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## ap\_fixedasian\_laplace

Output parameters:

- Price
- Delta

Fixed Asian options are priced with Laplace Transform method of [1] and [2]

```
/*Computation of Laplace transform*/

$$\mathcal{L}(f(x)) = F(\lambda) = \int_0^\infty \exp(-\lambda x) f(x) dx = \frac{\int_0^{\frac{1}{2q}} \exp(-u)(1-2qu)^{\left(\frac{\mu+\nu}{2}+1\right)} u^{\left(\frac{\mu-\nu}{2}-2\right)} du}{\lambda(\lambda-2-2\nu)\Gamma\left(\frac{\mu-\nu}{2}-1\right)},$$


$$\mu = \sqrt{2\lambda + \nu^2}, \quad q = \frac{\sigma^2}{4S(t)} \{k * (T - t)\}, \quad \nu = \frac{2y}{\sigma^2} - 1, \quad S_{INC}(t) = S(t)(1 + INC)$$


$$INC = 10^{-8}, \quad p = q * \frac{1}{1+INC}$$

/* Integral Computation */
```

This formula is from [2]

```

$$\int_0^{\frac{1}{2q}} \exp(-u)(1-2qu)^{\left(\frac{\mu+\nu}{2}+1\right)} u^{\left(\frac{\mu-\nu}{2}-2\right)} du$$

/* Rieman sums */
```

Here, we compute the integral with the rieman sum.

$$\sum_{j=1}^{j=999} \frac{1}{1000} * \exp\left(-\frac{u_j}{2q}\right) (1 - u_j)^{\left(\frac{\mu+\nu}{2}+1\right)} \left(\frac{u_j}{2q}\right)^{\left(\frac{\mu-\nu}{2}-2\right)}$$

$$u_j = j * \left(\frac{1}{1000}\right).$$

$$\Theta_q(\lambda) = \frac{\sum_{j=1}^{j=999} \frac{1}{1000} * \exp\left(-\frac{u_j}{2q}\right) (1 - u_j)^{\left(\frac{\mu+\nu}{2}+1\right)} \left(\frac{u_j}{2q}\right)^{\left(\frac{\mu-\nu}{2}-2\right)}}{\lambda(\lambda-2-2\nu)\Gamma\left(\frac{\mu-\nu}{2}-1\right)}$$

```
/*Inversion parameters*/
```

Using the algorithm [3]

$A = 19.1, N = 15, M = 11,$

```
/* INVERSION */
```

We should remind that the inversion is made throw  $h$ .

We compute  $sum = \frac{h}{e^{\frac{A}{2}}} * s(t) = \frac{F_q(\frac{A}{2h})}{2}$  and  $sum1 = \frac{h}{e^{\frac{A}{2}}} * s_{INC}(t) = \frac{F_p(\frac{A}{2h})}{2}$

/\* Computation of  $S[1] = s(N)$  and  $Q[1] = s_{INC}(N)$  which approximate  $f(t)$  \*/

$$S[1] = \frac{h}{e^{\frac{A}{2}}} * s(N) = \frac{F_q(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k Re \left( F_q \left( \frac{A+2ik\pi}{2h} \right) \right)$$

$$Q[1] = \frac{h}{e^{\frac{A}{2}}} * s_{INC}(N) = \frac{F_p(\frac{A}{2h})}{2} + \sum_{k=1}^{k=N} (-1)^k Re \left( F_p \left( \frac{A+2ik\pi}{2h} \right) \right)$$

/\* Computation of  $s(N+j)$ ,  $s_{INC}(N+j)$   $j \leq M+1$  for Euler approximations \*/

$$S[j] = S[j-1] + (-1)^{N+j} * Re \left( F_q \left( \frac{A+2(N+j)k\pi}{2h} \right) \right);$$

$$Q[j] = Q[j-1] + (-1)^{N+j} * Re \left( F_p \left( \frac{A+2(N+j)k\pi}{2h} \right) \right);$$

/\* Computation of Euler approximations \*/

$$Avg = Avg + Cnp(M, i) * s(N+i);$$

$$Avg1 = Avg1 + Cnp(M, i) * s_{INC}(N+i);$$

/\* f(h) value \*/

Then we have the value of the inversion of the Laplace Transform.

$$Fun = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg;$$

$$Fun1 = \frac{e^{\frac{A}{2}}}{h} * 2^{-M} * Avg1;$$

/\* Call Price \*/

Taking the Call price formula from [2]

$$C_{T,t}(K) = \frac{\exp(-r*(T-t))*4.0*S(t)}{(T-t)\sigma^2} C(h, q)$$

/\* Put Price from Parity \*/

Simple calculus give the call-put parity relationship

$$P_{T,t}(K) = C_{T,t}(K) - K * \exp(-r*(T-t)) - S(t) * \exp(-r*(T-t)) * (\exp(-(r-divid)*(T-t)) - 1) * \frac{1}{(T-t)*(r-divid)}$$

/\*Delta for call option\*/

Here we derive the formula from [2] with respect to the variable  $S(t)$

$$\Delta_C = \left( \frac{\exp(-r*(T-t))*4.0*S_{INC}(t)}{(T-t)\sigma^2} C(h, p) - \frac{\exp(-r*(T-t))*4.0*S(t)}{(T-t)\sigma^2} C(h, q) \right) * \frac{1}{S(t)*INC}$$

/\*Delta for put option\*/

We use again the call-put parity relation

$$\Delta_P = \Delta_C - \exp(-r*(T-t)) * (\exp(-(r-divid)*(T-t)) - 1) * \frac{1}{(T-t)*(r-divid)}$$

/\*Price\*/

/\*Delta \*/

## References

- [1] M.YOR. On some exponential functionals of brownian motion. *Adv. Appl. Pro.*, 24:509–531, 1992. 1
- [2] H.GEMAN M.YOR. Bessel processes, asian options, and perpetuities. *Mathematical finance*, 3:349–375, 1993. 1, 2
- [3] J.ABATE W.WHITT. Numerical inversion of laplace transform of probability distribution. *ORSA Journal of Computing*, 7(1), Winter 1995. 1