Case Studies in Financial Modelling: Jim Gatheral Homework 2

A Special Case of Heston

Following the derivation in the notes, we see that if $\rho = \pm 1$, the formula I presented for local variance should be pretty good (modulo some ansatz-related error). We will now show this numerically by computing one year European options using the parameters:

$$\begin{aligned} v &= 0.04 \\ \bar{v} &= 0.04 \\ \lambda &= 10 \\ \eta &= 1 \\ \rho &= -1 \end{aligned}$$

Specifically, your assignment is to amend the official Monte Carlo code posted on the web to price these one year options under both models and then to use the official implied volatility calculator to graph implied volatilities of one year European options with strikes from 0.8 to 1.2. Note that, because $\rho = -1$, both Monte Carlo computations are only one-factor.

From the notes, the local variance should be given by

$$\begin{aligned} v_{loc}(x_T, T) &= \hat{v}'_T - \eta \frac{x_T}{w_T} \int_0^T \hat{v}_s \, e^{-\lambda'(T-s)} ds \\ &= (v - \bar{v}')e^{-\lambda' T} + \bar{v}' - \eta \, x_T \left\{ \frac{1 - e^{-\lambda' T}}{\lambda' T} \right\} \end{aligned}$$

with $\lambda' = \lambda + \frac{\eta}{2}$, $\bar{v}' = \bar{v} \frac{\lambda}{\lambda'}$. The whole expression is bounded below by zero – all stock prices above the critical stock price at which the local variance reaches zero are unattainable.

Intuition

From the results of your computation, you can see that the local volatility model and the stochastic volatility model price one year European options almost identically. Also both models are single-factor, depending only on stock prices. Are there any differences between the two models? If so, what are these differences?