

ap_fixedasian_zhang

Output parameters:

- Price
- Delta

Description: By a well chosen change of variables, Zhang [1] obtains that the price at time t of the arithmetic average asian call option with fixed strike K , maturity T , spot $S = S_t$ and integral $I = \int_0^t S_u$ is given by $e^{-\delta\tau} \frac{S}{T} f(\xi, \tau)$ where

$$\begin{cases} \tau = T - t \\ \xi = \frac{TK - I}{S} e^{-(r-\delta)\tau} - \frac{(1 - e^{-(r-\delta)\tau})}{r - \delta} \end{cases}$$

and the function f satisfies the following PDE :

$$\begin{cases} \frac{\partial f}{\partial \tau} - \frac{\sigma^2}{2} \left(\xi + \frac{1 - e^{-(r-\delta)\tau}}{r - \delta} \right)^2 \frac{\partial^2 f}{\partial \xi^2} = 0 \quad \forall (\tau, \xi) \in [0, T] \times \mathbb{R} \\ \text{with the initial condition } f(\xi, 0) = (-\xi)^+. \end{cases}$$

The analytical approximation is based on the explicit solution f_0 of the same PDE but with diffusion coefficient $\frac{\sigma^2}{2} \left(\frac{1 - e^{-(r-\delta)\tau}}{r - \delta} \right)^2$ independent of ξ :

$$f_0(\eta, \xi) = -\xi N \left(-\frac{\xi}{\sqrt{2\eta}} \right) + \sqrt{\frac{\eta}{\pi}} e^{-\frac{\xi^2}{4\eta}}$$

where $\eta = \frac{\sigma^2}{4(r-\delta)^3} (-3 + 2(r-\delta) + 4e^{-(r-\delta)\tau} - e^{-2(r-\delta)\tau})$. The corresponding approximations for the price and the delta of the Asian Option are :

$$\begin{aligned} C_0(S, I, t) &= e^{-\delta\tau} \frac{S}{T} \left(-\xi N \left(-\frac{\xi}{\sqrt{2\eta}} \right) + \sqrt{\frac{\eta}{\pi}} e^{-\frac{\xi^2}{4\eta}} \right) \\ \Delta_0 &= e^{-\delta\tau} \left(\frac{1 - e^{-(r-\delta)\tau}}{(r - \delta)T} N \left(-\frac{\xi}{\sqrt{2\eta}} \right) + \frac{1}{T} \sqrt{\frac{\eta}{\pi}} e^{-\frac{\xi^2}{4\eta}} \right). \end{aligned}$$

The PDE satisfied by the correction term $f_1(\xi, \tau) = f(\xi, \tau) - f_0(\xi, \tau)$

$$\begin{cases} \frac{\partial f_1}{\partial \tau} - c(\xi, \tau) \frac{\partial^2 f_1}{\partial \xi^2} = R(\xi, \tau) \\ f_1(\xi, \tau = 0) = 0 \end{cases} \quad (1)$$

where

$$c(\xi, \tau) = \frac{\sigma^2}{2} \left(\xi + \frac{1 - e^{-(r-\delta)\tau}}{r - \delta} \right)^2 \quad \text{and} \quad R(\xi, \tau) = \frac{\sigma^2 \xi}{4\sqrt{\pi\eta}} \left(\xi + \frac{2}{r - \delta} \left(1 - e^{(r-\delta)\tau} \right) \right)$$

is solved by the finite differences method.

*/*Computation of the price*/*

computes the approximate price and the corrected price.

*/*Computation of the delta*/*

computes the approximate delta and the corrected delta.

*/*Diffusion coefficient C(t,x)*/*

computes the diffusion coefficient $C(t, x)$ in (1).

*/*right-hand-side R(t,x)*/*

computes the right-hand-side $R(t, x)$ in (1).

*/*Computation of f_0 */* */*derivative of f_0 w.r.t. S */*

compute f_0 and its derivative w.r.t. S .

*/*Tridiagonal matrix*/*

computes the tridiagonal matrix involved in the finite differences scheme for (1).

*/*resolution of the system*/*

At each time-step a system is solved by Gauss method.

*/*correction delta*/*

computes the correction of the delta.

References

- [1] J.E.ZHANG. A semy-analytical method for pricing and hedging continuously-sampled arithmetic average rate options. *preprint*, September 2000. **1**