Case Studies in Financial Modelling: Jim Gatheral Homework 1

Implied Volatility Calculator

Implement an implied volatility calculator. To simplify the problem, assume zero interest rates and dividends. Then, given the stock price, strike, expiration in years and option price, your calculator should return the Black-Scholes implied volatility of a European option.

Closed-Form Heston Solution

Write a program that implements the Heston formula. I suggest implementing the detailed computation exactly as laid out in the course notes.

I recommend that you do this exercise using Matlab, Mathematica, Maple or similar symbolic manipulation program to keep implementation problems to a minimum. If you choose to code in C, the numerical integration can be carried out using a standard numerical integration package although some work might be needed to deal with complex numbers. On the other hand, software like Matlab or Maple will do the numerical integration for you.

Using the implied volatility calculator you implemented earlier, plot the implied volatility of one year European options with strikes from 0.8 to 1.2 assuming a stock price of 1.0 and the following Heston parameters (from Bakshi, Cao and Chen (BCC)):

$$\begin{aligned} v &= 0.04 \\ \bar{v} &= 0.04 \\ \lambda &= 1.15 \\ \eta &= 0.39 \\ \rho &= -0.64 \end{aligned}$$

Heston Monte Carlo solution

Write a 2-dimensional Monte Carlo routine (in C probably) to price European options with Heston model assumptions. In order to solve this problem, you will need to specify what happens if the discretized variance process becomes negative.

Try both the absorbing assumption: if $v < 0$ then $v = 0$ and the reflecting assumption: if $v < 0$ then $v = -v$.

Once again, plot the implied volatility of one year European options with strikes from 0.8 to 1.2 using the BCC parameters.

Examine the behaviour of your Monte Carlo solution as you increase the number of time steps and compare with results obtained using the closed-form Heston solution.

Benefits of closed-form solutions

Based on your findings, what do you think are the benefits of closed-form solutions if any? What are the limitations of closed-form solutions?