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fd_multigrid_euro

Input parameters:

- Number of grids l
- TimeStepNumber M

Output parameters:

- Price
- Delta1
- Delta2

We have to solve the heat equation in 2D after change of variables. We use multigrid method. We refer to Hackbusch [1] for a detailed presentation of multigrid methods.

/*SpaceStepNumber N*/
$N = nn(l) + 1$ where $nn(l)$ calculates the number of points in each direction in the grid of level l.

/*Memory Allocation*/

/*Covariance Matrix*/

/*Space localisation/*
Define the integration domain $D = [-limit, limit]^2$ using probabilistic estimation.

/*Space Step/*

Define the space step $h = \dfrac{2 * limit}{N}$.

/*Time Step*/

Define the time step $k = \dfrac{T}{M}$.

/*Terminal Values/*

Put the value of the payoff into a vector P.

/*Homegenous Dirichlet Conditions/*

/*Finite difference Cycle/*

At any time step, we have to solve the linear discrete problem which can be written in the form

$$L^l u^l = f^l. \tag{1}$$

/*Init Rhs*/

/*Multigrid Method*/

We solve the linear discrete problem using the Multigrid method. The multigrid iteration (V-cycle) at level l for solving 1 is defined by the following recursive procedure:

$$v_l \leftarrow MGM(l, v^l, f^l)$$

Step 1 **/*Factor of scheme*/**
Initialize the matrix L^l issued from the discretization of the operator A in the case of Dirichlet Boundary conditions.
Relax 2 times on $L^l u^l = f^l$ with a given initial guess v^l.

Step 2 if Ω^l is the coarsest grid (l=0) then go to step 4.
Else

$$\begin{aligned} &f^{l-1} \leftarrow I_l^{l-1}(f^l - L^l v^l). \\ &v^{l-1} \leftarrow 0. \\ &v^{l-1} \leftarrow MGM(l-1, v^{l-1}, f^{l-1}) \end{aligned}$$

Step 3 Correct $v^l \leftarrow v^l + I_{l-1}^l v^{l-1}$.

Step 4 Relax 2 times on $L^l u^l = f^l$ with initial guess v^l

where I_{l-1}^l is the linear interpolation operator and I_l^{l-1} the restriction operator. I_{l-1}^l is defined by the rule $I_{l-1}^l v^{l-1} = v^l$ where

$$
\begin{aligned}
&v_{2i,2j}^l = v_{i,j}^{l-1}, \\
&v_{2i+1,2j+1}^l = \tfrac{1}{2}(v_{i,j}^{l-1} + v_{i+1,j}^{l-1}), \\
&v_{2i,2j+1}^l = \tfrac{1}{2}(v_{i,j}^{l-1} + v_{i,j+1}^l), \\
&v_{2i+1,2j+1}^l = \tfrac{1}{4}(v_{i,j}^{l-1} + v_{i+1,j}^{l-1} + v_{i,j+1}^{l-1} + v_{i+1,j+1}^{l-1}), 0 \le i,j \le \tfrac{n}{2} - 1.
\end{aligned}
$$

The restriction operator is defined by $I_l^{l-1} v^l = v^{l-1}$, where

$$v_{i,j}^{l-1} = v_{2i,2j}^l.$$

/*Price*/

/*Delta*/

/*Memory Desallocation*/

References

[1] Hackbusch, W.: Multi-grid methods and applications. Springer-Verlag. (1985) 1