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## fd\_howard

Input parameters:

- SpaceStepNumber  $N$
- TimeStepNumber  $M$
- Theta  $\frac{1}{2} \leq \theta \leq 1$
- Epsilon

Output parameters:

- Price
- Delta

The algorithm of Howard has been introduced by Howard in [1].

**/\*Memory Allocation\*/**

**/\*Time Step\*/**

Define the time step  $k = \frac{T}{M}$ .

**/\*Space localisation\*/**

Define the integration domain  $D = [-l, l]$  using the probabilistic estimate [there](#).

**/\*Space Step\*/**

Define the space step  $h = \frac{2l}{N}$ .

**/\*Peclet Condition\*/**

If  $|r - \delta| / \sigma^2$  is not small, then a more stable finite difference approximation is used. cf [there](#).

**/\*Lhs factor of theta scheme\*/**

Initialize the matrix  $M^h$  issued from the discretization of the operator  $A$  in the case of Dirichlet Boundary conditions. cf [there](#).

**/\*Rhs factor of theta scheme\*/**

Initialize the matrix  $N$  issued from the  $\theta$ -scheme method in the cases of Dirichlet Boundary conditions. [there](#)

**/\*Terminal values\*/**

After a logarithmic transformation, put the value of the payoff into a vector  $P$  which will be used to save the option value.

**/\*Finite difference Cycle\*/**

At any time step, we have to solve the linear complementarity problem cf. [there](#).

**/\*Init Control\*/**

We initialize the control  $pp$ .

**/\*Howard cycle\*/**

We solve the linear complementarity problem using the Howard algorithm, which consists in constructing a convergent sequence  $u^p$  whose limit is  $u$ .

Let  $\epsilon > 0$  be given.

**Step 1** Let  $u^k$  be given, we compute  $i \rightarrow pp^k[i] = \argmin(M^{pp}u^k(i) - f^{pp}[i])$  where  $pp = 0$  or  $1$  (the domain is divided into 2 regions: the continuation region and the exercise region),  $M^0$  is the matrix  $M^h$  issued from the discretization of the operator  $A$ ,  $M^1 = Id$ ,  $f^0 = R$ ,  $f^1 = Obst$ .

**Step 2** We solve the linear system  $M^{pp^k}u = G^{pp^k}$  by the Gauss factorization. It gives  $u^{k+1}$ .

The stopping criteria is

$$\|u^{k+1} - u^k\|_\infty < \epsilon. \quad (1)$$

**/\*Price\*/**

**/\*Delta\*/**

**/\*Memory Desallocation\*/**

## References

- [1] Howard, R.A.: Dynamic Programming and Markov Process. (MIT Press. 1960) 1