cf_fixed_putlookback

Let

- T = maturity date $\quad (T > t)$
- K = strike price
- x = spot price
- m = current minimum $\mathrm{m_{0,t}}$
- t = pricing date
- σ = volatility
- r = interest rate
- δ = dividend yields
- $\theta = T - t$
- $b = r - \delta$

Floating strike lookback options can be priced using Goldman-Sosin-Gatto formula [1] while fixed strike lookback options can be priced using Conze-Viswanathen formula[2].

We set, as $0 \leq u \leq v \leq T$,

$$M_{u,v} = \sup_{u \leq \tau \leq v} S_\tau \qquad \text{and} \qquad m_{u,v} = \inf_{u \leq \tau \leq v} S_\tau$$

and

- $d_1 = \dfrac{\log\left(\frac{x}{K}\right) + \left(b + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \qquad d_2 = d_1 - \sigma\sqrt{\theta}$
- $e_1 = \dfrac{\log\left(\frac{x}{M_{0,t}}\right) + \left(b + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \qquad e_2 = e_1 - \sigma\sqrt{\theta}$
- $f_1 = \dfrac{\log\left(\frac{x}{M_{0,t}}\right) + \left(b + \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \qquad f_2 = f_1 - \sigma\sqrt{\theta}$

Fixed Lookback Put Option

Payoff $\quad P_T = (K - m_{t,T})_+$

Both price and delta depend on K and $m_{0,t}$.

- if $K < m_{0,t}$ then

Price
$$P(t,x) = Ke^{-r\theta}N(-d_2) - xe^{-\delta\theta}N(-d_1)$$
$$+xe^{-r\theta}\frac{\sigma^2}{2b}\left[\left(\frac{x}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(-d_1 + \frac{2b}{\sigma}\sqrt{\theta}\right) - e^{b\theta}N(-d_1)\right]$$

Delta
$$\frac{\partial P(t,x)}{\partial x} = e^{-\delta\theta}N(d_1)\left(1 + \frac{\sigma^2}{2b}\right) + e^{-\delta\theta}\frac{n(d_1)}{\sigma\sqrt{\theta}} - e^{-r\theta}\frac{K}{x}\frac{n(d_2)}{\sigma\sqrt{\theta}}$$
$$+e^{-r\theta}\left(\frac{x}{K}\right)^{-\frac{2b}{\sigma^2}} N\left(-d_1 + \tfrac{2b}{\sigma}\sqrt{t}\right)\left(\tfrac{\sigma^2}{2b} - 1\right) - e^{-\delta\theta}\left(\frac{\sigma^2}{2b} - 1\right)$$

- if $K \geq m_{0,t}$ then

Price
$$P(t,x) = e^{-r\theta}(K - m_{0,t}) - xe^{-\delta\theta}N(-f_1) + m_{0,t}e^{-r\theta}N(-f_2)$$
$$+xe^{-r\theta}\frac{\sigma^2}{2b}\left[\left(\frac{x}{m_{0,t}}\right)^{-\frac{2b}{\sigma^2}} N\left(-f_1 + \frac{2b}{\sigma}\sqrt{\theta}\right) - e^{b\theta}N(-f_1)\right]$$

Delta
$$\frac{\partial P(t,x)}{\partial x} = e^{-\delta\theta}\left(1 + \frac{\sigma^2}{2b}\right)(N(f_1) - 1) + e^{-\delta\theta}\frac{n(f_1)}{\sigma\sqrt{\theta}} - e^{-r\theta}\frac{M_{0,t}}{x}\frac{n(d_2)}{\sigma\sqrt{\theta}}$$
$$+e^{-r\theta}\left(\frac{x}{M_{0,t}}\right)^{-\frac{2b}{\sigma^2}} N\left(-f_1 + \tfrac{2b}{\sigma}\sqrt{t}\right)\left(\frac{\sigma^2}{2b} - 1\right)$$

References

[1] B.M.GOLDMAN H.B.SOSIN M.A.GATTO. Path dependent options: buy at low, sell at high. *J. of Finance*, 34:111–127, 1979. 1

[2] A.CONZE R.VISWANATHAN. Path dependent options: the case of lookback options. *J. of Finance*, 46:1893–1907, 1992. 1