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fd_galerkin_discfem

Input parameters:

- SpaceStepNumber N
- TimeStepNumber M
- Theta $\frac{1}{2} \leq \theta \leq 1$
- Omega $1 \leq \omega \leq 2$
- Epsilon

Output parameters:

- Price
- Delta

We consider the European Black-Scholes Problem:

$$\left\{ \begin{array}{c} \frac{\partial u}{\partial t} + u = f \text{ on }]0,T] \times \Omega_l \\ u(t,.) = 0 \text{ on } \partial\Omega_l \\ u(0,.) = \psi \text{ on } \Omega_l \end{array} \right. \tag{1}$$

where $\Omega_l = (-l,l)$, f is a smooth function and is a second order linear operator assumed to be elliptic (i.e. $(v,v) \geq 0$ for every function v in $H^1(\Omega_l)$). For instance, in the Black-Scholes model, the stock price process satisfies the following stochastic differential equation:

$$dS_t S_t = \mu(S_t)dt + \sigma(S_t)dB_t,$$

where $(B_t)_{0 \leq t \leq T}$ is a standard Brownian motion, μ is a continuous function and σ is a C^1 function uniformly bounded below. We have,

$$u(x) = \sigma^2(x)2\frac{\partial^2 u}{\partial x^2}(x) + \mu(x)\frac{\partial u}{\partial x}(x) - ru(x).$$

The discretisation of problem (1) is made by a discontinuous Galerkin finite element discretisation in space and time. We split the time interval (0,T) into subintervals $I_n = (t_{n-1}, t_n]$, where $0 = t_0 < \ldots < t_N = T$, with lenght $k_n = t_n - t_{n-1}$.
At each time step, we split the space interval Ω_l into M_n subintervals (x_i^n, x_{i+1}^n), where $-l = x_0^n < \ldots < x_{M_n}^n = l$, with lenght $h_i^n = x_1^n - x_{i-1}^n$. Let $(\phi_i^n)_{1\leq i\leq M_n-1}$ the functions defined by

$$\phi_i^n(x) = \begin{cases} \frac{x-x_{i-1}^n}{h_i^n} & \text{if } x_{i-1}^n \leq x \leq x_i^n \\ \\ \frac{x_{i+1}^n-x}{h_{i+1}^n} & \text{if } x_1^n \leq x \leq x_{i+1}^n \end{cases}$$

and let V_n the vector space generated by the function ϕ_i^n.
Let us define the space-time finite element space $W_{N,q} \subset L^2(0,T; H_0^1(\Omega_l))$ by

$$W_{N,q} = \left\{ v \in L^2(0,T; H_0^1(\Omega_l))\,;\, v(t,x) =_{n=1}^{N} \left(V_0^n(x) + V_1^n(x)\frac{t-t_{n-1}}{k_n} \right)_{I_n}(t) \text{ with } V_0^n, V_1^n \in V_n \right\}$$

For functions from this space, let us denote by

$$\begin{aligned} [v]_n &= v_n^+ - v_n^- \\ &= \lim_{s\to 0^+} v(t_n+s) - v(t_n) \end{aligned}$$

This discretization is based on a variational formulation which allows the use of discontinuous function in time. This method determines approximation U in $W_{N,q}$ by

$$B(U,V) = F(V) \text{ for } V \in W_{N,q}, \tag{2}$$

with the bilinear form

$$B(v,w) = \sum_{n=1}^{N} \int_{I_n} \left\{ (\frac{\partial v}{\partial t}, w) + a(v,w) \right\} + \sum_{n=2}^{N} ([v]_{n-1}, w_{n-1}^+) + (v_0^+, w_0^+),$$

and the linear functional

$$F(w) = (f,w) + (\psi, w_0^+).$$

References