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Input parameters:

- StepNumber N

Output parameters:

- Price
- Delta

This is the archetype of a Tree routine. It is described in [1]. The dynamic of the underlying under the risk-neutral probability in the BS1D model:

$$S_t = S_0 e^{(r-\delta)t+\sigma W_t}$$

is replaced by the following: for $i = 0, 1, \ldots, N-1$

$$S_{(i+1)} = S_i e^{\epsilon\sigma\sqrt{h}}$$

where $h = \frac{T}{N}$, T is the maturity of the option, $\epsilon = 1$ with probability $p_\star$, $\epsilon = -1$ with probability $1-p_\star$ where the (conditional) risk-neutral probability $p_\star$ is chosen so that

$$E\left[e^{-(r-\delta)h} S_{(i+1)} I S_i\right] = S_i \tag{1}$$

where δ is the dividend rate.
The price of the european option P satisfies

$$E\left[e^{-rh} P_{(i+1)} I P_i\right] = P_i \tag{2}$$

whereas the price of the american option is given in any state j at time i by

$$P_{(i+1),j} = \max\left(\phi(x), E\left[e^{-rh} P_{(i+1)} I P_{ij}\right]\right) \tag{3}$$

where ϕ is the payoff of the option.
The algorithm is a backward computation of the option price based on (2)

or (3 on the page before), after a forward computation of the possible values of the underlying at maturity $S_N j$ for $j = 0, 1, \ldots, N$. Since the tree is a flat tree, it is easily seen that the value of the underlying at time i and level j (starting from below) is the same as that at time $i+2$ and level $j+1$. In particular there are only $2N+1$ possible values of the underlying between time 0 and time N. For computational purpose it is clever in the american case to compute only once the corresponding value of the intrinsic value of the option, at the beginning of the algorithm. The routine looks as follows (cf Source):

/*Memory Allocation: Price, Intrinsic Value arrays*/

/*Up and Down factors*/
Here $u = e^{\sigma\sqrt{h}}$,$d = e^{-\sigma\sqrt{h}}$.

/*Risk-Neutral Probability*/
Computation of $p_\star = \frac{e^{(r-\delta)h} - d}{u-d}$ (the value computed from (1 on the preceding page)).

/*Intrinsic Value computation*/
Storage of the $2N+1$ possible values of the intrinsic value.

/*Price initialization*/
The price of the option at maturity. It involves only the values $iv\,[2 * j]$.

/*Backward Resolution*/
Note that we don't re-compute the intrinsic value.

/*Delta*/
The delta here is the right hedging delta in the binomial model (cf The Generalized CRR model). There may be a more clever way to approximate the continuous-time Black&Scholes delta.

/*First time step*/

/*Price*/

/*Desallocation*/

References

[1] J.COX S.ROSS M.RUBINSTEIN. Option pricing: a simplified approach. *J. of Economics*, January 1978. 1