As usual, assume zero interest rates and dividends for the following assignments. Further assume a hazard rate $\lambda = 0.01$ and volatility $\sigma = 0.2$.

Pricing European options in Merton's model of default

Show by referring to equation (51) that the value of a European option with strike price K and expiration T is given by

$$e^{-\lambda T}\, C_{BS}(S\, e^{\lambda T}, K, \sigma, T)$$

where C_{BS} is the usual Black-Scholes formula.

Price one year European options with strikes 0.8, 0.9, 1.0, 1.1 and 1.2 with this formula.

Local volatility in Merton's model of default

Using the Dupire formula (5) from Lecture 1 and equation (51) for the Merton model of default, derive the following formula for the local volatility in that model:

$$\sigma^2_{loc}(k,t) = \sigma^2 - 2\lambda\, \sigma \sqrt{t}\, \frac{N(d_2)}{N'(d_2)}$$

where $N(.)$ is the cumulative Normal distribution function, $k = \log(K/S)$ and

$$d_2 = \frac{-k + \lambda t}{\sigma\sqrt{t}} - \frac{\sigma\sqrt{t}}{2}$$

Local Volatility Monte Carlo

Amend the official Monte Carlo code posted on the web and use the local variance formula you just derived to compute the same set of five one-year European option prices as above.

Jump-Diffusion Monte Carlo

Amend the official Monte Carlo code posted on the web to price European options under jump-diffusion with a jump to zero (the so-called *jump to ruin*). The parameters as as before: $\lambda = 0.01$ and volatility $\sigma = 0.2$. Verify that you get the same five one year European option prices back.

The baseball trade

Finally, amend your new local volatility and jump-diffusion Monte Carlo codes to price a one year baseball trade under both local volatility and jump-diffusion assumptions. With an initial spot of 100 and an initial range of $95 - 105$, assume that the range is always reset to be a symmetric $\pm 5\%$ around the stock price at the reset time.

Intuition

Explain the intuition behind your results in a form suitable to be presented to a non-technical manager.