

Gatheral's homework 3: *solution by Andrew Flint*

First, congratulations on discovering a potential new profit stream for the firm. Second, I'll explain why the competition is pricing these things all wrong and how we're going to make money from it.

As you know, the (quarterly) digital cliquet product is equivalent to a series of at-the-money short-dated European digital calls. The payout at the first reset period of the cliquet is clearly equivalent to that of a 3-month at-the-money digital. And at each reset, the subsequent strike is set at the new spot. Since the index has no drift (beyond the risk-free rate), our expectation is that each forward digital will be struck at today's spot level.

Very simply stated, the price of the cliquet is the basket price of today's at-the-money digital, over and over again, at each reset date out to the cliquet expiration, and then just adjusted for discounting.

Now, as you know, the price of even a single digital call is quite sensitive to the implied volatility skew. If anyone in this market was naive enough to think that it was a Black-Scholes world, and that skew was flat, they'd long ago had their pockets emptied in the digital options market. (In fact, they'd have been wiped out in the vanilla market long before that!)

So everyone knows there is vol skew. Our competitors know it, and we know it. That said, we probably do agree on the price of short-dated digitals.

What makes our cliquet price right (and the competition's wrong) is the price of those forward digitals at the middle and tail end of the cliquet. Using their easy-to-use (but wrong) local volatility models, they implicitly get a flattening of vol skew way out in the future. It's as if they believed the future was a Black-Scholes world. Of course, it is not.

Our stochastic vol methods maintain a future with definite vol skew, and hence our far-out digitals are (appropriately) more expensive, which makes our cliquet more expensive.

Now for the best part. Your awareness of the market can make us all some easy money. Next time you find us losing a deal for a digital cliquet, you call me. We'll buy what we know to be underpriced, and our guys can hedge them back to lock in the profit. As for our clients buying those unrealistically cheap cliquets, you'll just tell them they're getting a great deal.