Rogers and Stapelton propose to approximate the logarithm of the stock price $(X_t = X_0 + \sigma W_t + (r - \delta - \sigma^2/2)t)_t$ by the random walk $(\xi_n)_n$ where for some fixed $\Delta x > 0$,

$$\begin{cases} \xi_n = X_{\tau_n} \\ \tau_0 = 0,\ \tau_{n+1} = \inf\{t > \tau_n,\ |X_t - X_{\tau_n}| > \Delta x\} \end{cases}$$

The price of the European option with payoff function φ, maturity T, up and out barrier b^* (resp. down and out barrier b_*) and rebate R is approximated by

$$\sum_{n \geq 0} \mathbb{P}(\nu = n) \mathbb{E} \left(e^{-r\eta T/n} R 1_{\{\eta < n\}} + e^{-rT} \varphi(\xi_n) 1_{\{\eta \geq n\}} \middle| \nu = n \right) \tag{1}$$

where $\nu = \sup\{n : \tau_n \leq T\}$ and $\eta = \inf\{n : \xi_n \geq \log(b^*)\}$ (resp $\inf\{n : \xi_n \leq \log(b_*)\}$).

/*Up and Down probabilities*/

Computes the probabilities of an up step $\mathbb{P}(\xi_n = x + \Delta x | \xi_n = x)$ and a down step $\mathbb{P}(\xi_n = x - \Delta x | \xi_n = x)$. For $x = x^*$ the grid point immediately below the up-barrier b^* or $x = x_*$ the grid point immediately above the down-barrier b_*, modified probabilities are calculated.

/*moments of tau1*/

Computes the three first moments of the random variable τ_1.

/*Initialization*/

Initializes the variables before the

/*recursion on the number of time-steps*/

The price is computed according to (1) thanks to a recursion on the number n of time-steps.

/*computation of the probability of nu=n*/

Computation of the probability $\mathbb{P}(\nu = n)$ thanks to a refinement of the Central Limit Theorem. When this probability which appears as a multiplicative coefficient in (1) is smaller than 0.000005, the conditional expectation $\mathbb{E} \left(e^{-r\eta T/n} R 1_{\{\eta < n\}} + e^{-rT} \varphi(\xi_n) 1_{\{\eta \geq n\}} \middle| \nu = n \right)$ is not calculated. Otherwise,

/*contribution for a fixed number of time-steps*/

it is computed thanks to a backward resolution on a tree with n time steps. Two cases are distinguished to construct the tree. If $\log(b^*) \leq X_0+n\Delta x$ (resp. $\log(b_*) \geq X_0 - n\Delta x$) the barrier can be hit (/*Barrier hit*/). Otherwise it cannot and the construction is simpler. (/*Barrier not hit*/).

/*end of the recursion*/

The recursion on the number n on time-steps ends when the cumulative probability $Q = \sum_{m=0}^{n} \mathbb{P}(\nu = m)$ becomes greater than 0.99999.