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fd_multigrid_euro

Input parameters:

- Number of grids l
- TimeStepNumber M
- Theta $\frac{1}{2} \leq \theta \leq 1$

Output parameters:

- Price
- Delta

We refer to Hackbusch [1] for a detailed presentation of multigrid methods.

/*SpaceStepNumber N*/
$N = nn(l) + 1$ where $nn(l)$ calculates the number of points in the grid of level l.

/*Memory Allocation*/

/*Time Step*/
Define the time step $k = \dfrac{T}{M}$.

/*Space localisation*/
Define the integration domain $D = [-limit, limit]$ using the probabilistic estimate there.

/*Space Step*/
Define the space step $h = \dfrac{2 * limit}{M}$.

/*Peclet Condition*/
If $|r-\delta|/\sigma^2$ is not small, then a more stable finite difference approximation is used. cf there.

/*Rhs factor of theta scheme*/
Initialize the matrix N issued from the θ-scheme method in the cases of Dirichlet Boundary conditions. there

/*Terminal value*/
After a logarithmic transformation, put the value of the payoff into a vector P which will be used to save the option value.

/*Finite difference Cycle*/
At any time step, we have to solve the linear discrete problem cf there which can be written in the following form

$$L^l u^l = f^l \tag{1}$$

(where f^l denotes Rhs).

/*Init Rhs*/

/*Multigrid Method*/

We solve the linear discrete problem using the Multigrid method. The multigrid iteration (V-cycle) at level l for solving 1 is defined by the following recursive procedure:

$$v_l \leftarrow MGM(l, v^l, f^l)$$

Step 1 **/*factor of theta scheme*/**
Initialize the matrix L^l issued from the discretization of the operator A in the case of Dirichlet Boundary conditions.
Relax 2 times on $L^l u^l = f^l$ with a given initial guess v^l.

Step 2 if Ω^l is the coarsest grid (l=0) then go to step 4.
Else

$$f^{l-1} \leftarrow I_l^{l-1}(f^l - L^l v^l).$$
$$v^{l-1} \leftarrow 0.$$
$$v^{l-1} \leftarrow MGM(l-1, v^{l-1}, f^{l-1})$$

Step 3 Correct $v^l \leftarrow v^l + I_{l-1}^l v^{l-1}$.

Step 4 Relax 2 times on $L^l u^l = f^l$ with initial guess v^l, where I_{l-1}^l is the linear interpolation operator and I_l^{l-1} the restriction operator. I_{l-1}^l is defined by the rule $I_{l-1}^l v^{l-1} = v^l$ where

$$\begin{array}{l} v_{2j}^l = v_j^{l-1} \\ v_{2j+1}^l = \frac{1}{2}(v_j^{l-1} + v_{j+1}^{l-1}), 0 \leq j \leq \frac{n}{2} - 1. \end{array}$$

The restriction operator is defined by $I_l^{l-1} v^l = v^{l-1}$, where

$$v_j^{l-1} = v_{2j}^l.$$

/*Price*/

/*Delta*/

/*Memory Desallocation*/

References

[1] Hackbusch, W.: Multi-grid methods and applications. Springer-Verlag. (1985) 1