mc_rogers

Input parameters:

- Number of iterations N
- Generator_Type
- Increment *inc*
- Confidence Value

Output parameters:

- Price P
- Error Price σ_P
- Delta δ
- Error delta σ_δ
- Price Confidence Interval: IC_P =[Inf Price, Sup Price]
- Delta Confidence Interval: IC_δ =[Inf Delta, Sup Delta]

Description: In [1], Rogers proves that the initial price Y_0 of the American put is equal to $Y_0(S_0) = \inf_{\lambda \in R, M \in H_0^1} \mathrm{E}\left[\sup_{0 \leq t \leq T}\left(Z_t - \lambda M_t\right)\right]$ where $Z_t = e^{-rt}(K - S_t)^+$ is the discounted payoff process and H_0^1 the space of L^1 martingales vanishing at zero. For the good choice $dM_t = \mathrm{I}_{\{t^* \leq t\}} d\tilde{P}(t, S_t)$ with $t^* = \inf\{0 \leq t : S_t \leq K\}$ and $\tilde{P}(t, S_t)$ the discounted price of the European put, $\inf_{\lambda \in R} \mathrm{E}\left[\sup_{0 \leq t \leq T}\left(Z_t - \lambda M_t\right)\right]$ gives an accurate upper-bound of Y_0 which can be evaluated by the Monte Carlo method. The first step is devoted to the computation of $\hat{\lambda}$ which realizes the infimum. The second step consists in calculating the Monte Carlo approximation $\hat{Y}_0$ of $\mathrm{E}\left[\sup_{0 \leq t \leq T}\left(Z_t - \hat{\lambda} M_t\right)\right]$ over N simulated paths. All the simulated paths are taken with n time-steps.

/*The parameters of the method*/

given by the user are the number N_p of simulated paths used for the computation of $\hat{\lambda}$, the number N of simulated paths to calculate the price and the number n of time-step on each of these paths. Rogers proposes $N_p = 300$, $N = 30\,000$ and $n = 40$.

/*The standard normal cumulative distribution function*/

gives the value $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du$ using an approximation with precision 10^{-7}.

/*Simulation of random number*/

returns a random variable with standart normal distribution.

/*The price of the european put*/

returns the Black and Scholes price.

/*Computation of $\hat{\lambda}$ */

For this first step we use N_p simulated paths to compute by dichotomy $\hat{\lambda}$ a zero of a finite difference approximation of the derivative of the convex function $\lambda \mapsto \frac{1}{N_p} \sum_{i=1}^{N_p} \sup_{0 \le k \le n} \left(Z^i_{\frac{kT}{n}} - \lambda M^i_{\frac{kT}{n}} \right)$.

/*The computation of the bound*/

is done secondly with N simulated paths. We calculate concurrently and with the same random numbers $\hat{Y}_0(S_0)$ on each path starting from the initial spot S_0 and $\hat{Y}_0(S_0 + h)$ on each path starting from the initial spot $S_0 + h$. The upper-bound obtained is

$$\hat{Y}_0(S_0) = \frac{1}{N} \sum_{i=1}^{N} \sup_{0 \le k \le n} \left(Z^i_{\frac{kT}{n}} - \hat{\lambda} M^i_{\frac{kT}{n}} \right). \tag{1}$$

/*The end of the program*/

gives the approximated price $\hat{Y}_0(S_0)$ and the approximated delta $\frac{\hat{Y}_0(S_0+h)-\hat{Y}_0(S_0)}{h}$.

- /* Price Confidence Interval */

The confidence interval is given as:

$$IC_P = [P - z_\alpha \sigma_P; P + z_\alpha \sigma_P]$$

with z_α computed from the confidence value.

- /* Delta Confidence Interval */

The confidence interval is given as:

$$IC_\delta = [\delta - z_\alpha \sigma_\delta; \delta + z_\alpha \sigma_\delta]$$

with z_α computed from the confidence value.

Confidence intervals are always computed, but for a QMC simulation they don't work, thus they don't appear in the results.

References

[1] L.C.G. Rogers. Montecarlo valuation of american option. *Preprint*, 2000. 1