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Monte Carlo valuation of the european call in a general Black-Scholes model

This procedure is dedicated to the computation of the price and delta of the european call when the underlying evolves according to the following stochastic differential equation:

$dS_t = S_t((r-q)dt + \sigma(t,S_t)dW_t)$ for $0 < t < T$

where r is the risk free annual interest rate, q the annual dividend yield and σ the volatility measure. This dynamics generalizes the standard Black Scholes model in the following sense : the volatility $\sigma(t,S_t)$ is no longer a constant but may depend on the time t and the stock S_t.

Instead of discretizing this stochastic differential equation, it is better to deal with the logarithm of the stock $X_t = \ln(S_t)$ which satisfies:

$$dX_t = \sigma(t,e^{X_t})dW_t + (r-q-\sigma^2(t,e^{X_t})/2)dt$$

The corresponding Euler scheme is defined inductively by

$$\begin{cases} \hat{X}_0^x = x \\ \hat{X}_{(k+1)\Delta t}^x = \hat{X}_{k\Delta t}^x + \sigma(k\Delta t, e^{\hat{X}_{k\Delta t}^x})(W_{(k+1)\Delta t} - W_{k\Delta t}) \\ +(r-q-\sigma^2(k\Delta t, e^{\hat{X}_{k\Delta t}^x}/2))\Delta t \end{cases} \tag{1}$$

The time step Δt is given by t/M where t is the time to maturity for the Call option and the number M is fixed by the user. The approximate price and delta of the call option with strike K are respectively obtained by computing

$$E[e^{-rt}(e^{\hat{X}_T^{\ln S_0}} - K)^+]$$

$$E[e^{-rt}(e^{\hat{X}_T^{\ln(S_0+h)}} - K)^+ - e^{\hat{X}_T^{\ln S_0}} - K)^+)/h]$$

by the Monte-Carlo method. The finite difference step used in the approximation of the delta is h=0.0001.

To improve the precision, we use a variance reduction method. We compute $\bar{\sigma}$:

$$\bar{\sigma} = 1/T(10S_0 - S_0/10)\int_0^T\int_{S_0/10}^{10S_0}\sigma(t,x)dxdt$$

and construct control variables based on the standard Black Scholes model with constant volatility $\bar{\sigma}$.

Let :

/* Explicit Euler scheme */

we compute $\hat{X}_T^{\ln S_0}$ and $\hat{X}_T^{\ln(S_0+h)}$ by the Euler scheme. Of course, the same brownian increments $W_{(k+1)\Delta t} - W_{k\Delta t}$ are used in the calculation of $\hat{X}_T^{\ln S_0}$ and $\hat{X}_T^{\ln(S_0+h)}$ and their sum $\sum_{k=1}^{M}(W_{(k+1)\Delta t} - W_{k\Delta t})$ is stored in order to compute $\bar{X}_t$.

/* Monte-Carlo's method */

The expectation (2) and (3) are approximated by the Monte-Carlo method. $\bar{X}_t$ is calculated for each iteration. The number N of iterations in this method is fixed by the user:

/* reduction variance method */

Then to obtain the calculated call option price, we use the reduction variance method as explained before.

/* the confidence interval, confidence upper bound, and confidence lower bound given at 95 per cent */

Finally, we use the central limit theorem to give a confidence interval at 95 per cent for the price and the delta of the european call.