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mc_antithetic

Input parameters:

- Number of iterations N
- Generator_Type
- Increment inc
- Confidence Value

Output parameters:

- Price P
- Error Price σ_P
- Delta δ
- Error delta σ_δ
- Price Confidence Interval: $IC_P = [\text{Inf Price}, \text{Sup Price}]$
- Delta Confidence Interval: $IC_\delta = [\text{Inf Delta}, \text{Sup Delta}]$

Description:

Computation for a Call - Put - CallSpread or Digit European Option of its Price and its Delta with the [Antithetic Variables](#) method with [Monte Carlo](#) or [Quasi-Monte Carlo](#) simulation. In the case of Monte Carlo simulation, the method also provides an estimation for the integration error and a confidence interval.

The underlying asset price evolves according to the Black and Scholes model, that is:

$$dS_u = S_u((r - d)du + \sigma dB_u), \quad S_{T-t} = s$$

then

$$S_T = s \exp \left(\left(r - d - \frac{\sigma^2}{2} \right) t \right) \exp(\sigma B_t)$$

where S_T denotes the spot at maturity T , s is the initial spot, t is the time to maturity.

The Price of an option at $T - t$ is:

$$P = E [\exp(-rt) f(K, S_T, R)]$$

where f denotes the payoff of the option, K the strike and R the rebate (for Digit option only).

The Delta is given by:

$$\delta = \frac{\partial}{\partial s} E [\exp(-rt) f(K, S_T, R)]$$

Estimators with antithetic variables are expressed as:

$$\tilde{P} = \frac{1}{2N} \exp(-rt) \sum_{i=1}^N (P(i) + P'(i))$$

$$\tilde{\delta} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{\partial}{\partial s} P(i) = \frac{1}{2N} \exp(-rt) \sum_{i=1}^N (\delta(i) + \delta'(i))$$

$P(i)$ and $P'(i)$ are antithetic payoffs obtained from $S_T(i)$ and $S'_T(i)$ given by:

$$\begin{cases} S_T(i) = s \exp \left[\left(r - d - \frac{\sigma^2}{2} \right) t \right] \exp(\sigma B_t(i)) \\ S'_T(i) = s \exp \left[\left(r - d - \frac{\sigma^2}{2} \right) t \right] \exp(\sigma B'_t(i)) \end{cases}$$

where $B_t(i) = \sqrt{t}g_i$ and $B'_t(i) = -\sqrt{t}g_i$ are antithetic brownian motions. g_i is a standard gaussian variable.

Values for $P(i)$ and $\delta(i)$ are detailed for each option in the following points.

Values for $P'(i)$ and $\delta'(i)$ are obtained in the same way.

- **Put:** The payoff is $(K - S_T)^+$. We have:

$$P(i) = (K - S_T(i))^+$$

$$\delta(i) = \begin{cases} -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if } P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **Call:** The payoff is $(S_T - K)^+$.

The *Call-Put Parity* relations for price and delta are expressed by:

$$C = P + s \exp(-dt) - K \exp(-rt)$$

$$\delta_C = \delta_P + \exp(-dt)$$

where C and P respectively denotes the Call and the Put prices.

They will be used for a Call simulation (which corresponds to a method of control variate and leads generally to a reduced variance for the estimator).

- **CallSpread:** The payoff is $(S_T - K_1)^+ - (S_T - K_2)^+$.

We have:

$$P(i) = [(S_T(i) - K_1)^+ - (S_T(i) - K_2)^+]$$

$$\delta(i) = \begin{cases} \frac{\partial S_T(i)}{\partial s} = \frac{S_T(i)}{s} & \text{if } S_T(i) > K_1 \text{ and } S_T(i) < K_2 \\ -\frac{\partial S_T(i)}{\partial s} = -\frac{S_T(i)}{s} & \text{if } S_T(i) > K_2 \text{ and } S_T(i) < K_1 \\ 0 & \text{otherwise} \end{cases}$$

- **Digit:** The payoff is $R1_{\{S_T - K \geq 0\}}$.

We have:

$$P(i) = R1_{\{S_T(i) - K \geq 0\}}$$

To have an estimation of the Delta in the case of a Digit option, we need to use the increment value *inc* at each iteration i as:

$$\delta_i = \begin{cases} \frac{R}{2s \cdot inc} & \text{if } S_T(i)(s(1 + inc)) > K \text{ and } S_T(i)(s(1 - inc)) < K \\ 0 & \text{otherwise} \end{cases}$$

$S_T(i)(s(1 + inc))$ is the spot value at T with initial value $s(1 + inc)$.

$S_T(i)(s(1 + inc))$ and $S_T(i)(s(1 - inc))$ are computed with the same brownian motion for each iteration. Thus we always have $S_T(i)(s(1 + inc)) > S_T(i)(s(1 - inc))$ and there is only one case for which $\delta_i > 0$.

Algorithm:

```
/* Value to construct the confidence interval */
For example if the confidence value is equal to 95% then the value  $z_\alpha$  used
to construct the confidence interval is 1.96. This parameter is taken into
account only for MC simulation and not for QMC simulation.
/*Initialization*/
/*Call-Spread*/
```

Strike K_1 and K_2 used for a Call-Spread option.

/*Median forward stock and delta values*/

Computation of intermediate values we use several times in the program.

/* Change a Call into a Put to apply the Call-Put parity */

In case of Call, we modify parameters of the option; they will be reinitialized at the end of the simulation program. Simulation will be done as for a put.

• /*MC sampling*/

Initialization of the simulation: generator type, dimension, size N of the sample

/* Test after initialization for the generator */

Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences). For standard Monte Carlo in the one-dimensional Black and Scholes model, we never have any problem with the dimension, fixed to 1 at the beginning of the program.

Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm: for simulation of a gaussian variable and in results in the simulation.

/* Begin N iterations */

- /* Simulation of a gaussian variable according to the generator type, that is Monte Carlo or Quasi Monte Carlo. */

Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method.

- /*Price*/

At the iteration i , we obtain $S_T(i)$ and $S'_T(i)$ from a simulation according to the selected generator of antithetics brownian motions $B_t(i)$ as $\sqrt{t}g_i$ and $B'_t(i)$ as $-\sqrt{t}g_i$ where g_i is a standard gaussian variable.

$$P(i) = \text{Payoff}(S_T(i), K)$$

$$P'(i) = \text{Payoff}(S'_T(i), K)$$

- /*Delta*/

Calculation of Delta δ_i and δ'_i with formula previously detailed for each option.

```
/*Digit*/
/*CallSpread*/
/*Call-Put*/
```

```
/*Sum*/
```

Computation of the sums $\sum \frac{1}{2}(P(i) + P'(i))$ and $\sum \frac{1}{2}(\delta(i) + \delta'(i))$ for the mean price and the mean delta.

```
/*Sum of squares*/
```

Computation of the sums $\sum (P(i) + P'(i))^2$ and $\sum (\delta(i) + \delta'(i))^2$ necessary for the variance price and the variance delta estimations. (finally only used for MC estimation)

```
/* End N iterations */
```

```
• /*Price*/
```

The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{1}{2} (P(i) + P'(i))$$

The error estimator is σ_P with :

$$\sigma_P^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^N \frac{1}{4} (P(i) + P'(i))^2 - P^2 \right)$$

```
• /*Delta*/
```

$$\delta = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{1}{2} (\delta(i) + \delta'(i))$$

The error estimator is σ_δ with:

$$\sigma_\delta^2 = \frac{1}{N-1} \left(\frac{1}{N} \exp(-2rt) \sum_{i=1}^N \frac{1}{4} (\delta(i) + \delta'(i))^2 - \delta^2 \right)$$

```
• /* Call Price and Delta with the Call Put Parity */
```

We now compute the price and the delta in case of a call, because call was considered as a put until now.

Parameters of the option are reinitialized. This step is necessary: if you want to begin an other call simulation just after a first one with the antithetic

method, the parameters must have been modified to specify that we really consider a Call and not a Put.

- /* Price Confidence Interval */

The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with z_α computed from the confidence value.

- /* Delta Confidence Interval */

The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta; \delta + z_\alpha \sigma_\delta]$$

with z_α computed from the confidence value.

Confidence intervals are always computed, but for a QMC simulation they don't work, thus they don't appear in the results.

References