

Case Study on Malliavin Calculus, HW due to Oct-07

1. Assume log-normal dynamics as in Chapter 4 of my notes. Consider a *range option* which pays \$1 if

$$S_T \in [K_{\min}, K_{\max}]$$

at maturity. Implement a Monte Carlo simulation to compute Delta with (a) a finite difference, (b) with formula (18) in my notes. Use the following Black & Scholes parameters

$$S_0 = 100, T = 1 \text{ year}, \sigma = 20\%, r = 5\%, K_{\min} = 105, K_{\max} = 110.$$

To see how Malliavin Calculus helps the Monte Carlo method to converge faster, plot delta against the number of paths used, both for (a) and (b).

2. Do the same thing for a standard European Option struck at $K = 105$.
3. Conclusions? Without doing simulations, would a Malliavin-type formula as (19) help with the Gamma for a standard digital? What about the Delta for a square-call with payoff

$$(\max(S_T - K, 0))^2 \quad ?$$