cf_call_heston

This model is given by, $\mathrm{dS}_t = rS_t dt + \sqrt{v_t} S_t dW_t^1$,
$dv_t = k(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2$, where W^1 and W^2 are two correlated brownian motions with $\langle W^1, W^2\rangle_t = \rho t$, and k, θ and σ are constants. Discretizing with an *Euler scheme* leads to $\mathrm{S}_{T_{i+1}} = S_{T_i}(1 + r\Delta t + \sqrt{\sigma_i \Delta t} Z_i)$,
$v_{T_{i+1}} = v_{T_i} + k(\theta - v_{T_i})\Delta t + \sigma\sqrt{\Delta t v_{T_i}}(\rho Z_i + \sqrt{1-\rho^2} Z_{m+i})$, where $(Z_i)_{i\geq 1}$ is a sequence of independent Gaussian variables with mean 0 and variance 1. In our implementations we have taken the stochastic input to model to be the single vector $(Z_1, \ldots, Z_{2m})$. In some respects, it might be more natural to think of two separate vectors, each of length m.
In the case of a European call option, Heston guessed a solution of the form

$$C(S, v, t) = SP_1 - Ke^{-r(T-t)}P_2,$$

by analogy with the Black-Scholes formula. The first term in this formula is the present value of the spot price while the second term is the present value of the strike-price payment.
Using this model, Heston has given a closed form solution to the pricing of a European call option by the caracteristic functions technique. For more details one can see [1]

References

[1] S.L.HESTON. A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2):327–343, 1993. 1