Funding, Collateral and IR Curves

New Challenges

Igor Smirnov

Head of Fixed Income Quants Europe

Santander
GLOBAL BANKING & MARKETS

Important Information

Roadmap

- Symmetric Collateralised Funding
  - Margin interest as funding rate
  - Switch options, clearing
- Uncollateralised Funding
  - Institutional funding costs
  - Derivatives Funding
- Beyond Curves
  - Asymmetry, Custodians, Collateral Substitution
- To Price or Not To Price?
  - Pricing & Risk in new paradigm

Symmetric Collateralised Funding

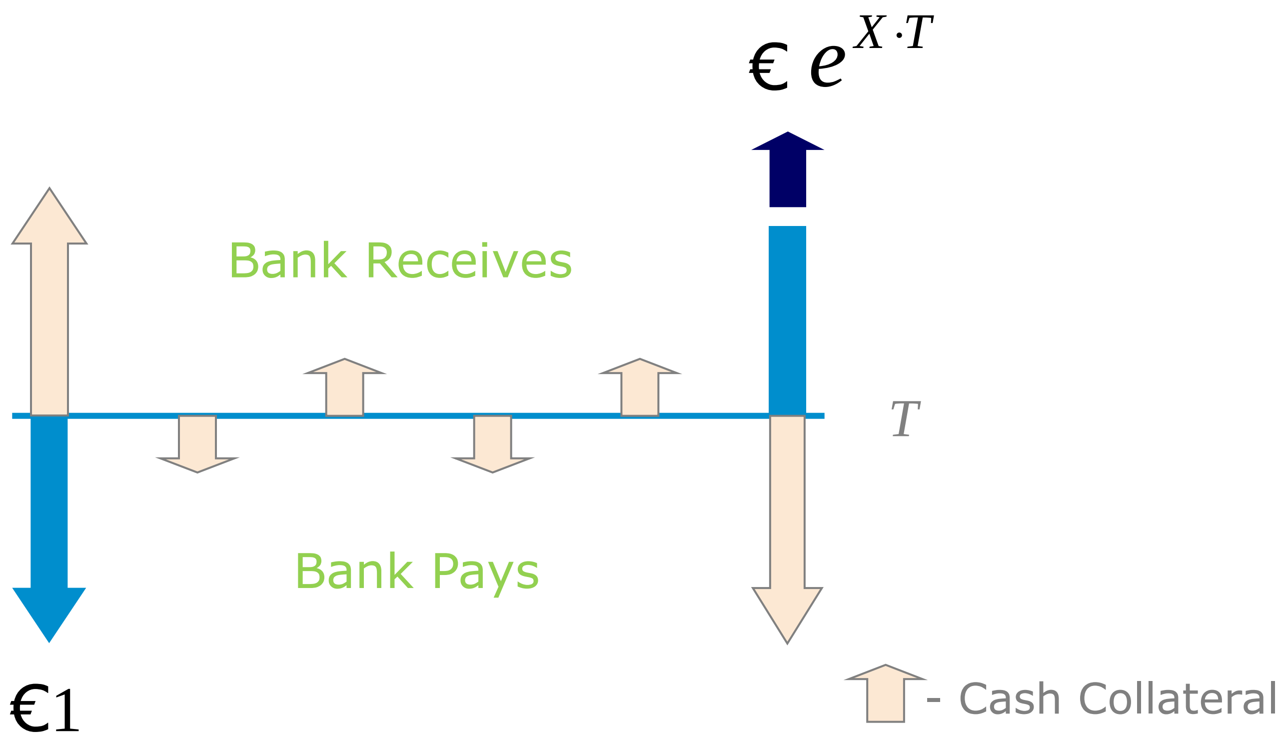
The most well-understood case for "OIS discounting"

- If we assume
  - Zero no-collateral threshold
  - Symmetric, cash-only collateral, margin pays OIS
  - Daily collateral settlement
  - No additional complicating terms
  - Market valuation convention used for settlement

can demonstrate OIS is the correct funding rate.

Without repeating details of the arguments

- Consider a loan paying cts rate X under the CSA



Idealised Bilateral CSA's III

Then if at any point $t \in [0, T)$

$$x(t) = r_C(t) = r_C(0)$$

where

- r_C is the collateral return rate (OIS here)
- $X \cdot T = E\left[\int_0^T x(t)\,dt\right]$

we see that the collateral portfolio and the loan margin interest are offsetting perfectly, and compound OIS gives the fair value loan rate.

Idealised Bilateral CSA's IV

More formally[1] we can consider, following HJM,

$$\frac{dP_C(t,T)}{P_C(t,T)} = r_C(t)dt - \sigma_C(t,T)dW(t)$$

giving the collateralised discounting dynamics, and that the underlying of our derivative follows (Black-Scholes)

$$\frac{dS(t)}{S(t)} = \mu_S(t)dt - \sigma_S(t)dW(t)$$

Then proceeding as normal, consider the delta hedge portfolio including (potentially partial) collateral and financing obtained using return (r_C) of underlying, with any excess / shortfall funded at uncollateralised rate r_F

[1] *see Piterbarg (2010)*

Idealised Bilateral CSA's V

After some algebra (using Itô, etc...) we obtain

$$V(t) = E_t\left[e^{-\int_t^T r_C(u)\,du} \cdot V(T) - \int_t^T e^{-\int_t^u r_C(v)\,dv} \cdot \left(r_F(u) - r_C(u)\right) \cdot \left(V(u) - C(u)\right) du \right]$$

where V is the value of the derivative and C is the collateral due (potentially different to V, due to market conventions, no-collateral windows, asymmetric terms, etc)

In particular if V=C we have $V(t) = E_t\left[e^{-\int_t^T r_C(u)\,du} \cdot V(T) \right]$

i.e. we are discounting at the collateral margin rate.

So far so good…

- If collateral is in a given currency, discount curves in other currencies are implied via FX arbitrage considerations
- E.g. for a period t we can obtain the implied funding rate

$$R_{USD}(0,t)=\left[\left(1+R_{EUR}(0,t)\cdot\Delta(0,t)\right)\cdot FX_{EUR->USD}(t)/FX_{EUR->USD}(0)-1\right]/\Delta(0,t)$$

  - To be accurate, should also worry about collateral of FX fwds

➢ In many circumstances most of the assumptions we made along the way are false or not strictly applicable…

Significance of the Overnight Rate

Collateral-implied funding is often termed “OIS Discounting”

- Important to note that choice of OIS not fundamental
  - The only significance of the OIS rate is that it is a handy, realistic benchmark of overnight bank lending rates
  - So a convenient “fair” return on cash deposited overnight as margin to another bank or clearing house...
  - And so CSA terms often reference OIS in one or more currencies
- Any interest rate would work as well, including zero
- OIS can be far from “fair” return for long duration collateralised exposures
- OIS is not chosen “because it is the risk free rate”!

Multiple Collateral Types

- Typical CSA's will allow a range of assets that may be posted as collateral – cash in different currencies, government bonds, perhaps even corporate or mortgage backed securities.

- As the cash funding terms are usually tied to the corresponding currency's overnight rate (EONIA, SONIA, FF effective, etc.), choice of which currency to post collateral in can materially alter expected return.

- Some agreements allow the posting counterparty to substitute collateral on demand, others do not - or require consent, which is materially the same as a rational counterparty would not want to accept collateral giving inferior return.

Collateral Optimisation

- Let's first consider agreements where collateral substitution is permitted. Then we can swap types collateral we post to maximise our return, and our counterparty would do likewise.
- So we can express the implied optimal funding curve by

$$R^*_{OPT}(0,t)\cdot t = E\left[\int_{x=0}^{t}\max\left[r_i(x)\middle|\, i\in S\right]dx\right]$$

- where we optimize over the set S of all available instantaneous returns.
  - For multiple currencies, these are implied returns in the currency of the trade given by the margin index curve and the FX markets.

Non-Cash Assets

- Just as we can choose to post cash in different currencies, we can optimise over other assets
  - for example accept a suitable bond as repo collateral and re-hypothecate as collateral under a CSA
- The choice over possible cash or non-cash assets will yield the optimal curve given the earlier formula, but care must be taken to account for both market and CSA-stipulated haircuts.
- For a single bond, haircut adjusted return $R = \frac{1 - H_M}{1 - H_{CSA}} \cdot \text{Repo}$ can be compared to that of posting cash or other bonds, and implied funding rate determined.
  - Balance sheet considerations, technical issues are a factor
  - CSA haircuts are fixed and, will differ from market levels

Central Clearing

- For a number of reasons, there has been a push towards using central clearers for derivative transactions where possible
- Booking a derivative transaction through a clearer such as LCH has a number of implications impacting value, including default fund contributions, initial margin posting, etc
- But we can notice that variation margin terms have the same properties as a symmetric CSA, with the margin interest rate (usually local OIS in major currencies) implying funding as before
- The clearers themselves are recognizing this, with LCH now beginning to use OIS discounting to bring their valuations closer to those of participating dealers

Local OIS Discounting

- One of the approaches (and the one currently followed by LCH) is to require collateral to be posted in the currency of the deal, attracting margin interest at the local overnight rate

- Apart from practical issues (no liquid reference in many currencies, etc) this has an important consequence – valuing cross-currency instruments in this framework gives rise to arbitrage. This is not an immediate issue for LCH, which at the moment does not clear cross-currency products

- If no-arbitrage valuation of multi-currency products is required, collateral terms must be defined in a way respecting the FX markets – e.g. by admissible collateral always being in one chosen currency or a basket of currencies, irrespective of trade currency

Uncollateralised Funding

Uncollateralised Transactions

- We have already referred to uncollateralised deals and funding rates for uncollateralised derivative exposures.
- What is this uncollateralised rate? Broadly speaking one can consider one of the following approaches:
  - It is the **cost of funding of the institution**, that is manifest in the borrowing costs it has to pay in the market
  - It is an **exogenously specified funding rate** specific to derivative exposures, possibly very different from above
- The former approach has been extensively explored recently, with a large number of papers (for example Morini [2009]) breaking out constituent risk factors.
  - *Unfortunately, for many institutions reality is better described by the latter approach*

Uncollateralised Transactions II

- Firstly let us consider that the rate we want relates to institution's term funding costs. Then, we can interpret it in several ways, for example as

➢ **Risk free rate + liquidity premium**. This approach is consistent with use of unilateral CVA, and valuation conditional on no own default.

➢ **Risk free rate + credit risk charge + liquidity premium**. This approach is consistent with use of bilateral CVA, and valuation that allows economic benefit to be derived from one's own default.

- Note that both of the approaches will, by definition, match the value of the institutions outstanding liabilities – the difference lies in interpretation of liquidity / credit premia components of that value.

Avoiding double-counting

- Typically uncollateralised transaction value is expressed as
- **Reference Value + CVA + LVA**

- The first will vary from institution to institution, and can refer to risk-free value, collateralised deal value, etc.
- CVA accounts for counterparty risk related losses, and potentially own default risk gains (for bilateral CVA).
- Liquidity Value Adjustment, or LVA, accounts for liquidity related costs over the reference index that not already accounted for by the CVA.

Avoiding double-counting II

- So we can see, it is not appropriate to both use bilateral CVA and apply LVA based on market funding rates.

- Neglecting recovery, we can write our funding rate as

$$r_U = r_{RF} + \lambda + l$$

where the last two terms are default intensity and liquidity.

- For unilateral CVA, this is simply

$$r_U = r_{RF} + l$$

as the default intensity term disappears.

Uncollateralised Transactions III

- Now suppose that uncollateralised rate is an exogenously specified funding rate specific to derivative exposures (approach preferred at many institutions):

- Considering own default as a factor is a problem
- Suppose we do – hazard rate may be higher than the actual funding rate!
- There is little justification in linking the two, unlike the term funding case
- Unilateral CVA + LVA approach (funding = Risk Free Rate + liquidity) still makes sense

Uncollateralised Transactions IV

- There's just one problem – we are using the unknown Risk Free Rate along the way. Do we really need it?
  - CDS contracts are collateralised, allowing us to obtain default intensities (for ourselves or the counterparty) without referencing the Risk Free Rate
  - If we now revisit the previous slides, that means we can simply talk about
    - the (observable) overall funding rate r_U for unilateral CVA
    - Credit-adjusted funding rate $r_U - \lambda$ for bilateral CVA*
- Newly armed with the uncollateralised funding rate we can go back to pricing.

* for proper treatment we need to also take into account recovery

Fair Value & Uncollateralised Exposures

Consider $ funding costs of two banks, A and B, and UST

Institution	5Y Funding	5Y CDS (bp)	$r_{RF}+l$
Bank A	3.5%	250	1.0%
Bank B	3.0%	180	1.2%
US Treasury	1.0%	50	0.5%

- Suppose, for simplicity, zero recovery for all, no collateral, no other exposures to net against
  - What is the risk free rate?
  - At what rates should A and B lend to each other?

Fair Value & Uncollateralised Exposures II

- If A buys B's bonds or B buys A's bonds, they will provide funding at 3% and 3.5% respectively
  - If A buys B's bonds and finances them with own bond issuance rather than repo, he has negative carry
  - If repo markets are used, carry depends on repo levels
- What about derivative exposure they may take on to each other, say an in the money FRA? Under unilateral CVA + LVA

Liquidity	A	B
Borrow	3.5%	3.0%
Lend	5.3%	5.5%

if we regard derivative value as deterministic.

Fair Value & Uncollateralised Exposures III

- Similarly under bilateral CVA (but each using own liquidity)

Liquidity	A	B
Borrow	3.5%	3.0%
Lend	2.8%	3.7%

  - Unlike before, we have one feasible trade (A lending to B)
  - Moreover A is happy to finance B below its bond-implied funding costs...
  - ... While B, though having lower overall cost of funds and credit risk, is still not happy to finance A

- Full 2-way market only possible if $r_{RF}+l$ terms match (or if we properly consider recovery, if r_{RF} and l match individually)

Fair Value & Uncollateralised Exposures IV

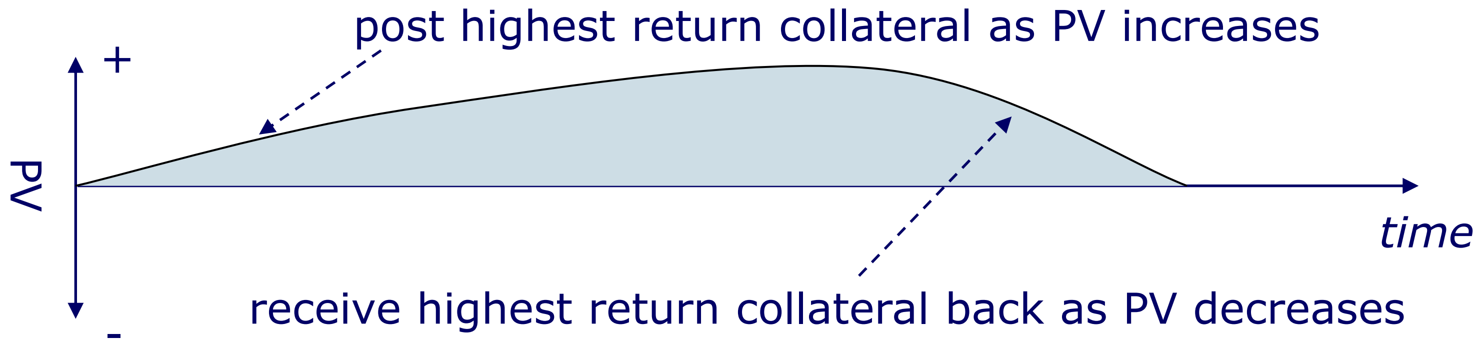
- Risk free rate is difficult to estimate, let alone agree on
- So is the recovery we need to assume to estimate λ
- Therefore so is the liquidity premium l for each party

- Price agreement is unlikely even when both sides in reality have the same liquidity premium and use bilateral CVA
- Use of bilateral CVA raises a number of other issues

Beyond Curves

Restricted Optimisation

- Let us return to the issue of collateral optimisation. If substitution on demand is not permitted, a degree of optimisation is still possible
- As portfolio fluctuates in value, each party can optimise their return by posting (or returning if receiver) the highest yielding collateral
- In such situation valuation becomes path-dependent, so a simulation is required for valuation even if the agreement is otherwise fully symmetric



Asymmetric CSA Terms

Some collateral agreements contain asymmetric terms. The lack of symmetry may arise from

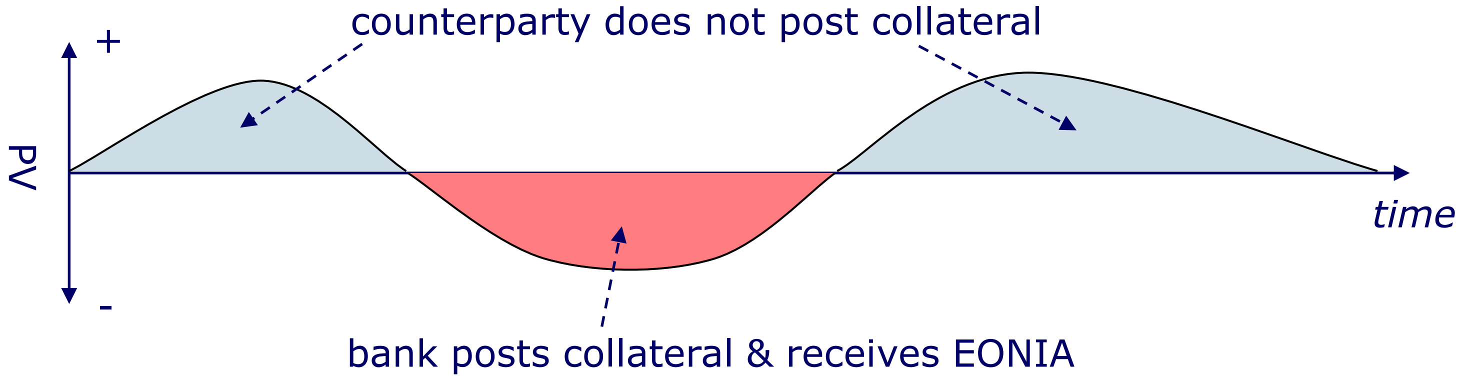
- Rating-dependent conditions (such as threshold sizes), coupled with a difference in ratings
- Legal requirements meaning counterparties are not allowed to post collateral (some investment funds)
- Often linked to above, rehypothecation restrictions or use of custodians to hold collateral posted (the latter not truly asymmetric but results in asymmetry in funding needs)

- A number of sovereign states and supranational entities also insist on receiving collateral while refusing to post it.

Asymmetric CSA Terms II

- When CSA terms are symmetric, valuation impact can be calculated by using a suitable funding curve as discussed earlier.
- When the terms are asymmetric (for example, one side has to post EUR cash collateral with EONIA margin interest, the other is not required to post at all), no such single funding curve exists.

Consider an exposure with following PV profile:



Asymmetric CSA Terms III

Funding rate at any point in the future depends on whether the entire portfolio of trades has +ve or –ve PV

- For any +ve PV period, uncollateralised funding rate applies
- For any –ve PV period, CSA implied rate (EONIA above) applies

- In general, this is very similar to calculating CVA, and a simulation is usually required to value any given trade given current total exposure.
- Not additive
- Uncollateralised funding rates are specific to each entity so value need not be the same from each counterparty's point of view, as we will discuss shortly

Rating-Dependent Provisions

- As well as rating-dependent thresholds, some agreements contain other provisions linked to rating conditions, including
  - Restrictions on acceptable collateral
  - Charges based on outstanding nominal (such as requiring an "Independent Amount" to be posted)
  - Provisions for forced novation or change of collateral custody
  - Termination on downgrade (individual deals or ISDA level)

- In many situations, effect on value can be counterintuitive:
  - Termination of uncollateralised deal with collateralised reference value (substitution cost in interdealer market) increases duration rather than lowering it…
  - IA can be triggered by a party deeply out of the money…

Rating-Dependent Provisions II

- Rating model is usually necessary to model any of the effects discussed on previous slide
- Even if the conditions are symmetric, difference in ratings & their volatility and correlation result in asymmetry
- Since this is usually computationally intensive, the “adjustment” due to rating transitions may be calculated less frequently

To Price or not to Price?

- Modelling all details of funding and credit factors massively increases computational and management complexity
  - When do we worry about each of the risk factors?
  - Who do we ask to manage related risks, including crosses?
  - Can we design our systems to be fast enough?
  - How do we obtain difficult to estimate parameters (uncollateralised funding curve etc)?
- Computationally, not a new set of problems (consider Stoch Vol etc) – just a question of what to assume locally constant
- Management organisation determines possible system options (what risks are managed daily / realtime, vs. managed monthly)

References

- “Funding beyond discounting: collateral agreements and derivatives pricing”, Piterbarg, February 2010 RISK Magazine
- “Being Two-Faced Over Counterparty Credit Risk”, Gregory, March 2009 RISK Magazine
- “Solving the puzzle in the interest rate market”, Morini, 2009, SSRN#1506046
- “Risky funding: a unified framework for counterparty and liquidity charges”, Morini, Prampolini, SSRN#1669930
- “Does a Central Clearing Counterparty Reduce Counterparty Risk?”, Duffie & Zhu, 2009

Santander

GLOBAL BANKING & MARKETS

Dow Jones Sustainability Indexes
Member 2008/09

FTSE4Good Index Series