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Current Developments

It has recently been agreed between **Deutsche Bank** and **Microsoft** that Global Quantitative Research will collaborate in the Enterprise Early Adopter program for the new .NET technology. This collaboration will extend throughout most of this year and is expected greatly to enhance our pricing server development programme. This Month's Selection on page 4 gives more details.

Thus far, the pricing server has found a quite number of applications around the equity division, and has been offered to clients as a way of delivering specialised services to major accounts. The group has initiated a consultation exercise with the sales force globally (for information, contact Karen James) to determine how we can best use this promising technology for sales. In future, we anticipate that it will be increasingly used by other applications such as Xavex Online and Imagine. To use a simple option pricer based on this technology, visit our external site www.dbquant.com/.

Our Theory and Practice article this month expands on the theme of credit spread treatment in convertible bond pricing, one of which was outlined in Quantessence Volume 1, Issue 6. In fact, this article was prompted by feedback from a reader, and we welcome and encourage suggestions for content in future issues.

Finally, the library now supports quanto and composite pricing in the Jump Diffusion framework (see *Heston Model Steps Further* in Quantessence Volume 1, Issue 7).

Publications and Presentations

BOOKS

Equity Derivatives: Theory and Applications
Andrew Ferraris, Thomas Knudsen, Frank Mao, Ross Milward, Laurent Nguyen-Ngoc, Marcus Overhaus, Gero Schindlmayr
Wiley, 2001.

Equity Derivatives and Market Risk Models
Oliver Brockhaus, Michael Farkas, Andrew Ferraris, Douglas Long, and Marcus Overhaus
Risk Books, 2000.

Modelling and Hedging Equity Derivatives
Oliver Brockhaus, Andrew Ferraris, Christoph Gallus, Douglas Long, Reiner Martin, and Marcus Overhaus
Risk Books, 1999.

ARTICLES

Generic Tools for Derivative Valuation
Andrew Ferraris and Reiner Martin
Journal of Computational Finance, submitted.

Working up a Lather Over SOAP
Michael Farkas and Ross Milward
Risk, October 2000.

Pricing European Options in a Stochastic-Volatility-Jump-Diffusion Model.
Thomas Knudsen and Laurent Nguyen-Ngoc
Journal of Financial and Quantitative Analysis, submitted.

Pricing Quanto and Composite Equity Derivatives in Skewed Markets with Discrete Dividends
Gero Schindlmayr
Journal of Computational Finance, submitted.

Inside this Issue and Contact Information

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Risk Premium Treatment in CB Models

Part A: THE THEORY

Convertible Bonds (CB's) are corporate bonds which pay the holder regular coupons and may be converted into the underlying shares at the holder's discretion. Here we focus on their exposure to the credit of the issuer. At low equity prices, when the equity optionality is worth little, the convertible is essentially a pure bond and it is clearly correct to price (i.e. discount cash flows) with the full credit spread of the issuer. However, it is generally held that a company's ability to issue stock is not strongly influenced by its credit rating. Accordingly, the value contributed to the bond by its conversion rights should not be subject to the same risky discounting as the fixed payments.

Within this basic framework, however, a number of algorithms may be defined which achieve this result. In the one-factor convertible bond model (GE_Convertible_Bond), we have implemented five, known as Risk Premium Treatments A, B, C, D and E. Bear in mind that the aim of all these (except C) is to use a single, constant, deterministic credit spread in such a way as to discount the fixed payments but not the equity value.

We assume throughout that the underlying stock S_t follows the log-normal process:

$$dS = (r - q)Sdt + \sigma SdW$$

where r is the risk-free interest rate, q the dividend yield and σ the volatility. The Risk Premium Treatments are defined as follows.

Risk Premium Treatment A: The quantity *risk neutral conversion probability* P_c is introduced, which is a derivative of the underlying stock S, satisfying the following PDE:

$$\frac{\partial P_c}{\partial t} + (r - q)S\frac{\partial P_c}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P_c}{\partial S^2} = 0$$

At maturity, P_c is set to 1 if the CB is converted, otherwise to 0. For any other time, P_c is set to one if the CB is optimal to convert or is called. The discount rate is equal to $r + (1 - P_c)s$, where s is the credit spread.

Risk Premium Treatment B: This treatment is the same as treatment A except the conversion probability P_c is re-set to 1 only when it is optimal to convert.

Risk Premium Treatment C: All the cash flows are discounted by the full risky rate ($r + s$).

Risk Premium Treatment D: This method is proposed by Tsiveriotis & Fernandes. In this approach, a cash only convertible bond (*COCB*) is defined. The holder of such a bond is entitled to all the cash flows from the bond part but not any equity cash flows. This means at time t, the value of *COCB* V_{co} is set to zero if it is optimal to convert. By definition, V_{co} is the function of the underlying stock price S. Therefore V_{co} should follow the Black-Scholes differential equation with the discount rate set to be the full risky rate ($r + s$):

$$\frac{\partial V_{co}}{\partial t} + (r - q)S\frac{\partial V_{co}}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V_{co}}{\partial S^2} = (r + s)V_{co}$$

The difference between the CB value V and V_{co} is the share related part of the bond (*SOCB*) and its price V_{so} naturally satisfies the Black-Scholes differential equation with the discount rate equal to the risk-free rate r:

$$\frac{\partial V_{so}}{\partial t} + (r - q)S\frac{\partial V_{so}}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V_{so}}{\partial S^2} = rV_{so}$$

From the above two equations, the convertible bond price $V = V_{co} + V_{so}$ satisfies:

$$\frac{\partial V}{\partial t} + (r - q)S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = (r + \hat{s})V$$

where $\hat{s} = s\dfrac{V_{co}}{V}$ is the conversion adjusted credit spread.

Therefore this model is similar to the probability conversion model with a different technique to adjust the discount rate. When the equity price is very high, the convertible is certain to convert, V_{co} is very small, the risk-free rate is used to discount the corresponding cash flows. On the other hand, if the equity price is very low, the convertible bond behaves like an ordinary debt bond and $\hat{s}$ is close to s, which means the full risky rate is used to discount the cash flows.

Risk Premium Treatment E: Assume the equity risk of the convertible bond V is hedged by shorting Δ number of shares, the portfolio $Y = V - \Delta S$ should, on the first order of approximation, be independent of the equity price. This means:

$$\frac{\partial Y}{\partial S} = \frac{\partial V}{\partial S} - \Delta = 0$$

From this equation, the number of shares needed to short is

$$\Delta = \frac{\partial V}{\partial S}$$

Therefore the convertible bond value V can be split into two parts: stock part ΔS, which is discounted by risk-free rate r and bond part (stock independent) Y by full risky rate ($r + s$). Again this model provides another alternative to adjust the credit spread for the discount rate. We can find the equivalent conversion adjusted credit spread

$$\hat{s} = \left(1 - \frac{\Delta S}{V}\right)s$$

Part B: IN PRACTICE

The convertible bond analysed here is a four-year bond with annual coupon 4%. The bond may be converted any time up to maturity. It is call-protected for the first two years and then callable at call price of 110 with trigger level at 130. The conversion ratio is 1.0. The market yield curve is assumed to be flat at 6% and the underlying volatility is 30%. In order to see how the models respond to the credit risk, we utilise 5% credit spread. The spot price is 80 and no dividends are considered for simplicity.

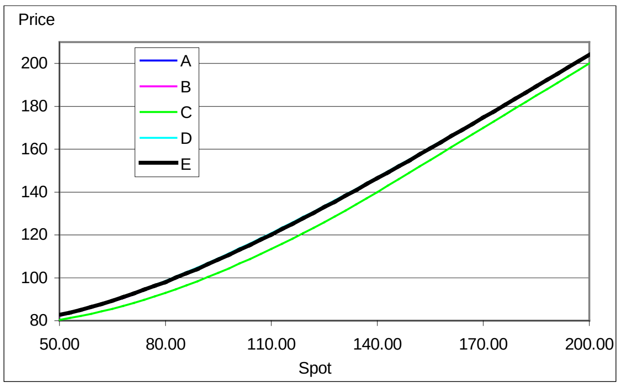


Fig. 1 CB Price vs. Spot

Fig. 1 shows the CB value vs. spot price with different risk treatments. The value from Treatment C is lower due to the higher discount rate used and the other methods are hardly distinguishable.

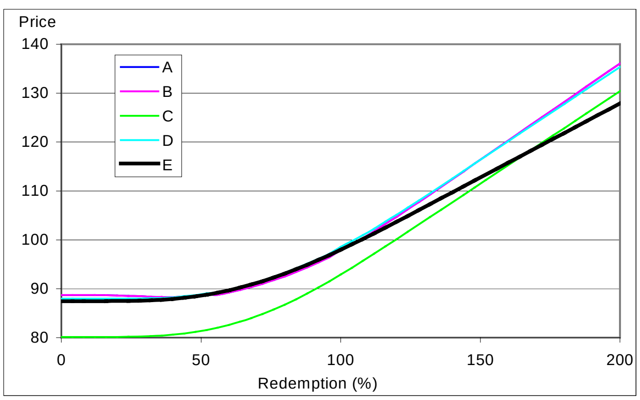


Fig. 2 CB Price vs. Redemption Rate

Fig. 2 shows how the CB value moves against the redemption rate. For Treatments A and B, the CB value drops when the redemption rate increases within the lower range (less than 50%). Clearly this is not reasonable in the real market, and this abnormal phenomenon is due to the credit risk processing. When the low redemption rate increases, the CB has lower conversion probability so the cash flows are discounted at higher rates. This will decrease the CB value. When the redemption rate increases further, its redeemed value increase overtakes the decrease from the higher discount rate.

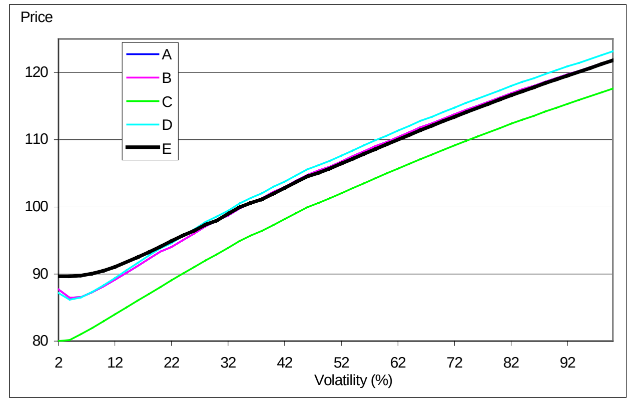


Fig. 3 CB Price vs. Volatility

Fig. 3 shows the relation between CB value and volatility. When the volatility is very low, Treatments A, B and D show negative vega. When the volatility starts to increase from a very low value, the option value clearly increases. On the other hand, the probability of exercise after a coupon date is increased, because the severe discounting decreases ones incentive to hold to the next coupon date. Volatility also increases the chances of the bond going out of the money, with the resulting fixed cash flows discounted at the full risky rate. In the very-low-volatility range, the option value increase is outweighed by the probability of coupon loss and higher discount rate. This abnormal phenomenon does not happen for Treatment E, as the hedged portfolio is insensitive to the volatility in the first order of approximation. Nor does it happen any method at higher spot prices. When the volatility gets higher, all the methods except C behave similarly.

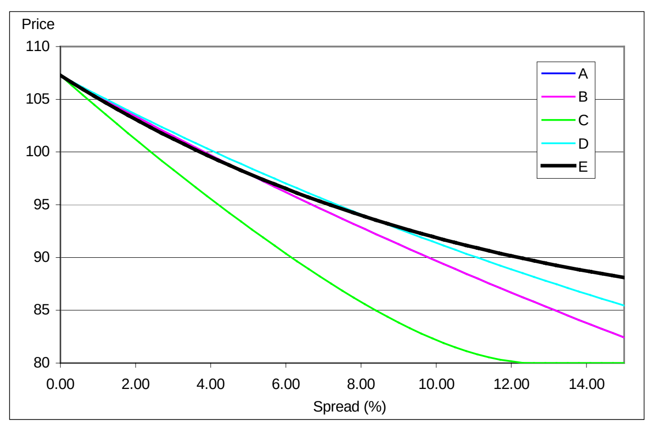


Fig. 4 CB Price vs. Credit Spread

Fig. 4 shows the CB price vs. credit spread. When the credit spread is low (<3%), the CB prices do not change very much with differenct treatments except Treatment C. When the credit spread is high, the differences become more apparent.

From the above discussion and results, Treatment E is the most reliable method to process the high credit spread with respect to redemption and volatility. It is now the default risk premium treatment in the GE Library.

Microsoft E2A Program

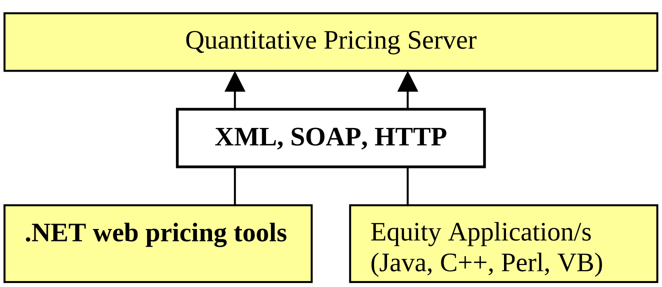
Introduction

As a result of the work achieved with SOAP and XML (as described in Quantessence Volume 1, Issue 6 & Issue 7, and Risk Magazine October 2000), Global Quantitative Research has been invited by **Microsoft** to participate in their Enterprise Early Adopter (E2A) program for the new **.NET** platform.

This participation will benefit the business through the development of web based pricing and simulation tools. Global Quantitative Research will gain an advantage via access to .NET programming tools and consulting expertise during the development of this new pricing infrastructure.

Architecture

As the Quantitative Pricing Server is based on SOAP and XML, the use of the .NET environment for developing web based infrastructure is an effective fit. The following diagram illustrates the relationship with the Pricing Server and the .NET pricing tools as well as other Equities Applications.

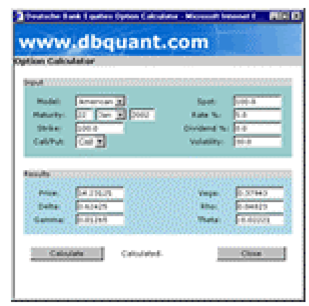


As the core Pricing Server remains platform neutral, the use of .NET for web based pricing tools in no way affects access from other languages such as Java.

Web based Pricing Clients

The web provides a good alternative to the current spreadsheet environment by simplifying access to many of the pricing models where the flexibility of a spreadsheet is not required. Also, by using a common web interface it is easier to ensure consistency among multiple users.

The first such tool for pricing American and European options is now available on our external web site www.dbquant.com. This was developed for the Convertible Bond desk which offer it as a service to their clients on www.dbconvertibles.com.



On the Internet, the user must supply their own data for the calculation, however within Equities it will be possible to add extra capabilities such as the loading of market data directly from Imagine. Currently in development is a reverse convertible calculator that will offer this functionality.

Conclusion

The E2A program will benefit the Equities division by leveraging the work already achieved with the Pricing Server and accelerating the rollout of web based pricing and simulation tools to the desk in 2001.

Other Information

Conferences

Jump Diffusion Models for Pricing and Hedging Equity Derivatives. - Marcus Overhaus: Risk 2001 Conference, April 2001.
Latest Advances for Modelling and Hedging Volatility -Marcus Overhaus: Risk 2000 Conference, April 2000.
Pricing, Hedging And Trading Exotic Derivatives - Thomas Knudsen: Risk Conference, December 2000.
Volatility forecasting and modelling techniques for accurate pricing, hedging and trading of derivatives -Marcus Overhaus: Risk Conference, November 2000.

Seminars

Bath, Jun99 & Dec99 (workshop in London); C. Rogers. Berlin Humboldt, Oct00 & Dec00; H. Foellmer, M. Schweizer.
Bonn, Jul99 & Feb01; S. Albeverio. London Birkbeck, Feb00; R. Kiesel. Oxford, July00 (workshop in London); N. Johnson.
Paris P&M Curie, Jan00, Sep00 & Dec00; M. Yor, N. El-Karoui. Pittsburgh, Oct00 & Apr01; S. Shreve, D. Heath.

Futures
Transactions in futures involve the obligation to make, or to take, delivery of the underlying asset of the contract at a future date, or in some cases to settle your position with cash. **They carry a high degree of risk**. The “gearing” or “leverage” often obtainable in futures trading means that a small deposit or down payment can lead to large losses as well as gains. It also means that a relatively small market movement can lead to a proportionately much larger movement in the value of your investment, and this can work against you as well as for you. **Futures transactions have a contingent liability, and you should be aware of the implications of this, in particular the margining requirements, which are set out in the paragraph below.**

Contingent liability transactions
Contingent liability transactions which are margined require you to make a series of payments against the purchase price, instead of paying the whole purchase price immediately.

If you trade in futures, contracts for differences or sell options you may sustain a total loss of the margin you deposit with your broker to establish or maintain a position. If the market moves against you, you may be called upon to pay substantial additional margin at short notice to maintain the position. If you fail to do so within the time required, your position may be liquidated at a loss and you will be liable for any resulting deficit.

Even if a transaction is not margined, it may still carry an obligation to make further payments in certain circumstances over and above any amount paid when you entered the contract.

Except in specific circumstances under SFA rules, your broker may only carry out margined or other contingent liability transactions with or for you if they are traded on or under the rules of a recognised or designated investment exchange. Contingent liability transactions which are not traded on or under the rules of a recognised or designated investment exchange may expose you to substantially greater risks.