

Counterparty risk and funding: putting things together

Interest in counterparty risk, along with counterparty risk itself, has exploded since the 2008-09 crisis, when it was realised that the resilience of a bank to major financial turmoil is largely determined by its ability to properly value and hedge its credit valuation adjustment (CVA). *Stéphane Crépey* proposes an integrated mathematical approach to the related pricing, hedging/collateralisation and funding issues of CVA

Counterparty risk is the risk of either party defaulting on an OTC derivative contract (or portfolio of contracts). It is the native form of credit risk, which affects any OTC transaction between two parties, as opposed to reference credit risk present in the cash-flows of credit derivatives. Counterparty risk exposure is the positive part of the mark-to-market of a position (assuming zero recoveries everywhere).

Therefore for a swapped contract (as opposed to, say, a long bond, which is always in the money), we can think of the loss associated with this risk as an optional feature, as in a contingent credit default swap. This means counterparty risk cannot be simply handled by the application of a credit spread.

Defining the basis of the calculation

Moreover, one must apply this thinking to a large portfolio of OTC derivatives between two counterparties. These two parties, referred to henceforth as “the bank” and “the investor”, are tied by a legal agreement, the credit support annex (CSA), prescribing the collateralisation scheme (implemented through margin calls) and closeout cash-flow in case of default of either party.

The aim of the agreement is to mitigate counterparty risk. However, from the modelling and computational point of view one must, in order to properly manage this risk, employ a dynamic model across all asset classes and run it over thousands of contracts at every point in time of every scenario.

A particular trading desk only has a detailed view of its own activity. It lacks the aggregated data needed to properly value CSA cash-flows. Therefore most banks are now moving to the use of a central CVA desk. It is in charge of collecting global information and valuing and hedging counterparty risk. The value-and-hedge of the contract is then obtained as the difference between the

“clean” value-and-hedge provided by the trading desk (this value is clean of counterparty risk and excess funding costs), and a value-and-hedge adjustment computed by the CVA desk.

In principle, the possibility of one’s own default should be accounted for by a suitable correction, actually standing as a benefit (the so-called DVA for debt valuation adjustment), to the value of the contract. There is a debate among practitioners regarding the relevance of accounting for one’s own credit risk as a benefit through bilateral counterparty risk valuation. But the practical justification for using a model of bilateral counterparty risk is that a unilateral valuation of counterparty risk induces a significant, irreconcilable gap between the CVAs computed by the two parties.

A related issue, especially when dealing with bilateral counterparty risk, is a proper accounting for the costs and benefits of funding one’s position into the contract, accounting for the existence of various funding opportunities with different growth rates (“multiple curves”). From the perspective of the bank (and symmetrically so for the investor), this introduces a third party into the scenario, namely the funder of the position of the bank.

This also gives rise to another close-out cash-flow in case the bank is indebted toward its funder at its time of default.

Developing an integrated approach

The allocation of tasks between the various industry trading desks of an investment bank and the central CVA desk motivates a mathematical CVA approach to the problem of valuing and hedging counterparty risk. Moreover this is done in a multiple curve setup, accounting for the various funding constraints (or costs) involved, allowing one to investigate the question of interaction between counterparty risk and funding.

Given multiple rates, it is not possible to exclude funding costs through discounting as in a classical one-curve setup. All cash-flows are therefore priced instead under an additive, flat extension of the classical multiplicative, discounted risk-neutral assumption.

Consistent with our initial remarks, we obtain a representation of CVA as an option, the so-called contingent credit default swap (CCDS), on the clean value of the contract. However, in a multiple curve set up this is a dividend-paying option, where the dividends correspond to funding costs.

At its most simple, the terminal payoff of a CCDS reads:

$$\mathbb{1}_{\tau=\theta} (M_{\tau}^+ + D_{\tau}^+) - \mathbb{1}_{\tau=\bar{\theta}} M_{\tau}^- \tag{1}$$

where θ and $\bar{\theta}$ are the default time of the bank and the investor, τ is the first (minimum) of the two, M is the clean value (mark-to-market) of the contract, D is the debt of the bank to its funder, and $x^{\pm} \geq 0$ refer to the positive/negative part of a number x , so that $x = x^+ - x^-$.

The task of the bank is then to properly value and dynamically hedge and/or mitigate (by margining) this pay off, where again by “contract” one must understand here a CSA portfolio of thousands of contracts across all asset classes. We develop a practical reduced form, backward stochastic differential equations (BSDEs) approach to this problem. To give a flavour of the BSDEs, let a function $u = u(t, x)$ solve a partial differential equation:

$$\begin{cases} u(T, x) = \phi(x), x \in \mathbb{R}^d \\ \frac{\partial u}{\partial t}(t, x) + Au(t, x) \\ + g(t, x, u(t, x), \nabla_x u(t, x))\sigma(t, x) \\ = 0, t < T, x \in \mathbb{R}^d \end{cases} \tag{2}$$

where A is the generator of a Markov process X and σ is its diffusion coefficient, and ϕ and g are terminal and running cost functions, corresponding to the above terminal payoff (1) and to the funding costs in our counterparty risk setup. BSDEs can be viewed as a probabilistic way to represent the solution of (2) as

$$\begin{cases} u(T, X_T) = \phi(X_T) \text{ and for } t < T, \\ g(t, X_t, u(t, X_t), \\ -du(t, X_t) = \nabla_x u(t, X_t)\sigma(t, X_t)dt \\ - \nabla_x u(t, X_t)\sigma(t, X_t)dW_t \end{cases} \tag{3}$$

where (3) stems from (2) by an application of the Itô formula. But BSDEs are actually more than that. They

offer much more generality and flexibility than partial differential equations (PDEs).

In a manner similar to that of the American Monte Carlo pricing methods, BSDEs also allow one to solve numerically problems in large dimensions d (as potentially is required with CVA applications), for which the numerical solution of a PDE is ruled out by Bellman’s curse of dimensionality.

The “curse of dimensionality”, is a term coined by Bellman to describe the problem caused by the exponential increase in computational volume associated with adding extra dimensions to a mathematical space. One implication of the curse of dimensionality is that some methods for numerical solution of the Bellman equation require vastly more computer time when there are more state variables in the value function.

Writing concrete recipes for risk management

Counterparty risk and funding corrections to the clean price-and-hedge of the contract are thus represented in our approach as the solution to a pre-default CVA BSDE, which is stated with respect to a reference filtration (the information structure in the model), in which the likelihood of default of the two parties only shows up through their default intensities.

In the end, we derive concrete recipes for risk-managing the contract as a whole (or just its CVA component). These are developed according to the bank’s wish to minimise the (risk-neutral for simplicity) variance of the market risk of the hedged contract, or of its CVA component, whilst achieving a perfect hedge of the jump-to-default exposure (or of the jump-to-default exposure to the investor only, not caring about its own jump-to-default exposure, in case a bilateral hedge is not practical or not required by the bank).

The preferred criterion can be optimised by solving (numerically if need be) the related BSDE (which is tantamount to pricing by Monte Carlo an American option), or (if found more efficient, in low dimension) by solving an equivalent semi-linear parabolic PDE.

Notes for this piece may be found online.



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