cf_putout_kunitomoikeda

Let

- T = maturity date $\quad (T > t)$
- K = strike price
- L = lower barrier
- U = upper barrier
- x = spot price
- t = pricing date
- σ = volatility
- r = interest rate
- δ = dividend yields
- $\theta = T - t$
- $b = r - \delta$

The exact value for double barrier call/put options is given by the Ikeda-Kunitomo formula [1], which allows to compute exactly the price when the boundaries suitably depend on the time variable t. More precisely, set

$$U(s) = Ue^{\delta_1 s} \qquad L(s) = Le^{\delta_2 s}$$

where the constants $U, L, \delta_1, \delta_2 \in$ are such that $L(s) < U(s)$, for every $s \in [t, T]$. The functions $U(s)$ and $L(s)$ play the role of *upper* and *lower* barrier respectively. δ_1 and δ_2 determine the curvature and the case of $\delta_1 = 0$ and $\delta_2 = 0$ corresponds to two flat boundaries.

In the software, we consider only flat boundaries.

The numerical studies suggest that in most cases it suffices to calculate the leading five terms of the series giving the price of the knock-out and knock-in double barrier call options.

Let τ stand for the first time at which the underlying asset price S reaches at least one barrier, i.e.

$$\tau = \inf\{s > t\,;\, S_s \leq L(s) \text{ or } S_s \geq U(s)\}.$$

We define the following coefficients:

- $$\mu_1 = 2\frac{b - \delta_2 - n(\delta_1 - \delta_2)}{\sigma^2} + 1$$
- $$\mu_2 = 2\frac{n(\delta_1 - \delta_2)}{\sigma^2}$$
- $$\mu_3 = 2\frac{b - \delta_2 + n(\delta_1 - \delta_2)}{\sigma^2} + 1$$

Knock-Out Put Option

Payoff $$P_T = \begin{cases} (K - S_T)_+ & \text{if} \quad \tau > T \\ 0 & \text{otherwise} \end{cases}$$

Price $$\begin{aligned} P(t,x) = Ke^{-r\theta} \sum_{n=-\infty}^{+\infty} & \left[\left(\frac{U^n}{L^n}\right)^{\mu_1 - 2} \left(\frac{L}{x}\right)^{\mu_2} \Big(N(y_1^-) - N(y_2^-)\Big) \right. \\ & \left. - \left(\frac{L^{n+1}}{xU^n}\right)^{\mu_3 - 2} \Big(N(y_3^-) - N(y_4^-)\Big) \right] \\ -xe^{-\delta\theta} \sum_{n=-\infty}^{+\infty} & \left[\left(\frac{U^n}{L^n}\right)^{\mu_1} \left(\frac{L}{x}\right)^{\mu_2} \Big(N(y_1^+) - N(y_2^+)\Big) \right. \\ & \left. - \left(\frac{L^{n+1}}{xU^n}\right)^{\mu_3} \Big(N(y_3^+) - N(y_4^+)\Big) \right] \end{aligned}$$

where $E = Le^{\delta_2\theta}$ and

$$y_1^\pm = \frac{\log(xU^{2n}/EL^{2n}) + \left(b \pm \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \qquad y_2^\pm = \frac{\log(xU^{2n}/KL^{2n}) + \left(b \pm \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}$$

$$y_3^\pm = \frac{\log(L^{2n+2}/ExU^{2n}) + \left(b \pm \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}} \qquad y_4^\pm = \frac{\log(L^{2n+2}/KxU^{2n}) + \left(b \pm \frac{\sigma^2}{2}\right)\theta}{\sigma\sqrt{\theta}}$$

References

[1] N.KUNIMOTO N.IKEDA. Pricing options with curved boundaries. *Mathematical finance*, 2:275–298, 1992. 1