5} + 0.7091\ SKEW_{t-1} \\
&\quad -0.8321\ SKEW_{t-2} + 0.6083\ SKEW_{t-3} - 0.4056\ SKEW_{t-4} \\
&\quad +0.3497\ SKEW_{t-5} - 0.2491\ SKEW_{t-6} + 0.2674\ SKEW_{t-7} \\
adjusted\ \bar{R}^2 &= 0.61
\end{aligned} \tag{C.2}
$$

It is noteworthy that the **VAR estimation gives the same results as Granger-causality tests**: in equation C.1 there are SKEW lagged terms and in C.2 there are also CDS lagged terms. Note also the **high value for the** $\bar{R}^2$ measure for equation C.1 and the not so high value for $\bar{R}^2$ in C.2. With the VAR regressions it is possible to examine the **impulse responses functions**. As already said, the ordering of the variables influences highly the results of the functions. Therefore, the analyses will be made in both directions: once CDS entering last and then entering first.

C.4 Innovation Accounting

The graphical output of the impulse responses when the SKEW enters first in the Cholesky decomposition are shown in Figure C.1. Since it is interesting to discover the impact of the skew measure in CDS, it makes sense that CDS enters last. The output of the impulse responses when the CDS enters first in the Cholesky decomposition are shown in Figure C.2. Since it is interesting to discover the impact of CDS in the skew measure, it makes sense that CDS enters first. The responses to the shock of one positive standard deviation in innovations for each equation are traced out for a period of 50 days. The standard deviations bands are displayed in dotted lines in the graphs.

Following the same systematic of the impulse response functions, Table C.4 Panel A shows the variance decomposition of CDS, when CDS enters last and first in the ordering, respectively. Panel B shows the variance decomposition of SKEW, when SKEW enters first and last in the ordering, respectively.

One first evidence drawn from the variance decomposition is that even with different ordering, CDS explains almost all of the forecast error variance of the CDS rates for the whole period. In fact, the skew measure has little contribution to the variance decomposition of the CDS rates. By contrast, there is a more balanced contribution of either CDS rates and skew measure to the SKEW variance decomposition, even changing the ordering of the variables.

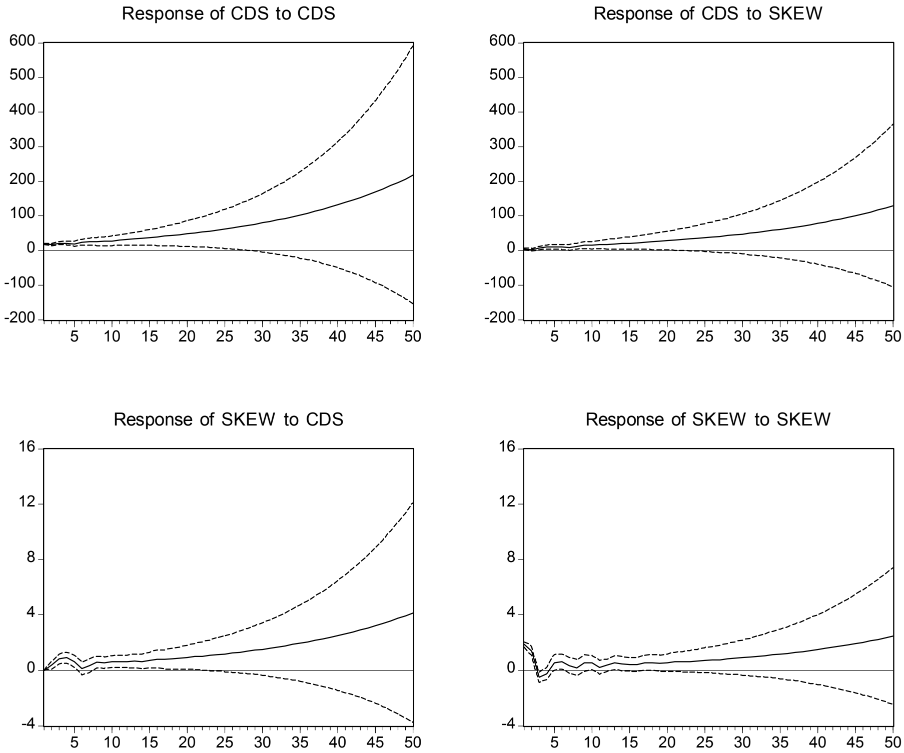

Figure C.1: Impulse Responses for CDS and SKEW with CDS entering last

Panel A **CDS variance decomposition**				
Horizon (days ahead)	**CDS**	**SKEW**	**CDS**	**SKEW**
5	99.45	0.55	99.27	0.73
10	99.40	0.60	99.12	0.88
25	98.42	1.58	97.95	2.05
35	98.07	1.93	97.54	2.46
50	97.86	2.14	97.30	2.70
Panel B **SKEW variance decomposition**				
Horizon (days ahead)	**CDS**	**SKEW**	**CDS**	**SKEW**
5	1.72	98.28	1.42	98.58
10	6.58	93.42	6.04	93.96
25	54.73	45.27	54.04	45.96
35	76.56	23.44	75.93	24.07
50	90.45	9.55	89.86	10.14

Table C.4: Variance Decomposition

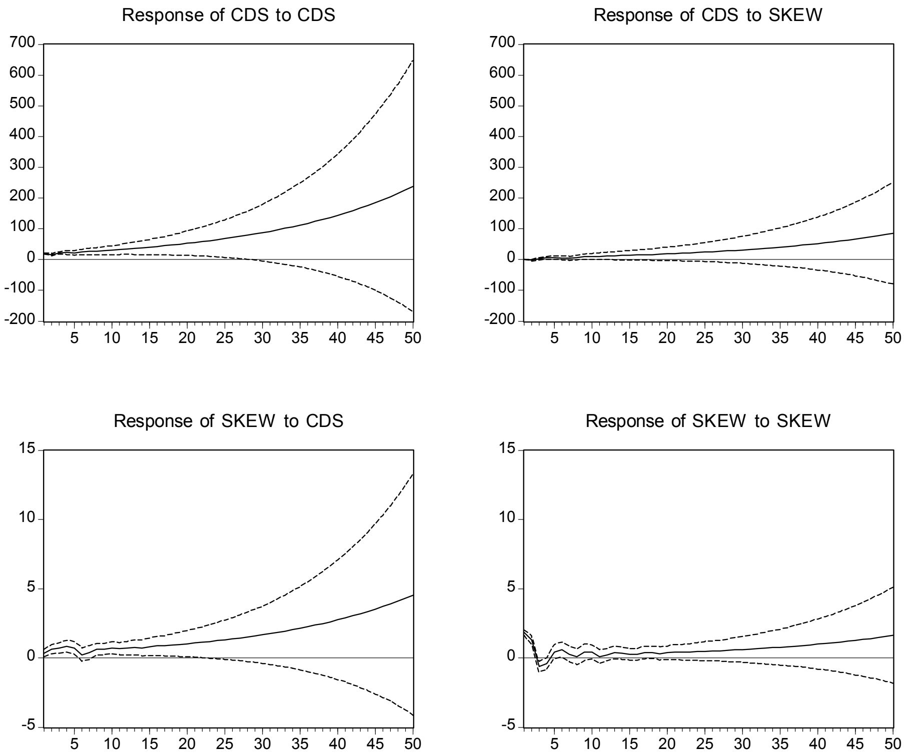

Figure C.2: Impulse Responses for CDS and SKEW with SKEW entering last

In fact, in the 25 days, about 50% of the variance is explained by the CDS rates and over time CDS contributes more to explain the variance in the skew measure.

C.5 Cointegration Tests

Since there is **a unit root in either variables** and they have the **same order of integration**, I(1), cointegration tests may be carried out. Using the assumption that the data-generation process contains a constant and a linear time trend, **Johansen's procedure** results is displayed in Table C.5 Panel A. Panel B shows the **normalized cointegrating vector**.

Panel A:Tests							
		Critical Values				**Critical Values**	
H_o	Trace	5%	1%	H_o	Max. Eigenvalue	5%	1%
$r = 0$	48.29	25.32	30.45	$r = 0$	41.39	18.96	23.65
$r \leq 1$	6.89	12.25	16.26	$r = 1$	6.89	12.25	16.26

Panel B: Normalised cointegrating vector				
	CDS	**SKEW**	**Trend**	**Constant**
Coefficient	1.00	-125.96	1.11	121.91

Table C.5: Johansen cointegration tests

The trace tests in Panel A reject the null hypothesis of no cointegration, $r = 0$, at all level of significance and fail to reject the null hypothesis of at most one cointegrating vector, $r \leq 1$. Similarly, the maximum eigenvalue tests reject the null hypothesis of no cointegration at all level of significance but fails to reject the other null hypothesis of at most one cointegrating vector. Therefore, **CDS and SKEW are cointegrated with one cointegrating vector**. In Panel B, all coefficients of the **normalized cointegrating vector** are significant at 1% level. Thus the vector is $\vec{\beta} = (1.00, -125.96, 1.11)'$.

Since the cointegration relationship can be found between CDS rates and the skew measure, there must exist a **representation of an error correction model (ECM)** which shows long and short run dynamics among the cointegrated variables, as described in equation 3.40. In equation 3.49 of section 3.4.2, the estimated VECM is presented with only significant coefficients at 10% level. The complete representation is shown in the next section of the Appendix.

C.6 VAR and VECM representations

The complete VAR and VECM representations with all the coefficients and its t-statistics of the analysis of CDS rates and volatility skew for the bubble cycle (section 3.4.2) are reported here. Table C.6 shows the 7-lags VAR estimation, where the first number in brackets are the standard errors and second the t-statistic. Table C.7 shows the 7-lag VECM representation, where the first number in brackets are the standard errors and second the t-statistic. In Table C.7, adjustment stands for the coefficient of adjustment of the cointegrating equation. The 10% critical level is 1.65. For the VAR estimation the adjusted sample is from 10/09/2001 to 26/06/2002, and the number of observations is 208, after adjusting endpoints. For the VECM representation the adjusted sample is from 11/09/2001 to 26/06/2002, and the number of observations is 207, after adjusting endpoints.

	CDS	**SKEW**
CDS(-1)	0.8559	0.0191
	(0.0777)	(0.0076)
	(11.0195)	(2.5160)
CDS(-2)	0.3678	0.0135
	(0.1005)	(0.0098)
	(3.6594)	(1.3785)
CDS(-3)	-0.1412	0.0011
	(0.1063)	(0.0104)
	(-1.3285)	(0.1021)
CDS(-4)	-0.2721	-0.0127
	(0.1085)	(0.0106)
	(-2.5076)	(-1.1923)
CDS(-5)	0.2128	-0.0214
	(0.1089)	(0.0107)
	(1.9544)	(-2.0066)
CDS(-6)	0.0572	(0.0163)
	(0.1098)	(0.0107)
	(0.5209)	(1.5222)
CDS(-7)	-0.0814	-0.0068
	(0.0889)	(0.0087)
	(-0.9154)	(-0.7862)
SKEW(-1)	-1.0641	0.7091
	(0.7441)	(0.0728)
	(1.4301)	(9.7440)
SKEW(-2)	2.8665	-0.8321
	(0.9469)	(0.0926)
	(3.0273)	(-8.9863)
SKEW(-3)	0.0402	0.6083
	(1.3213)	(0.1292)
	(0.0305)	(4.7073)
SKEW(-4)	0.7429	-0.4056
	(1.3676)	(0.1337)
	(0.5432)	(-3.0327)
SKEW(-5)	-0.8391	0.3497
	(1.3028)	(0.1274)
	(-0.6441)	(2.7453)

	CDS	**SKEW**
SKEW(-6)	-0.2018	-0.2491
	(1.2168)	(0.1190)
	(-0.1658)	(-2.0936)
SKEW(-7)	1.4665	0.2674
	(0.9379)	(0.0917)
	(1.5635)	(2.9154)
Constant	-8.6738	-0.2322
	(4.3055)	(0.4210)
	(-2.0146)	(-0.5515)
R-squared	0.9648	0.6364
Adj. R- squared	0.9623	0.6100
Sum sq. resids	70198.44	671.3411
S.E. equation	19.0715	1.8651
F-statistic	378.3247	24.1260
Log likelihood	-900.5797	-417.0001
Akaike AIC	8.8037	4.1538
Schwarz SC	9.0443	4.3945
Mean dependent	252.7019	3.9188
S.D. dependent	98.2128	2.9865
Determinant Residual Covariance		1219.3100
Log Likelihood (d.f. adjusted)		-1329.3070
Akaike Information Criteria		13.0703
Schwarz Criteria		13.5516

Table C.6: VAR Estimation

	ΔCDS	**ΔSKEW**
Adjustment[a]	-0.0227	0.0064
	(0.0121)	(0.0011)
	(-1.8753)	(5.6479)
ΔCDS(-1)	-0.0794	0.0107
	(0.0797)	(0.0074)
	(-0.9968)	(1.4408)
ΔCDS(-2)	0.2818	0.0234
	(0.0799)	(0.0074)
	(3.5287)	(3.1486)
ΔCDS(-3)	0.1106	0.0248
	(0.0902)	(0.0084)
	(1.2263)	(2.9518)
ΔCDS(-4)	-0.1662	0.0120
	(0.0929)	(0.0086)
	(-1.7905)	(1.3897)
ΔCDS(-5)	0.0532	-0.0045
	(0.0936)	(0.0087)
	(0.5682)	(-0.5152)
ΔCDS(-6)	0.0818	0.0154
	(0.0944)	(0.0088)
	(0.8665)	(1.7548)
ΔCDS(-7)	-0.0820	0.0201
	(0.0962)	(0.0090)
	(-0.8525)	(2.2460)
ΔSKEW(-1)	-3.7694	0.5120
	(1.4843)	(0.1381)
	(-2.5396)	(3.7087)
ΔSKEW(-2)	-1.0903	-0.2757
	(1.5308)	(0.1424)
	(-0.7123)	(-1.9363)
ΔSKEW(-3)	-0.4248	0.1966
	(1.4227)	(0.1323)
	(-0.2986)	(1.4856)
ΔSKEW(-4)	-0.2006	-0.2067
	(1.2481)	(0.1161)
	(-0.1607)	(-1.7809)
ΔSKEW(-5)	-0.4126	0.0702
	(1.2246)	(0.1139)
	(-0.3370)	(0.6164)
ΔSKEW(-6)	-1.0536	-0.2466
	(0.9579)	(0.0891)
	(-1.0999)	(-2.7673)
ΔSKEW(-7)	-0.6648	0.1271
	(0.9707)	(0.0903)
	(-0.6849)	(1.4073)
Constant	2.6372	-0.2325
	(1.4667)	(0.1364)
	(1.7980)	(-1.7045)
R-squared	0.2184	0.5643
Adj. R- squared	0.1570	0.5301
Sum sq. resids	71768.85	620.8674
S.E. equation	19.3844	1.8029
F-statistic	3.5574	16.4936
Log likelihood	-899.0387	-407.4045
Akaike AIC	8.8410	4.0909
Schwarz SC	9.0986	4.3485
Mean dependent	2.7633	-0.0420
S.D. dependent	21.1122	2.6302

Determinant Residual Covariance	1160.1520
Log Likelihood	-1301.1160
Log Likelihood (d.f. adjusted)	-1317.7680
Akaike Information Criteria	13.0702
Schwarz Criteria	13.6337

a: The cointegrating vector is shown in equation 3.49

Table C.7: VECM Representation

Appendix D

VAR and VECM representations of section 3.4.3

The complete VAR and VECM representations with all the coefficients and its t-statistics of the analysis of CDS rates and equity prices (section 3.4.3) are reported here. Table D.1 shows the 8-lags VAR estimation, where the first number in brackets are the standard errors and second the t-statistic. Table D.2 shows the 8-lag VECM representation, where the first number in brackets are the standard errors and second the t-statistic. In Table D.2, adjustment stands for the coefficient of adjustment of the cointegrating equation. The 5% critical level is 1.96. For the VAR estimation the adjusted sample is from 10/08/2001 to 28/07/2003, and the number of observations is 512, after adjusting endpoints. For the VECM representation the adjusted sample is from 13/08/2001 to 28/07/2003, and the number of observations is 511, after adjusting endpoints.

	CDS	**SKEW**
CDS(-1)	0.8864	0.009
	(0.0446)	(0.0005)
	(19.8652)	(1.5981)
CDS(-2)	0.2737	-0.0010
	(0.0592)	(0.0007)
	(4.6257)	(1.4775)
CDS(-3)	-0.1983	0.0016
	(0.0604)	(0.0007)
	(-3.2810)	(2.1836)
CDS(-4)	-0.0302	-0.0020
	(0.0610)	(0.0007)
	(-0.4950)	(-2.7219)
CDS(-5)	0.1267	0.0001
	(0.0613)	(0.0007)
	(2.0670)	(0.1703)
CDS(-6)	0.0148	(0.0007)
	(0.0607)	(0.0007)
	(0.2440)	(0.9479)
CDS(-7)	-0.2082	-0.0005
	(0.0596)	(0.0007)
	(-3.4928)	(0.7083)
CDS(-8)	0.1267	0.0005
	(0.0451)	(0.0005)
	(2.8107)	(0.9111)
SKEW(-1)	-1.0951	1.0049
	(3.0783)	(0.0371)
	(-0.3558)	(27.1147)
SKEW(-2)	-2.9033	-0.1038
	(4.3148)	(0.0520)
	(-0.6729)	(-1.9972)
SKEW(-3)	6.4267	-0.0301
	(4.3160)	(0.0520)
	(1.4891)	(-0.5783)
SKEW(-4)	-0.0394	0.0490
	(4.3058)	(0.0518)
	(0.0091)	(0.9452)
SKEW(-5)	-2.5973	0.1269
	(4.2943)	(0.0517)
	(-0.6048)	(2.4550)
SKEW(-6)	-2.6919	-0.0834
	(4.2979)	(0.0518)
	(-0.6263)	(1.6115)
SKEW(-7)	2.8199	-0.0299
	(4.2887)	(0.0516)
	(0.6575)	(-0.5785)
SKEW(-8)	0.1314	0.0295
	(3.0111)	(0.0363)
	(0.0437)	(0.8134)
Constant	1.9882	0.1018
	(3.4121)	(0.0411)
	(0.5827)	(2.4775)
R-squared	0.9830	0.9603
Adj. R- squared	0.9825	0.9590
Sum sq. resids	145370.20	21.0729
S.E. equation	17.1370	0.2063
F-statistic	1793.7450	748.8519
Log likelihood	-2172.5670	90.2293
Akaike AIC	8.5530	-0.2861
Schwarz SC	8.6937	-0.1453
Mean dependent	263.2949	4.2994
S.D. dependet	129.5324	1.0195

Determinant Residual Covariance	12.4732
Log Likelihood (d.f. adjusted)	-2099.0300
Akaike Information Criteria	8.3321
Schwarz Criteria	8.6136

Table D.1: VAR Estimation

	ΔCDS	**ΔSKEW**		**ΔCDS**	**ΔSKEW**
Adjustment[a]	-0.0188	0.0004	ΔSKEW(-4)	1.2670	-0.0245
	(0.0088)	(0.0001)		(3.1056)	(0.0373)
	(-2.1419)	(3.8758)		(0.4080)	(-6564)
ΔCDS(-1)	-0.0935	0.0006	ΔSKEW(-5)	-1.4260	0.1044
	(0.0451)	(0.0005)		(3.0853)	(0.0371)
	(-2.0760)	(1.0341)		(-0.4622)	(2.8178)
ΔCDS(-2)	0.1702	-0.0004	ΔSKEW(-6)	-3.7470	0.0132
	(0.0449)	(0.0005)		(3.0638)	(0.0368)
	(3.7929)	(-0.6510)		(-1.2230)	(0.3597)
ΔCDS(-3)	-0.0275	0.0012	ΔSKEW(-7)	-1.4448	-0.0106
	(0.0454)	(0.0006)		(3.0506)	(0.0366)
	(-0.6063)	(2.2182)		(-0.4736)	(-0.2885)
ΔCDS(-4)	-0.0544	-0.0009	SKEW(-8)	1.4717	0.0052
	(0.0455)	(0.0006)		(3.0384)	(0.0365)
	(-1.1935)	(-1.5703)		(0.4844)	(0.1421)
ΔCDS(-5)	0.0698	-0.0007	Constant	0.0254	0.0011
	(0.0455)	(0.0006)		(3.7599)	(0.0091)
	(1.5360)	(-1.2247)		(0.0335)	(0.1220)
ΔCDS(-6)	0.0796	(0.0001)	**R-squared**	0.0915	0.1030
	(0.0455)	(0.0006)	**Adj. R- squared**	0.0601	0.0721
	(1.7499)	(0.2366)	**Sum sq. resids**	144092.40	20.7829
			S.E. equation	17.0961	0.2053
ΔCDS(-7)	-0.1157	-0.0006	**F-statistic**	2.9194	3.3303
	(0.0451)	(0.0005)	**Log likelihood**	-2166.5680	93.0951
	(-2.5660)	(-1.0834)	**Akaike AIC**	8.5502	-0.2939
			Schwarz SC	8.6994	-0.1447
ΔCDS(-8)	0.0534	-0.0007	**Mean dependent**	-0.0450	0.0010
	(0.0452)	(0.0005)	**S.D. dependet**	17.6345	0.2131
	(1.1809)	(-1.2232)			
			Determinant Residual Covariance		12.3096
ΔSKEW(-1)	-2.8387	0.0740	**Log Likelihood**		-2073.2330
	(3.7684)	(0.0453)	**Log Likelihood (d.f. adjusted)**		-2073.5570
	(-0.7533)	(1.6358)	**Akaike Information Criteria**		8.3388
			Schwarz Criteria		8.6621
ΔSKEW(-2)	-5.5063	-0.0428			
	(3.0934)	(0.0372)			
	(-1.7801)	(-1.1528)			
ΔSKEW(-3)	0.9653	-0.0732			
	(3.1008)	(0.0372)			
	(0.3113)	(-1.9667)			

a: The cointegrating vector is shown in equation 3.46

Table D.2: VECM Representation

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