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## mc\_standard2d

### Input parameters:

- StepNumber  $N$
- Generator\_Type
- Confidence Value

### Output parameters:

- Price  $P$
- Error Price  $\sigma_P$
- Deltas  $\delta_1, \delta_2$
- Errors delta  $\sigma_{\delta_1}, \sigma_{\delta_2}$
- Price Confidence Interval:  $IC_P = [\text{Inf Price}, \text{Sup Price}]$
- Delta Confidence Intervals:  $IC_{\delta_j} = [\text{Inf Delta}, \text{Sup Delta}]$

### Description:

Computation for a Call on Maximum - Put on Minimum - Exchange or Best of European Option of its Price and its Delta with the [Standard Monte Carlo](#) or [Quasi-Monte Carlo simulation](#). In the case of Monte Carlo simulation, this method also provides an estimation for the integration error and a confidence interval.

- The underlying asset prices evolve according to the two-dimensional Black and Scholes model, that is:

$$\begin{cases} dS_u^1 = S_u^1((r - d_1)du + \sigma_1 dB_u^1), & S_{T-t}^1 = s^1 \\ dS_u^2 = S_u^2((r - d_2)du + \sigma_2 dB_u^2), & S_{T-t}^2 = s^2 \end{cases}$$

where  $S_T^j$  denotes the spot at maturity  $T$ ,  $s^j$  is the initial spot and  $(B_u^1, u \geq 0)$  and  $(B_u^2, u \geq 0)$  denote two real-valued Brownian motions with instantaneous correlation  $\rho$ . A description for correlated brownian motions and their simulation is given in the part about random variable simulation?????????.

Then we have:

$$\begin{cases} S_T^1 = s^1 \exp((r - d_1 - \frac{\sigma_1^2}{2})t) \exp(\sigma_{11}B_t^1) \\ S_T^2 = s^2 \exp((r - d_2 - \frac{\sigma_2^2}{2})t) \exp(\sigma_{21}B_t^1 + \sigma_{22}B_t^2) \end{cases}$$

where the parameters  $\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$  are given in the following matrix  $A$ :

$$\begin{vmatrix} \sigma_{1,1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2} \end{vmatrix} = \begin{vmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1 - \rho^2}\sigma_2 \end{vmatrix}$$

such that  $AA^t = \Gamma$  where  $\Gamma$  is the covariance matrix expressed by:

$$\begin{vmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{vmatrix}$$

- The price of an option is

$$P = E \left[ \exp(-rt) f(K, S_T^1, S_T^2) \right]$$

where  $f$  denotes the payoff of the option,  $K$  the strike and  $t$  time to maturity. The Deltas are given by:

$$\begin{aligned} \delta_1 &= \frac{\partial}{\partial s^1} E[\exp(-rt) f(K, S_T^1, S_T^2)] \\ \delta_2 &= \frac{\partial}{\partial s^2} E[\exp(-rt) f(K, S_T^1, S_T^2)] \end{aligned}$$

- Estimators are expressed as:

$$\tilde{P} = \frac{1}{N} \exp(-rt) \sum_{i=1}^N P(i)$$

$$\tilde{\delta}_j = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \frac{\partial}{\partial s^j} P(i) = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \delta_j(i)$$

The values for  $P(i)$  and  $\delta_j(i)$  are detailed for each option.

- **Put on the Minimum:** The payoff is  $(K - \min(S_1, S_2))^+$ .

$$P(i) = \left[ K - \min(S_T^1(i), S_T^2(i)) \right]^+$$

If  $P(i) > 0$  then:

$$\delta_1(i) = \begin{cases} -\frac{\partial S_T^1(i)}{\partial s^1} = -\frac{S_T^1(i)}{s^1} & \text{if } S_T^1(i) \leq S_T^2(i) \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} -\frac{\partial S_T^2(i)}{\partial s^2} = -\frac{S_T^2(i)}{s^2} & \text{if } S_T^2(i) \leq S_T^1(i) \\ 0 & \text{otherwise} \end{cases}$$

- **Call on the Maximum:** The payoff is  $(\max(S_1, S_2) - K)^+$ .

$$P(i) = [\max(S_T^1(i), S_T^2(i)) - K]^+$$

If  $P(i) > 0$  then:

$$\delta_1(i) = \begin{cases} \frac{\partial S_T^1(i)}{\partial s^1} = \frac{S_T^1(i)}{s^1} & \text{if } S_T^1(i) \geq S_T^2(i) \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} \frac{\partial S_T^2(i)}{\partial s^2} = \frac{S_T^2(i)}{s^2} & \text{if } S_T^1(i) \geq S_T^2(i) \\ 0 & \text{otherwise} \end{cases}$$

- **Exchange Option:** The payoff is  $(S_1 - \text{ratio} \times S_2)^+$ .

$$P(i) = (S_T^1(i) - \text{ratio} \times S_T^2(i))^+$$

$$\delta_1(i) = \begin{cases} \frac{S_T^1(i)}{s^1} & \text{if } P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} -\text{ratio} \times \frac{S_T^2(i)}{s^2} & \text{if } P(i) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- **BestOf Option:** The payoff is  $[\max(S_1 - K_1, S_2 - K_2)]^+$ .

$$P(i) = [\max(S_T^1(i) - K_1, S_T^2(i) - K_2)]^+$$

If  $P(i) > 0$  then:

$$\delta_1(i) = \begin{cases} \frac{S_T^1(i)}{s^1} & \text{if } S_T^1(i) - K_1 \geq S_T^2(i) - K_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_2(i) = \begin{cases} \frac{S_T^2(i)}{s^2} & \text{if } S_T^1(i) - K_1 \geq S_T^2(i) - K_2 \\ 0 & \text{otherwise} \end{cases}$$

**Algorithm:**

/\* Value to construct the confidence interval \*/  
 For example if the confidence value is equal to 95% then the value  $z_\alpha$  used to construct the confidence interval is 1.96. This parameter is taken into account only for MC simulation and not for QMC simulation.  
 /\*Initialization\*/  
 /\* Covariance Matrix \*/  
 /\* Coefficients of the matrix A such that  $AA^t = \Gamma$  \*/ This covariance matrix allows to generate the correlated two-dimensional brownian motions.  
 /\*Median forward stock and delta values\*/  
 Computation of intermediate values we use several times in the program.

- /\*MC sampling\*/  
 Initialization of the simulation: generator type, dimension, size  $N$  of the sample  
 /\* Test after initialization for the generator \*/  
 Test if the dimension of the simulation is compatible with the selected generator. (See remarks on QMC simulation, especially on dimension of low-discrepancy sequences). For standard Monte Carlo in the two-dimensional Black and Scholes model, we never have any problem with the dimension, fixed to 2 at the beginning of the program.  
 Definition of a parameter which exprimes if we realize a MC or QMC simulation. Some differences then appear in the algorithm for simulation of a gaussian variable and in results in the simulation.

/\* Begin N iterations \*/

- /\*Gaussian Random Variables\*/  
 Generation of 2 gaussian variables  $g_1$  and  $g_2$  used for the Brownian motions as  $\sqrt{t}g_j$ .  
 Simulation of independent gaussian variables according to the generator type, that is Monte Carlo or Quasi Monte Carlo.  
 Call to the appropriate function to generate a standard gaussian variable. See the part about simulation of random variables for explanations on this point. We just recall that for a MC simulation, we use the Gauss-Abramovitz algorithm, and for a QMC simulation we use an inverse method and a two-dimensional low-discrepancy sequence.

- /\*Price\*/

At the iteration  $i$ , we obtain

$$P(i) = \text{payoff}(K, S_T^1(i), S_T^2(i))$$

- /\*Delta\*/

Calculation of Delta  $\delta_1(i)$  and  $\delta_2(i)$  for the different cases with formula given previously.

/\*Call on the Maximum\*/

/\*Put on the Minimum\*/

/\*Best of\*/

/\*Exchange\*/

Formula were previously described.

/\*Sum\*/

Computation of the sums  $\sum P(i)$  and  $\sum \delta_j(i)$  for the mean price and the means delta.

/\*Sum of squares\*/

Computation of the sums  $\sum P(i)^2$  and  $\sum (\delta_j(i))^2$  necessary for the variance price and the variances delta estimations. (finally only used for MC estimation)

/\* End N iterations \*/

• /\*Price\*/

The price estimator is:

$$P = \frac{1}{N} \exp(-rt) \sum_{i=1}^N P(i)$$

The error estimator is  $\sigma_P$  with :

$$\sigma_P^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N P(i)^2 - P^2 \right)$$

The confidence interval is

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with  $z_\alpha$  computed from the confidence value.

• /\*Delta\*/  
 - /\* Delta1 estimator \*/

The delta estimator is:

$$\delta_1 = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \delta_1(i)$$

The error estimator is  $\sigma_{\delta_1}$  with:

$$\sigma_{\delta_1}^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N \delta_1^2(i) - \delta_1^2 \right)$$

The confidence interval is given as:

$$IC_{\delta_1} = [\delta_1 - z_\alpha \sigma_{\delta_1}; \delta_1 + z_\alpha \sigma_{\delta_1}]$$

with  $z_\alpha$  computed from the confidence value.

- /\* Delta2 estimator \*/  
 The delta estimator is:

$$\delta_2 = \frac{1}{N} \exp(-rt) \sum_{i=1}^N \delta_2(i)$$

The error estimator is  $\sigma_{\delta_2}$  with:

$$\sigma_{\delta_2}^2 = \frac{1}{N-1} \left( \frac{1}{N} \exp(-2rt) \sum_{i=1}^N \delta_2^2(i) - \delta_2^2 \right)$$

The confidence interval is given as:

$$IC_{\delta_2} = [\delta_2 - z_\alpha \sigma_{\delta_2}; \delta_2 + z_\alpha \sigma_{\delta_2}]$$

with  $z_\alpha$  computed from the confidence value.

## References