
ANATOMY OF OPTION FEATURES IN CONVERTIBLE BONDS

KA WO LAU
YUE KUEN KWOK*

Several earlier theoretical studies on the optimal issuer's calling policy of a convertible bond suggest that the issuer should call the bond as soon as the conversion value exceeds the call price. However, empirical studies on actual cases of calling by convertible bond issuers reveal that firms "delayed" calling their convertible bonds until the conversion value well exceeded the call price. In this paper, we construct valuation algorithms that price risky convertible bonds with embedded option features. In particular, we examine the impact of the soft call and hard call constraints, notice period requirement and other factors on the optimal issuer's calling policy. Our results show that the critical stock price at which the issuer should optimally call the convertible bond depends quite sensibly on these constraints and requirements. The so-called "delayed call phenomena" may be largely attributed to the underestimation of the critical call price due to inaccurate modeling of the contractual provisions. © 2004 Wiley Periodicals, Inc. *Jrl Fut Mark* 24:513–532, 2004

This research was supported by the Research Grants Council of Hong Kong, under the project HKUST6116/02H.

*Correspondence author, Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong, China; e-mail: maykwok@ust.hk

Received July 2002; Accepted August 2003

-
- *Ka Wo Lau is in the Department of Computer Science at the Hong Kong University of Science and Technology in Clear Water Bay, Hong Kong, China.*
 - *Yue Kuen Kwok is in the Department of Mathematics at the Hong Kong University of Science and Technology in Clear Water Bay, Hong Kong, China.*

INTRODUCTION

A convertible bond is a corporate bond that offers the holders the right but not the obligation to convert the bond at any time to a specific number of shares of the issuer's corporation or receive the par at maturity. The conversion right (equity component) gives the holder the possibility to benefit from future capital appreciation in the company's equity, while the fixed income component provides a return floor. Like usual corporate bonds, the bond issuer pays regular discrete coupon payments to the holders. Since the bondholders have the conversion right as an incentive, they may accept a lower coupon rate in the bond. In essence, the bond issuer shorts a conversion option to the holders. Since the conversion option increases in value with increasing volatility of the stock price, issuers of convertible bonds are usually risky, growth-oriented companies.

Most convertible bonds contain the call provision that could be used by the issuer to manage the debt-equity ratio of his company. Upon issuer's call, the holder can either redeem the bond at the call price or convert into shares. Under the call notice period requirement, the holders are allowed to make their decision to redeem or convert at the end of the notice period. Through this call provision, the issuer gains the flexibility to manage its debt-equity balance. This "delayed equity financing" feature is another important consideration why corporate issuers choose to raise capital through convertible bonds. On the other hand, forced conversion is undesirable for bondholders. To protect the conversion privilege from being called away too soon, the bond indenture commonly contains the hard call constraint that restrains the issuer to initiate the call during the early life of the bond. In addition to the hard call constraint, the soft call constraint further requires the stock price to be above certain trigger price (usually 30% to 50% higher than the conversion price) in order that the issuer can initiate the call. To avoid market manipulation by the issuer, the usual clause in the soft call constraint may require that the stock price has to stay above the trigger price for a consecutive or cumulative period, perhaps, 20 days out of the past 30 consecutive trading days. Below is an excerpt from the bond indenture of the convertible bond issued by the Bank of East Asia in Hong Kong (2% Convertible Bond due 2003):

"On or after July 19, 1998, the Issuer may redeem the Bonds at any time in whole or in part at the principal amount of each Bond, together with accrued interest, if for each of 30 consecutive Trading Days, the last of which Trading Days is not less than five nor more than 30 days prior to the day upon which the notice of redemption

is first published, the closing price of the Shares as quoted on the Hong Kong Stock Exchange shall have at least 130 percent of the Conversion Price in effect on such Trading Day.”

Besides the conversion feature and call provision, a convertible bond may have other embedded features, like the put feature that allows the holder to sell back the bond to the issuer at a preset put price and the reset feature that allows the holder to reset downward the conversion price according to some preset rules when certain conditions are met.

Ingersoll (1977a) and Brennan and Schwartz (1977; 1980) pioneered the use of contingent claim models to price convertible bonds and analyze the optimal call policies to be adopted by issuers. Under certain simplifying assumptions in their models (no call notice period requirement and soft call constraint), they both reached the conclusion that the issuer should call the bond as soon as the conversion value exceeds the call price. However, this theoretical prediction does not correlate with empirical observations. Empirical studies on calling by convertible bond issuers reveal that firms “delayed” calling their convertible bonds until the conversion value exceeded the call price by 83.5% on average (median 38.5%). A number of explanations on the “delayed call phenomena” have been proposed in the literature. These include the signaling hypothesis, yield advantage and after-tax-cash flow considerations, and safety premium hypothesis. The signaling hypothesis rationalizes delayed call by arguing that a call by the management is usually perceived by the market as a signal of unfavorable private information (Harris & Raviv, 1985). Firms may delay call if the dividend yield is higher than the coupon rate, and the loss of debt-tax advantage upon conversion of the convertible bond to equity (Asquith & Mullins, 1991). The safety premium theory hypothesizes that a firm may delay call until the convertible bond is deeper-in-the-money since there are chances that the stock price may drop significantly over the notice period (Jaffee & Shleifer, 1990). If this happens, the bondholders would choose to redeem the bond for cash. This may cause financial distress since it is costly to raise capital within a short period.

Besides the above corporate finance considerations, one would envision that the call notice period requirement and soft call constraint may have impact on the critical stock price at which the firm should call. Ingersoll (1977b) modified his contingent claim model to allow for the call notice period. He considered perpetual convertible bonds, and his findings suggested that the firm should call before the conversion value reaches the call price. Unfortunately, his result further deepens the gap

between theory and market reality. For finite-lived convertible bonds, Butler (2002) obtained results that are in contrast with the findings of Ingersoll (1977b). He showed that issuers delay calling their convertible bonds when a notice period exists, and this delay increases monotonically as the length of the notice period increases. However, his model is based on the simplified assumptions that the convertible bond value is the simple sum of the bond floor value and conversion option value, and conversion is allowed only at bond maturity. In a related study on callable warrants, Kwok and Wu (2000) showed that the critical stock price at which the issuer of the callable warrant should call optimally depends sensibly on the length of the notice period. The critical stock price increases quite significantly with the length of the notice period for moderate value of time to expiry. Moreover, the critical stock price first increases with time to expiry, reaches a maximum then decreases. Since callable warrants and callable convertible bonds share similar properties on optimal calling policy, this may explain the controversial result as reported by Ingersoll (1977b) that the critical stock price to call a perpetual convertible bonds decreases with the presence of notice period.

The impact of the soft call constraint on optimal calling has not been explored in the literature. The excursion time requirement in the soft call constraint is called the *Parisian feature*. In recent years, effective numerical methods for pricing the Parisian feature have been developed (Kwok & Lau, 2001). Similar numerical techniques can be adopted into the valuation algorithms for pricing convertible bonds.

Convertible bond valuation models have been quite extensively studied in the past decades (see Nyborg's 1996 paper for a survey of the models). These contingent claim models either use the firm value of the issuer or the stock price as the underlying state variable for modeling the equity component. The firm value model naturally incorporates the dilution effect upon conversion of the convertible bond. If the dilution effect is not significant (say, the particular convertible bond constitutes only a minute portion of the whole capital structure of the firm), then the use of stock price as the underlying state variable may be more appropriate. Compared to the firm value models, the stock price models avoid the prescription of the capital structure of the firm. Also, the estimation of the parameter values in the stock price model is easier. For example, the stock price volatility is more directly observable compared to the firm value volatility. Also, the conversion value and payoff structures of the convertible bond depend directly on the stock price.

The valuation models of convertible bonds can be broadly classified into one-factor models and two-factor models. In one-factor models, the

interest rate and default spread/hazard rate of default are assumed to be deterministic. Brennan and Schwartz (1980) have shown that the value of a convertible bond is not very sensitive to interest rate fluctuations. If we are mainly interested in the analysis of the conversion feature, which is related largely to the equity component, the simplification in one-factor models can be considered acceptable. To model the credit risk of a convertible bond, Tsiveriotis and Fernandes (1998) incorporated the issuer's debt spread into the pricing model by solving a set of coupled equations, one for the bond part of the convertible bond, and the other for the whole bond value, using different discount rates for the equity and bond components. Takahashi et al. (2001) incorporated the reduced form approach of modeling default as a Poisson arrival process into their convertible bond valuation model. Ayache, Forsyth, and Vetzal (2002) performed a comprehensive analysis of different forms of convertible bond models with credit risk. They concluded that Takahashi et al.'s model has better theoretical justification under the contingent claim pricing framework than the model proposed by Tsiveriotis and Fernandes.

In this paper, we construct numerical algorithms that accurately model the embedded features in a convertible bond and use them to explore the various factors that affect the optimal calling policy and conversion policy. The one-factor model proposed by Takahashi et al. is employed to examine the impact of these embedded features on the critical stock price at which it is optimal to call or convert. In the next section, we show the formulation of the one-factor convertible bond model with credit risk, where the arrival of default is modeled by a hazard rate process. The details of the valuation algorithms are presented, illustrating how to accommodate coupon payments, conversion, and call policies. In particular, we propose effective numerical techniques to deal with the soft call and hard call constraints, notice-period requirement, etc. We analyze the significance of conversion ratio, coupons, and soft call requirements on bond prices. We then analyze the interaction of the call and conversion policies, impact of soft call, hard call, and notice-period requirements on the optimal calling policies and provide conclusive summaries in the last section.

CONTINGENT CLAIMS MODEL

The two most common approaches of modeling the credit risk of risky corporate bonds are the firm value approach and the reduced form approach. The firm value approach models the credit risk exposed to the

bondholders as a put option granted to the issuer whereby the issuer has the right to put the firm for payment of bond par. The reduced form approach models the occurrence of default as a Poisson arrival process. To examine the optimal calling and conversion policies, it is more preferable to use the stock price rather than the firm value as the underlying state variable. When the firm value is not chosen as the state variable in the contingent claims model, the reduced form approach appears to be a more convenient choice to model the credit risk.

We adopt the one-factor contingent claims model for convertible bonds with credit risk. We assume constant interest rate and model the arrival of default by a Poisson arrival process with constant hazard rate. The stock price S is the underlying stochastic state variable, and in the risk neutral valuation framework, it is assumed to follow the lognormal process

$$\frac{dS}{S} = (r - q)dt + \sigma_S dZ_S \quad (1)$$

where r is the riskless interest rate, q and σ_S are the constant dividend yield and volatility of the stock price, respectively, dZ_S is the standard Wiener process. Conditional on no prior default up to time t , the probability of default within the time period $(t, t + dt)$ is $h dt$, where h is the constant hazard rate. By following the usual contingent claims arguments, Ayache et al. (2002) derived various forms of the contingent claims models for defaultable convertible bonds under different assumptions of default mechanisms and recovery upon default. Suppose we assume that upon default the bondholder receives the fraction R (recovery rate) of the bond value and the stock price drops to zero instantaneously, the corresponding governing equation for the convertible bond price function $V(S, t)$ is given by

$$\begin{aligned} \frac{\partial V}{\partial t} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} + (r - q + h)S \frac{\partial V}{\partial S} - [r + (1 - R)h]V + c(t) = 0, \\ 0 < S < S^*(\tau), 0 < t < T \end{aligned} \quad (2)$$

Note that the bond price function satisfies Equation (2) only in the continuation region $= \{(S, t) : 0 < S < S^*(t), 0 < t < T\}$, where the bond remains alive. Here, $S^*(\tau)$ denotes the critical stock price at which the bond ceases to exist either due to early conversion or calling, T is the bond maturity date and $c(t)$ is the source term due to the coupon payment stream. The external cash payout may be represented by

$c(t) = \sum_{i=1}^N c_i \delta(t - t_n)$, where c_i is the coupon payment paid on the discrete coupon payment dates t_i , $i = 1, 2, \dots, N$.

The embedded option features in a convertible bond are characterized by the prescription of the auxiliary conditions in the pricing model, the details of which are discussed below.

1. Terminal payoff on maturity date T .

The terminal value of V is given by

$$V(S, T) = (P + c_N)I_{\{P+c_N \geq nS\}} + nSI_{\{P+c_N < nS\}} \quad (3)$$

where I_A is the indicator function for the event A . Here, P denotes the par value of the bond, c_N is the last coupon payment and n is the number of units of stock to be exchanged for the bond upon conversion.

2. Conversion policy.

Since the bondholders have the right to convert the bond into n units of stock at any time, the intrinsic value of the convertible bond always stays equal or above the conversion value. Upon voluntary conversion, the value of the bond equals the conversion value identically. We then have

$$V(S, t) \geq nS \quad \text{when the convertible bond remains alive} \quad (4a)$$

$$V(S, \bar{t}) = nS \quad \text{when the convertible bond is converted} \quad (4b)$$

where \bar{t} is the optimal time of conversion chosen by the bondholders. It is a common practice in convertible bond indenture that the accrued interest will not be paid upon voluntary conversion. Such clause may inhibit bondholders to convert voluntarily when a coupon date is approaching.

3. Calling policy.

The convertible bond indenture usually contains the hard call provision where the bond cannot be called for redemption or conversion by the bond issuer in the early life of the bond. This serves as a protection for the bondholders so that the privilege of awaiting growth of the equity component will not be called away too soon. Let $[T_c, T]$, $T_c > 0$, denote the callable period, that is, the bond cannot be called during the earlier part of the bond life $[0, T_c]$. Upon calling, the bondholders can decide whether to redeem the bond for cash or convert into shares at the end of the notice period of t_n days. Let \hat{t} denote the

date of call so that $\hat{t} + t_n$ is the conversion decision date for the bondholders. The bondholders essentially replace the original bond at time \hat{t} by a new derivative that expires at the future time $\hat{t} + t_n$ and with terminal payoff $\max(nS, K + \hat{c})$, where \hat{c} is the accrued interest from the last coupon date to the time instant $\hat{t} + t_n$, and K is the pre-specified call price of the convertible bond. We write $V_{new}(S, t; K, t_n)$ as the value of this new derivative. When there is no soft call requirement (a constraint that is related to stock price movement over a short period prior to calling), the convertible bond value should be capped by V_{new} . The convertible bond should be called once its value reaches $V_{new}(S, t; K, t_n)$. We then have

$$V(S, t) \leq V_{new}(S, t; K, t_n) \quad \text{within the callable period} \quad (5a)$$

$$V(S, \hat{t}) = V_{new}(S, \hat{t}; K, t_n) \quad \text{at the calling moment} \quad (5b)$$

When there is a soft call requirement, it is possible that $V(S, t)$ stays above $V_{new}(S, t; K, t_n)$. The treatment of the soft call constraint in numerical valuation algorithms will be considered later.

4. Coupon payments.

By no arbitrage argument, there is a drop in bond value of an amount that equals the coupon payment c_i across a coupon payment date t_i , $i = 1, 2, \dots, N$. We have

$$V(S, t_i^+) = V(S, t_i^-) - c_i, \quad i = 1, 2, \dots, N \quad (6)$$

Remark

The interaction of the optimal conversion and calling policies determines the potential early termination of the convertible bond. The synergy of these two features can be treated effectively via dynamic programming procedure in numerical schemes.

NUMERICAL ALGORITHMS

There exists an arsenal of explicit and implicit finite difference methods to solve numerically the one-factor convertible bond model. The implicit schemes face less stringent time step constraint compared to the explicit schemes. However, explicit schemes are more popular in the financial engineering community due primarily to its relative ease in the design of computer programs. In this paper, we employ the explicit finite difference algorithm to compute the bond price function $V(S, t)$ [as governed by Equations (2a) and (2b) and subject to the auxiliary conditions Equations (3)–(6)].

We adopt the log-transformed variable $x = \ln S$, and define time to expiry $\tau = T - t$. Let V_j^m denote the numerical approximation of $V(x, \tau)$ at the grid point $x = j\Delta x$ and $\tau = m\Delta t$, where Δx and Δt are the respective stepwidth and time step. The explicit numerical scheme takes the following basic form

$$V_j^{m+1} = p_u V_{j+1}^m + p_m V_j^m + p_d V_{j-1}^m - [r + (1 - R)h]V_j^m + c_i I_{\{E_i\}} \quad (7)$$

The probabilities of upward jump, zero jump, and downward jump of the logarithm of the stock price, $x = \ln S$ are given by

$$p_u = \frac{1}{2\lambda^2} + \frac{(r - q + h - \frac{\sigma_S^2}{2})\sqrt{\Delta t}}{2\lambda\sigma_S}$$

$$p_m = 1 - \frac{1}{\lambda^2}, \quad p_d = \frac{1}{2\lambda^2} - \frac{(r - q + h - \frac{\sigma_S^2}{2})\sqrt{\Delta t}}{2\lambda\sigma_S} \quad (8)$$

respectively, and $\Delta x = \lambda\sigma_S\sqrt{\Delta t}$. Here, E_i denotes the event that the coupon payment c_i is paid at t_i . When the payment date t_i is bracketed between time levels $m\Delta t$ and $(m + 1)\Delta t$, the bond values V_j^{m+1} are increased by an extra amount c_i due to the coupon payment [see Equation (6)]. The values V_j^0 at time level $m = 0$, which correspond to terminal payoff values of the bond, are given by

$$V_j^0 = \begin{cases} P + c_N & \text{if } x_j \leq \ln \frac{P + c_N}{n} \\ ne^{x_j} & \text{otherwise} \end{cases} \quad (9)$$

Interaction of the Callable and Conversion Features

The most challenging part in the design of valuation algorithms for convertible bonds is the construction of the dynamic programming procedure applied at each lattice node that models the interaction of the callable and conversion features. Other intricacies include the notice-period requirement (as discussed by d'Halluin, Forsyth, Vetzal, & Labahn, 2001 in their pricing algorithms for callable bonds), and soft or hard call constraints.

Recall that upon the issuance of the notice of call, the bondholder essentially receives a new derivative that replaces the original bond. This new derivative has maturity life equals the length of the notice period and par value equals the call price K plus the accrued interest amount \hat{c} . The conversion ratio remains the same but there are no intermediate

conversion right and coupon payment. With the provision of the early conversion privilege, the bondholder chooses the maximum of the continuation value V_{cont} and conversion value $V_{conv} = nS$ if there is no recall. Upon recall of the bond, the original bond becomes the above new derivative. The issuer adopts the optimal policy of either to recall or abstain from recalling so as to minimize the bond value with reference to the possible actions of the bondholder. The following dynamic programming procedure effectively summarizes the above arguments

$$V_j^m = \min(V_{new}, \max(V_{cont}, V_{conv})) \tag{10}$$

where V_{cont} is the continuation value as computed by numerical scheme, Equation (7). When the calling right is non-operative (say, during the period under the hard call constraint) and only conversion right exists, the above dynamic programming procedure reduces to

$$V_j^m = \max(V_{cont}, V_{conv}) \tag{11}$$

To incorporate the soft call requirement, we model the associated Parisian feature using the forward shooting grid approach proposed by Kwok and Lau (2000), where an extra dimension is added to capture the excursion of the stock price beyond some predetermined threshold level B . With the inclusion of the path dependence of the stock price associated with the soft call requirement, Equation (7) is modified as follows:

$$V_{j,k}^{m+1} = p_u V_{j+1,g(k,j+1)}^m + p_m V_{j,g(k,j)}^m + p_d V_{j-1,g(k,j-1)}^m - [r + (1 - R)h]V_{j,g(k,j)}^m + c_i I_{\{E_j\}} \tag{12}$$

For example, the grid evolution function assumes the form (Kwok & Lau, 2000)

$$g_{cum}(k, j) = k + I_{\{x_j > \ln B\}} \tag{13}$$

for cumulative counting of number of days that the stock price has been staying above the level B . Suppose M cumulative days of breaching is required to activate the calling right, then the dynamic programming procedure in Equation (10) is applied only when the condition $g_{cum} \geq M$ has been satisfied.

The Kwok–Lau algorithm is most effective in dealing with either consecutive or cumulative counting of breaching days. However, the most common form of soft call requirement corresponds to the situation where the stock price stays above the trigger price for a certain proportion

of the moving window of past daily closing stock prices (like the East Asia Bank example quoted earlier). If the moving window spans m days, the computational complexity of the Kwok–Lau algorithm increases by a factor of 2^m . Recently, Grau (2003) proposed a more efficient algorithm that combines the lattice method with Monte Carlo simulations to deal with the moving window Parisian feature.

SIGNIFICANCE OF CONVERSION NUMBER, COUPONS, AND SOFT CALL REQUIREMENTS ON CONVERTIBLE BOND PRICES

Using the one-factor defaultable convertible bond pricing model, we would like to explore the dependence of the convertible bond value on coupon payment streams, conversion number, and soft call constraint. In the sample calculations of the convertible bond pricing model presented below, the parameter values that are adopted in the pricing calculations (unless otherwise specified) are listed in Table I.

In Figure 1, we plot the convertible bond value against time corresponding to different stock price levels. The bond value always exhibits a drop in value that equals the size of the coupon payment across a coupon date. Within the time period between successive coupon payment dates (except for the last period right before maturity), the bond value (evaluated at fixed stock price level) increases as time increases, mainly due to the effect of accrued interests. Within the last coupon period, the bond value may increase, decrease, or stay almost at constant level, depending on the moneyness of the conversion right. The lower dotted curve shows the bond value against time corresponding to stock price $S = 70$. At this low stock price level (30% below the conversion price), the value of the

TABLE I

List of Parameter Values Used in the Sample Calculations of the Convertible Bond Pricing Model

Par value, P	100
Annualized volatility, σ	20%
Annualized dividend yield, q	2%
Maturity date, T	5 years
Coupon rate, c	2% per annum, paid semi-annually
Conversion number, n	1
Call period	Starting 1.0 years from now until maturity
Conversion period	Throughout the whole life
Call price	140
Riskless interest rate, r	Flat at 5% per annum
Hazard rate, h	0.02
Recovery rate, R	0.8

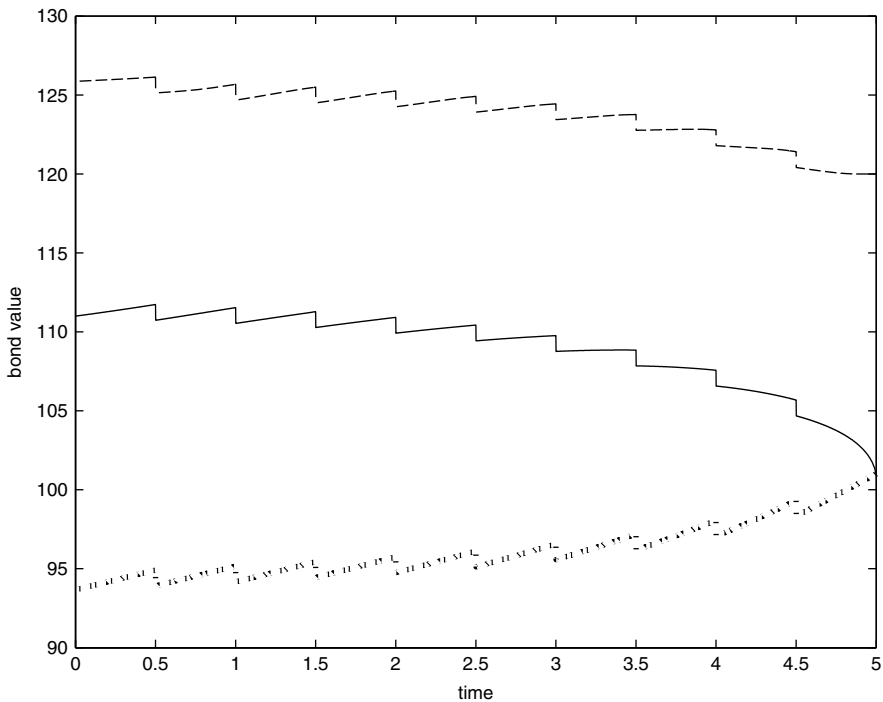


FIGURE 1

Plot of convertible bond value against time at different levels of stock price (dotted curve corresponds to $S = 70$, solid curve corresponds to $S = 100$, and dashed curve corresponds to $S = 120$).

equity component is negligibly small. The bond value shows a general trend of increase with increasing time. The convertible bond behaves like a simple coupon bond, and its value increases with time since the riskless interest rate is higher than the coupon rate. At maturity, the bond value matches the total value of par plus last coupon. At the stock price level $S = 100$ (same as conversion price), the convertible bond drops in value within the last coupon period (see middle solid curve). The drop in value may be attributed primarily to the higher rate of decrease in the value of the conversion option at times close to maturity. At a higher stock price level $S = 120$ (20% above the conversion price), the bond value shows a trend of slight decrease with increasing time (see upper dashed curve). However, the bond value stays almost at constant value within the last coupon period. The value of a deep-in-the-money convertible bond is dominated by its equity component since the bond is almost sure to be converted into shares at maturity, so the time-dependent effect of accrued interest of the bond component is negligible.

TABLE II
Convertible Bond Values Corresponding to Different Conversion Numbers
and Stock Price Levels

<i>Stock Price</i>	<i>Conversion Number</i>						
	<i>0.7</i>	<i>0.8</i>	<i>0.9</i>	<i>1.0</i>	<i>1.1</i>	<i>1.2</i>	<i>1.3</i>
50	85.30	85.67	86.29	87.19	88.41	89.97	91.87
100	94.10	99.47	105.90	113.18	121.12	129.56	138.37
120	101.93	110.18	119.49	129.56	140.16	151.14	162.37
130	106.59	116.29	126.98	138.37	150.21	162.37	174.73
140	111.67	122.77	134.81	147.45	160.48	173.77	187.23
150	117.08	129.56	142.88	156.73	170.91	185.30	199.81

In Table II, we demonstrate the dependence of the bond value on the conversion number and stock price level (with the issuer's call provision excluded). At a low stock price level, the bond value is not quite sensitive to an increase in conversion number. Similarly, the bond value is also insensitive to an increase in stock price when the conversion number is low. Both phenomena are due to the low value of the equity component of the convertible bond. The data also reveal that the delta of the bond value increases with higher conversion number, due to increased weight in the equity component.

With regard to the numerical accuracy of the bond values shown in Table II, we performed the numerical calculations using a varying number of time steps in the trinomial scheme to assess the numerical accuracy of the results. In most cases, the numerical bond values obtained using 20 and 40 time steps per year differ by less than one penny.

Next, we examined the effects of the soft call requirement on the convertible bond value. In the two right columns in Table III, we list

TABLE III
Bond Values Subject to Varying Levels of Trigger Price^a

<i>Trigger Price</i>	<i>Consecutive Counting</i>	<i>Cumulative Counting</i>
130	136.01	135.83
140	136.64	136.08
150	137.89	137.13
160	138.93	138.32
180	140.65	140.30
200	141.81	141.60

^aUnder the rules of consecutive counting and cumulative counting of the number of days of breaching the trigger price.

the bond values with varying levels of trigger price and under different rules of counting the number of days that the stock price rises above the trigger price. In the calculations, the current stock price is taken to be 130 and the annualized dividend yield is set to be 1%. We specify that the issuer can initiate the call only if the stock price stays above the trigger price consecutively or cumulatively for 30 days. For the purpose of comparison, the convertible bond value is found to be equal to 144.17 if there is no call feature and equal to 135.71 if there is no soft protection requirement. These two values serve as the respective upper and lower bounds for the value of the bond subject to the soft call requirement.

We also examined how the bond value is affected by the number of days required for the stock price to breach the trigger price to activate the call provision. In Figure 2, we show the dependence of the convertible bond value on the number of breaching days, according to either

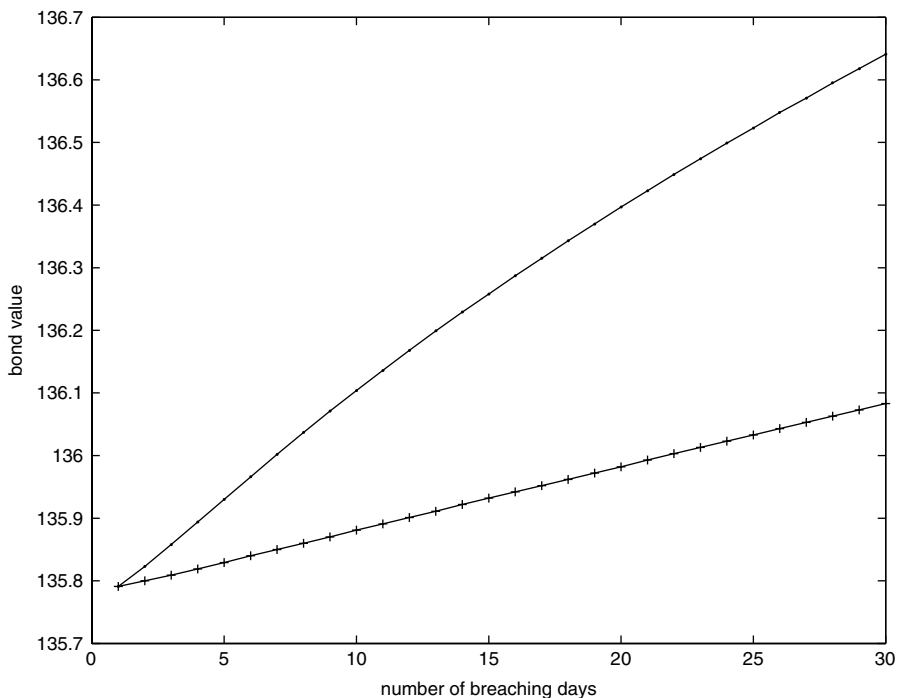


FIGURE 2

The two curves illustrate the dependence of the bond value on the soft call requirement. The calling right is activated only when the stock price stays above the trigger level of 140 either consecutively (corresponds to upper curve with dots '.') or cumulatively (corresponds to the lower curve with crosses '+') for a given number of breaching days.

consecutive or cumulative counting rules. In the calculations, the trigger price is taken to be 140. Since it becomes harder for the issuer to initiate the call when more days of breaching are required, the bond value is an increasing function of the number of breaching days.

By examining the results shown in Table III and Figure 2, the following conclusions can be drawn:

1. The bond value increases with increasing trigger price. This is obvious since it becomes harder for the issuer to initiate the call when calling is constrained by a higher trigger price.
2. The impact of the length of breaching period on the bond value is, in general, not quite significant.
3. The bond value becomes higher when the soft call requirement is more stringent. This is because bondholders have better protection against calling by issuer. Also, this explains why the convertible bond has higher value under the rule of consecutive counting compared to cumulative counting.

OPTIMAL CONVERSION AND CALLING POLICIES

The early termination of a convertible bond may arise from either voluntary conversion by the bondholder or optimal calling by the issuer. Our objectives were to understand how the recovery rate, hazard rate, coupon payments, and dividend yield affect the optimal conversion and calling policies, and to examine the interaction of the conversion and callable features. In particular, we sought to explore the impact of the notice-period requirement on the critical call price.

Optimal Conversion Policies With No Call Feature

First, we examined the optimal conversion policies adopted by the holder when there is no calling right granted to the issuer. Let S_{conv}^* denote the critical stock price at which the holder should optimally exercise the conversion right. The stopping region corresponds to the region $S \geq S_{conv}^*(t)$; and upon conversion, $V = nS$.

We performed sample calculations to reveal the behaviors of $S_{conv}^*(t)$, and the plot of $S_{conv}^*(t)$ against t is shown in Figure 3. The parameter values used in the calculations are: $S = 100$, $\sigma = 30\%$, $q = 3\%$, $T = 2$, $n = 1$ and discrete coupons of cash amount 2 are paid semi-annually; and the other parameter values are listed in Table I. In

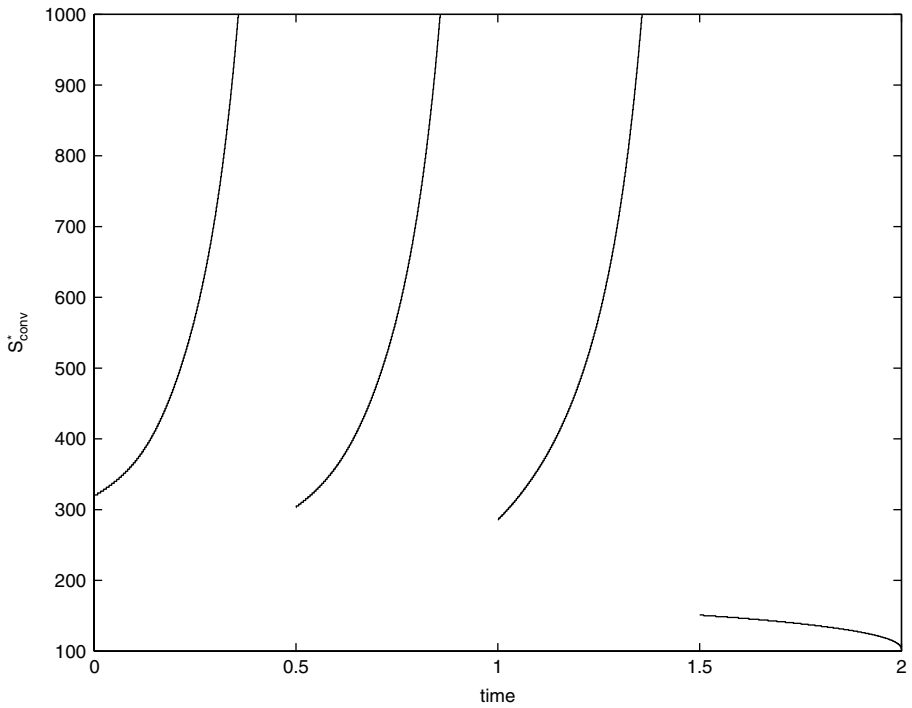


FIGURE 3

Assuming that the issuer cannot call, the curves show the plot of the critical conversion price S_{conv}^* against time. Within the last coupon payment period, S_{conv}^* decreases with time. At times right before a coupon date, S_{conv}^* tends to infinite value.

Figure 3, during the last coupon period (1.5, 2.0), $S_{conv}^*(t)$ is seen to decrease as time is approaching maturity. This is because the chance of regret of early conversion becomes less as time comes closer to maturity. Similar to the optimal exercise policy of American put with discrete dividends, the holder of a convertible bond should restrain from early conversion when a coupon date is approaching. We expect that $S_{conv}^*(t)$ tends to infinite value at times right before a coupon date. Also, we observe that $S_{conv}^*(t)$ increases with t within coupon dates (except during the last coupon period).

Interaction of Optimal Conversion and Calling Policies

We performed sample calculations to reveal the interaction of the optimal conversion and calling policies. The convertible bond is assumed to have a maturity life of 2 years, and the bond cannot be called (hard call

provision) within the first year. After then, the bond can be called by the issuer at the call price 120 (assuming no-notice period requirement). Other parameter values in the model are the same as those used in Figure 3. In Figure 4, we plot the critical stock price against time. During the hard call protection period $(0, 1)$, the premature termination of the convertible bond is only caused by early conversion. We observe that the critical conversion price $S_{conv}^*(t)$ decreases monotonically in time over $(0.5, 1)$. At $t = 1^-$, the instant right before the lifting of the hard call provision, we obtain $S_{conv}^*(1^-) = 122$. Right after the lifting of the hard call provision, the issuer will choose to call optimally at $S_{call}^*(1^+) = 120$. Over the time period $(1, 2)$, the issuer is allowed to call. For most of the time period $(1, 2)$, optimal calling commences at a lower stock price than that of optimal conversion so that the critical stock price is equal to S_{call}^* . We observe that $S_{call}^*(t)$ increases slowly over time due to the effect of accrued interest, and exhibits a drop across a coupon date. In particular, we have $S_{call}^*(1.5^-) = 122$ and $S_{call}^*(1.5^+) = 120$. When the time is approaching maturity, $S_{conv}^*(t)$ may become less than $S_{call}^*(t)$; and the bond is terminated due to voluntary conversion by the holder.

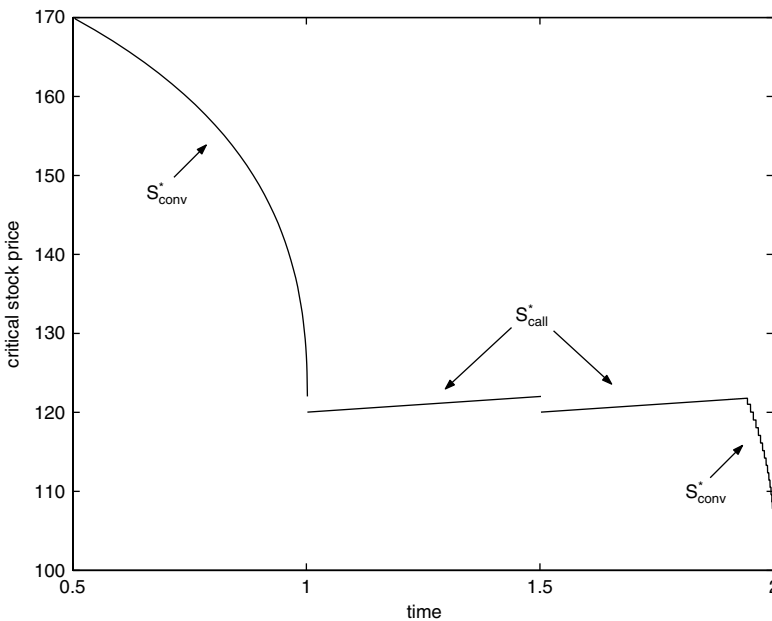


FIGURE 4

We plot the time dependence of the critical stock price. During the hard call protection period $(0, 1)$, the bond is terminated prematurely by early conversion only. The critical conversion price S_{conv}^* decreases over time, and $S_{conv}^*(1^-) = 122$. Over the time period $(1, 2)$, $S_{call}^*(t)$ increases slowly over time and exhibits a drop across a coupon date. At times close to maturity, the bond is terminated due to early conversion.

Notice-Period Requirement

In the earlier theoretical works on optimal calling policies, Ingersoll (1977a; 1977b) and Brennan and Schwartz (1977) claimed that the bond issuer should call the bond whenever the convertible bond value reaches the call price. We demonstrate in our sample calculations that the notice-period requirement may have profound impact on the critical call price, S_{call}^* . This is because the bondholder receives upon calling a more valuable short-lived option (whose maturity date coincides with the ending of the notice period), rather than the cash amount that equals the sum of call price plus accrued interest.

Let X denote the sum of call price plus accrued interest. We compute the time-averaged value of the ratio of S_{call}^* over X with varying length of the notice period, and these average values of the ratio S_{call}^*/X are listed in Table IV. Unless specified otherwise, the parameter values used in the calculations are: $S = 100$, $q = 3\%$, $\sigma = 30\%$, conversion number = 1, call price = 150, coupon rate = 4%, and the remaining parameter values are listed in Table I. Also, there is no hard call protection period. When

TABLE IV
The Impact of the Notice-Period Requirement on the
Theoretical Critical Call Price, S_{call}^* ^a

		<i>Length of Notice Period (Days)</i>			
		0	15	30	45
Volatility	20%	1.006	1.049	1.073	1.093
	30%	1.007	1.061	1.093	1.122
	40%	1.008	1.067	1.101	1.136
Interest rate	2%	1.003	1.043	1.069	1.088
	5%	1.007	1.061	1.093	1.122
	8%	1.010	1.077	1.112	1.145
Coupon rate	1%	1.004	1.106	1.161	1.208
	3%	1.008	1.073	1.110	1.145
	5%	1.006	1.045	1.077	1.102
Call price	120	1.007	1.061	1.093	1.122
	150	1.012	1.090	1.135	1.174
	180	1.015	1.108	1.158	1.199
Hazard rate	0.01	1.008	1.065	1.103	1.135
	0.03	1.006	1.051	1.079	1.108
	0.05	1.004	1.046	1.068	1.086
Recovery rate	0.2	1.010	1.078	1.118	1.150
	0.5	1.009	1.068	1.107	1.135
	0.8	1.007	1.061	1.093	1.122

^aThe time-averaged values of the ratio S_{call}^*/X are obtained under varying length of the notice period and a different set of parameter values.

there is no notice-period requirement, the time-averaged value of the ratio is very close to one. However, the ratio increases quite significantly with increasing length of the notice period. The percentage increase of S_{call}^*/X may range from a few percentages to more than 10%. This provides a partial answer to the “delayed call phenomena.” There may be other corporate finance considerations that lead to delayed call decision by the bond issuer. However, the amount of call delay should be assessed based on a more accurate theoretical S_{call}^* . From Table IV, we also observe that S_{call}^* is an increasing function of volatility, interest rate, and call price, but a decreasing function of coupon rate, hazard rate, and recovery rate.

CONCLUSION

In this paper, we propose a valuation algorithm for pricing one-factor contingent claims models for convertible bonds with credit risk. Compared to earlier algorithms in the literature, our algorithm enables us to pursue more detailed investigation into the interaction of different embedded features that affect the optimal conversion and call policies in convertible bonds. We examine the effects of conversion number, coupons, and soft call requirement on the value of a convertible bond. The time-dependent behaviors of the critical stock price at which the convertible bond should be called by issuer or converted into shares by bondholders are seen to depend sensibly on various features in the bond indenture. In particular, we show that the notice-period requirement and coupon payments have profound impact on the value of the critical stock price.

Our sample calculations reveal that the so called “delayed call phenomena” may be largely attributed to the under estimation of the critical call price at which the issuer should call the bond optimally. A large portion of the “amount of call delay” may be eliminated when more careful contingent claims pricing calculations are performed. There may be other rationales from corporate finance perspectives (say, taxes, asymmetric information) that explain why issuers choose to delay their calls. We recommend that in future empirical studies on assessing the amount of call delay due to corporate finance considerations, the more accurate theoretical critical stock price should be computed.

BIBLIOGRAPHY

- Asquith, P., & Mullins, D. W. Jr. (1991). Convertible debt: Corporate call policy and voluntary conversion. *Journal of Finance*, 46, 1273–1289.
- Ayache, E., Forsyth, P. A., & Vetzal, K. R. (2002, December). Next generation models for convertible bonds with credit risk. *Wilmott Magazine*, 68–77.

- Brennan, M. J., & Schwartz, E. S. (1977). Convertible bonds: Valuation and optimal strategies for call and conversion. *Journal of Finance*, 32, 1699–1715.
- Brennan, M. J., & Schwartz, E. S. (1980). Analyzing convertible bonds. *Journal of Financial and Quantitative Analysis*, 15, 907–929.
- Butler, A. W. (2002, January–February). Revisiting optimal call policy for convertibles. *Financial Analyst Journal*, 58(1), 50–55.
- D'Halluin, Y., Forsyth, P. A., Vetzal, K. R., & Labahn, G. (2001). A numerical PDE approach for pricing callable bonds. *Applied Mathematical Finance*, 8, 49–77.
- Grau, A. H. (2003). Moving windows. Unpublished manuscript, University of Waterloo, Canada.
- Harris, M., & Raviv, A. (1985). A sequential signaling model of convertible debt call policy. *Journal of Finance*, 40, 1263–1281.
- Ingersoll, J. E. (1977a). A contingent claims valuation of convertible securities. *Journal of Financial Economics*, 4, 289–322.
- Ingersoll, J. E. (1977b). An examination of corporate call policies on convertible securities. *Journal of Finance*, 32, 463–478.
- Jaffee, D., & Shleifer, A. (1990). Costs of financial distress, delayed calls of convertible bonds, and the role of investment banks. *Journal of Business*, 63, 107–123.
- Kwok, Y. K., & Lau, K. W. (2001). Pricing algorithms for options with exotic path dependence. *Journal of Derivatives*, 9, 28–38.
- Kwok, Y. K., & Wu, L. (2000). Effects of callable feature on early exercise feature. *Review of Derivatives Research*, 4, 189–211.
- Nyborg, K. G. (1996). The use and pricing of convertible bonds. *Applied Mathematical Finance*, 3, 167–190.
- Takahashi, A., Kobayashi, T., & Nakagawa, N. (2001, December). Pricing convertible bonds with default risk. *Journal of Fixed Income*, 11, 20–29.
- Tsiveriotis, K., & Fernandes, C. (1998, September). Valuing convertible bonds with credit risk. *Journal of Fixed Income*, 8, 95–102.