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## cf\_putmin

Let

- $T$  = maturity date ( $T > t$ )
- $K$  = strike price
- $x1$  = spot1 price
- $x2$  = spot2 price
- $t$  = pricing date
- $\sigma1$  = volatility1
- $\sigma2$  = volatility2
- $\rho$  = correlation
- $r$  = interest rate
- $\delta1$  = dividend1 yields
- $\delta2$  = dividend2 yields
- $\theta = T - t$

Here, closed formulas due to Johnson and Stulz are presented [1],[2].

We set

- $d = \frac{\log \frac{x_1}{x_2} + \left( \delta_2 - \delta_1 + \frac{\sigma_2^2}{2} \right) \theta}{\sigma \sqrt{\theta}}$
- $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$
- $d_i = \frac{\log \left( \frac{x_i}{K} \right) + \left( r - \delta_i + \frac{\sigma_i^2}{2} \right) \theta}{\sigma_1 \sqrt{\theta}}, \quad i = 1, 2$
- $\rho_1 = \frac{\sigma_1 - \rho\sigma_2}{\sigma}$
- $\rho_2 = \frac{\sigma_2 - \rho\sigma_1}{\sigma}$

and  $M$  as the cumulative bivariate normal distribution function:

$$M(a, b; \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \int_{-\infty}^a \int_{-\infty}^b e^{-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}} dx dy.$$

# Put On Minimum Option

PAYOFF	$P_T = (K - \min(S_T^1, S_T^2))_+$
PRICE	$P(t, x_1, x_2) = Ke^{-r\theta} - c_0 + c_1$
DELTA	$\frac{\partial P(t, x_1, x_2)}{\partial x_1} = e^{-\delta_1\theta}(1 - N(d)) + e^{-\delta_1\theta}M(d_1, -d; -\rho_1)$ $\frac{\partial P(t, x_1, x_2)}{\partial x_2} = e^{-\delta_2\theta}N(d - \sigma) + e^{-\delta_2\theta}M(d_2, d - \sigma\sqrt{\theta}; -\rho_2)$

where

- $c_0 = x_1e^{-\delta_1\theta}(1 - N(d)) + x_2e^{-\delta_2\theta}N(d - \sigma\sqrt{\theta})$
- $c_1 = x_1e^{-\delta_1\theta}M(d_1, -d, -\rho_1) + x_2e^{-\delta_2\theta}M(d_2, d - \sigma\sqrt{\theta}; -\rho_2) - Ke^{-r\theta}M(d_1 - \sigma_1\sqrt{\theta}, d_2 - \sigma_2\sqrt{\theta}; \rho)$

## References

- [1] H.JOHNSON. Options on the maximum of the minimum of several assets. *J.Of Finance and Quantitative Analysis*, 22:227–283, 1987. 1
- [2] R.STULZ. Options on the minimum or the maximum of two risky assets. *J. of Finance*, 10:161–185, 1992. 1