The Whys of the LOIS: Credit Skew and Funding Spread Volatility

Stéphane Crépey[1,*] and Raphaël Douady[2]

[1] Laboratoire Analyse et Probabilités
Université d'Évry Val d'Essonne
91037 Évry, France

[2] Centre d'économie de la Sorbonne
CNRS/University Paris 1 Pantheon Sorbonne

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Abstract

The 2007 subprime crisis has induced a persistent disconnection between the Libor derivative markets of different tenors and the OIS market. Commonly proposed explanations for the corresponding spreads are a combination of credit risk and liquidity risk. However in the literature the meaning of liquidity is either not precisely stated, or it is simply defined as a residual spread after removal of a credit component. In this paper we propose a stylized equilibrium model in which a Libor-OIS spread (LOIS) emerges as a consequence of a credit component determined by the skew of the CDS curve of a representative Libor panelist (playing the role of the "borrower" in an interbank loan) and a liquidity component corresponding to a volatility of the spread between the refinancing (or funding) rate of a representative Libor panelist (playing the role of the "lender") and the overnight interbank rate. The credit component is thus in fact a credit skew component, whilst the relevant notion of liquidity appears as the optionality, valued by the aforementioned volatility, of dynamically adjusting through time the amount of a rolling overnight loan, as opposed to lending a fixed amount up to the tenor horizon on Libor. "At-the-money" when the funding rate of the lender and the overnight interbank rate match on average, this results, under diffusive features, in a square root term structure of the LOIS, with a square root coefficient given by the above-mentioned volatility. Empirical observations reveal a square root term structure of the LOIS consistent with this theoretical analysis, with, on the EUR market studied in this paper on the period half-2007 half-2012, LOIS explained in a balanced way by credit and liquidity until the beginning of 2009 and dominantly explained by liquidity since then.

Keywords: Libor, OIS, Funding, Credit, Liquidity.

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1 Introduction

The main reference rate for a variety of fixed income derivatives is the Libor in the USD market and the Euribor in the EUR market. Libor (resp. Euribor) is computed daily as an average of the rates at which a designated panel of banks believe they (resp. a prime bank) can obtain unsecured funding, for periods of length up to one year. From now on we shall use the term Libor and the letter L to denote any of these two rates. Following the recent credit crunch, which severely impacted trust between financial institutions, overnight interest rate swaps (OIS) became more and more popular. In these financial instruments, the floating rate is obtained by compounding an overnight interbank rate (O/N) r_t, i.e. a rate at which overnight unsecured loans can be obtained in the interbank market (the Federal Funds rate in the USD fixed income market and the EONIA rate in the EUR market). As a result, an OIS rate R [1] can be interpreted as a suitable average of r (we denote by a single letter r the whole random process $r_t(\omega)$, as will be used further down with "n", "α", "β" and "c").

In theory, arbitrage relations imply that $L = R$. However, the interbank loan market has been severely impacted since the 2007 subprime crisis and the ensuing liquidity squeeze. The reference interbank rate remains the Libor, though, which still underlies most vanilla interest-rate derivatives like Swaps, FRA, IRS, cap/floor and swaptions. The resulting situation where an underlying to financial derivatives has become in a sense arbitrarily fixed by a panel of key players in the derivatives market poses insider issues, as illustrated by the recent manipulation Libor affairs (Wheatley 2012). But first of all, it poses a crucial funding issue as, on the one hand, in parallel to the drying up of the interbank loan market, Libor got disconnected from OIS rates (see Fig. 1); whilst on the other hand, as more and more trades are collateralized, their effective funding rate is the corresponding collateral rate, which is typically indexed to the O/N r_t. This creates a situation where the price of an interest-rate product, even the simplest flow instrument like a FRA, involves (at least) two curves, a Libor fixing curve and an OIS discount curve, as well as the related convexity adjustment, which in the case of optional products can be significant (cf. (Mercurio 2010)). Via the relations between counterparty risk and funding this also has some important CVA implications; see (Crépey 2012a; Crépey 2012b; Pallavicini, Perini, and Brigo 2011).

Commonly advanced explanations for the Libor-OIS spreads ($L-R$), often called LOIS in the market, are a combination of credit risk and/or liquidity risk.See (Bean 2007; Brunnermeier and Pedersen 2009; Smith 2010; Crépey, Grbac, and Nguyen 2012; Eisenschmidt and Tapking 2009; Filipović and Trolle 2011; Morini 2009). Nevertheless, in these explanations the meaning of liquidity is either not precisely stated, or it is simply defined as a residue after removal of a credit component. In this paper we propose a stylized equilibrium model to evaluate at which rate does a bank find it interesting to lend at a given tenor horizon, as opposed to rolling an overnight loan which it can cancel at any moment. In this setup LOIS emerges as a consequence of the skew (as measured by λ_t below) of the credit curve of a representative Libor panelist (playing the role of the borrower in an interbank loan), and of the volatility of the spread $c_t = \alpha_t - r_t$ between the refinancing rate α_t of a representative Libor panelist (playing the role of the lender) and the O/N r_t.

We illustrate our study using 07/07/2005 to 16/04/2012 EUR market Euribor/Eonia-swap Bloomberg data, covering both out-of-crisis and in-crisis data (2007-09 credit crisis and ongoing Eurozone sovereign debt crisis). Note that the EUR market is even larger than the USD market; moreover, the Euribor and Eonia bank panels are the same, whereas the

[1] See e.g. the formula (62) in Subsection 4.3 of (Crépey, Grbac, and Nguyen 2012) for an exact definition of an "OIS rate".

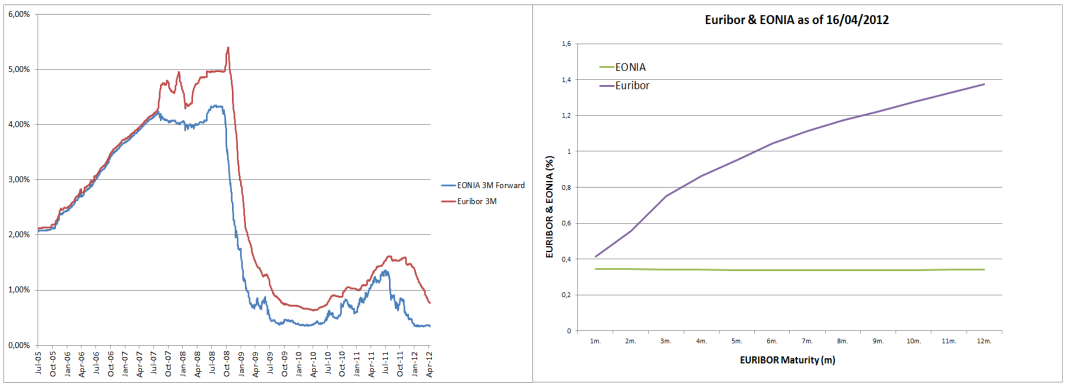

Figure 1: Divergence Euribor ("L") / EONIA-swap ("R") rates. *Left*: Sudden divergence between the 3m Euribor and the 3m EONIA-swap rate that occurred on Aug 6 2007; *Right*: Term structure of Euribor vs EONIA-swap rates, 16 April 2012.

FF rate panel is larger than the USD Libor one. On related topics (Filipović and Trolle 2011) conducted empirical studies on both EUR and USD markets and obtained very similar results in both cases.

2 Equilibrium Model

We assume that the funding rate (or refinancing rate, funding rate, cost-of-capital, cost-of-liquidity...) of a lending bank with a short term debt D ("immediately callable debt" like short term certificate deposits, say more broadly $\leq 1y$-liabilities) is given by a random function $\rho_t(D)$. In particular, denoting by D_t the time-t short term debt of the bank, $\alpha_t = \rho_t(D_t)$ is the annualized interest rate charged to the bank on the last euro that she borrowed for funding her loan (current refinancing or funding rate of the bank). Note that this time-t funding rate α_t is a complex output possibly impacted by, depending on the treasury management of the bank, beyond the level of the time-t short term debt D_t of the bank (as compared to its immediately repositable capital C_t), various factors including the O/N r_t, the bank's CDS spread as well as other macro-economic global variables. More broadly, $\rho_t(D_t + x)$ is the annualized interest rate charged to the bank on the last euro of a total of €$(D_t + x)$ borrowed by the bank for funding her loan. Here the variable x represents the putative extra amount of capital borrowed by the bank to refund a loan (rolled overnight or on the Libor market). For liquidity reasons, $\rho_t(D)$ is increasing in the debt D (or $\rho_t(D_t + x)$ is increasing in x; see (4) for a linear specification), so that "the next euro borrowed costs more than the previous one". Since the marginal cost $\rho_t(D_t + x)$ needs to be integrated over x from 0 to N in order to give the global refinancing cost for the bank of lending a given global amount N, therefore this global refinancing cost is convex in N (as the integral of an increasing function). This convexity reflects the optionality feature of the funding liquidity issue.

We denote by $\mathbb{P}$ and $\mathbb{E}$ the actuarial probability measure and the related expectation. Let n_t represent the amount of notional that the bank is willing to lend at the O/N rate r_t between t and $t + dt$. The problem of the bank lending overnight is modeled as: maximizing over the whole stochastic process n the expected profit of the bank, which we put in mathematical form as

$$\mathcal{U}(r;n) = \frac{1}{T}\mathbb{E}\left(\int_0^T n_t r_t dt - \int_0^T \int_0^{n_t} \rho_t(D_t + x)dxdt\right) \longleftarrow \max n \tag{1}$$

By contrast to this situation, when lending at Libor over a whole period of length T, a bank cannot modify a notional amount N which is locked between 0 and T. As the composition of the Libor panel is updated at regular time intervals, there is, during the life of a Libor loan, increasing credit risk as compared with an overnight loan rolled over the same period. Indeed, as time goes on, the refreshment mechanism of the panel guarantees a sustained credit quality of the names underlying the rolling overnight loan, whereas the LIBOR loan is contracted once for all with the initial panelists; see (Filipović and Trolle 2011) for a detailed analysis, and see further developments following (6) below. Accordingly, let a stylized default time τ of the borrower reflect the deterioration of the average credit quality of the Libor contributors during the length of the tenor. This deterioration only affects the lender when lending at Libor, in other words, the stylized default time τ only corresponds to those default events that could have been avoided, had the loan be made as rolling overnight. It represents the "survivorship bias" that overnight loans benefit from. Let a constant N represents the amount of notional that the bank is willing to lend at Libor rate L over the period $[0,T]$. The related optimization problem of the bank can then be modeled as: maximizing with respect to the constant amount N the expected profit of the bank, which we put in mathematical form as

$$\mathcal{V}(L;N) = \frac{1}{T}\mathbb{E}\left(NL(T\wedge\tau) - \int_0^{T\wedge\tau}\int_0^N \rho(D_t + x)dxdt - \mathbb{1}_{\tau<T}N\right) \longleftarrow \max N \tag{2}$$

As we are dealing with short-term debt, we assume no recovery in case of default.

We stress that r and n represent stochastic processes in (1) whereas L and N are constants in (2). The utility functions of the bank which are implicit in (1)-(2) are taken in a standard economic equilibrium formalism of Legendre transforms U or V of the OIS and Libor cost functions, represented by the integrals over x in the right-hand side of (1)-(2); see (Karatzas and Shreve 1998) regarding the standard economic equilibrium formalism of utility functions and Legendre transforms. These utility functions are linear to reflect the general risk-neutral behavior of banks when lending, in which gains and losses are assessed in terms of actuarial expectations. In other word, the choice of banks to lend is less driven by preferences than by an optimization of the cost-of-capital and credit protection. One could incorporate a concavely distorted utility function to account for risk aversion. Such a distortion would appear in our model as an increased volatility of capital needs and of the corresponding borrowing rate. However, we believe that short term lending decisions are more driven by the estimated cost-of-capital than by a trade-off between interest returns and default risk.

As we are dealing with short term debt with $T \leq 1$ yr, we did not introduce a discount factor. Discount factors would make no significant difference at this time horizon and would only obscure the analysis in the financial qualitative perspective of this paper. Extending the model to longer term debt might require a correction in this regard. We stick to the stylized formulation (1)-(2) for tractability issues, and also in order to emphasize that the volatility terms which appear in the end-formula (11) are convexity adjustments reflecting an inherent optionality of LOIS, even without any risk premia.

Letting $U(r) = \max_n \mathcal{U}(r;n)$ and $V(L) = \max_N \mathcal{V}(L;N)$ represent the best utilities a bank can achieve by lending OIS or Libor, respectively, our approach for explaining the LOIS consists, given the O/N process r, in solving the following equation for L:

$$V(L) = U(r) \tag{3}$$

This equation expresses an equilibrium relation between the utility of lending rolling overnight versus Libor for a bank involved in both markets (indifference value at the optimal amounts prescribed by the solution to the corresponding optimization problems). A second optimization problem, left aside in this article, would be in the O/N process r, depending on the supply/demand of liquidities and on base rates from the central bank.

To summarize at this point, the key features differentiating the two strategies are: on the one hand, the deterioration in credit worthiness of a representative Libor "borrower"; on the other hand, the funding liquidity of a representative Libor "lender". Note that other issues like central banks' policies or possible manipulations of the rates are not explicitly stated in the analysis. However, to some extent, these can be reflected in the model parametrization that we will specify now (see, in particular, the last paragraph of Subs. 2.1 and the discussion of Fig. 2). Also note that in our equilibrium analysis we only consider a rolling overnight loan and a Libor loan of a given maturity (tenor) T, equating in (3) the expected profits from optimally lending in each case, in order to eventually derive our LOIS formula (11). It appears that this formula does show a (square root) dependence in the tenor T. This relation doesn't need to consider the possibility of lending at Libor over different tenors T, but only rolled overnight versus at one fixed Libor of given maturity T.

2.1 Credit and Funding Costs Specification

For tractability we assume henceforth that the funding rate ρ is linear in D, i.e.

$$\rho_t(D_t + x) = \alpha_t + \beta_t x \tag{4}$$

where $\alpha_t = \rho_t(D_t)$ is the time-t cost-of-capital of the lending bank and the coefficient β_t (positive in spirit) represents the marginal cost of borrowing one more unit of notional for the bank already indebted at the level D_t. For instance, $\alpha_t = 2\%$ and $\beta_t = 50$bp means that the last euro borrowed by the bank was at an annualized interest charge of 2 cents, whereas if the bank would be indebted by €100 more, the next euro to be borrowed by the bank would be at an annualized interest charge of 2.5 cents.

By (4), we have

$$\mathcal{U}(r;n) = \frac{1}{T}\mathbb{E}\int_0^T \left((r_t - \alpha_t)n_t - \frac{1}{2}\beta_t {n_t}^2\right) dt \tag{5}$$

Denoting by λ_t the intensity of τ and letting $\gamma_t = \alpha_t + \lambda_t$ and $\ell_t = e^{-\int_0^t \lambda_s ds}$, we also have:

$$\mathcal{V}(L;N) = \frac{1}{T}\mathbb{E}\int_0^T \left((L - \gamma_t)N - \frac{1}{2}\beta_t N^2\right) \ell_t dt \tag{6}$$

where a standard credit risk computation was used to get rid of the default indicator functions in (6) (see for instance (Bielecki and Rutkowski 2002)). As explained after (2), the stylized default time τ reflects the deterioration of the average credit quality of a Libor representative borrower during the length of the tenor. Recall the classical argumentation of (Merton 1974), according to which a high-quality credit-name has a decreasing CDS curve reflecting the expected deterioration of his credit. Consistent with this interpretation, the intensity λ_t of τ can thus be proxyed by the slope of the credit curve of the Libor representative (and therefore high-quality) borrower (differential between the borrower's 1y CDS spread and the spread of her short term certificate deposits, currently 10 to a few tens of bp for major banks). Accordingly we call λ_t the credit skew of a Libor representative borrower.

Note that central banks' liquidity policies can be reflected in the α_t and β_t components of the cost-of-liquidity ρ in (4). A possible manipulation effect, or incentive for a Libor contributor to bias its borrowing rate estimate in order to appear in a better condition than it is in reality (Wheatley 2012), can be included as a spread in the borrower's credit risk skew component λ.

3 LOIS Formula

Problems (1), (5) and (2), (6) are respectively solved, for given r and L, as follows. Writing $c_t := \alpha_t - r_t$, the OIS problem (1), (5) is resolved independently at each date t according to

$$u_t(r_t; n_t) = c_t n_t - \frac{1}{2}\beta_t n_t^2 \longleftarrow \max n_t$$

hence

$$n_t^* = \frac{c_t}{\beta_t} \quad \text{and} \quad u_t(r_t; n_t^*) = \frac{c_t^2}{2\beta_t}$$

The expected profit of the bank over the period $[0, T]$ is

$$U(r) = \mathcal{U}(r; n^*) = \mathbb{E}\left(\frac{1}{T}\int_0^T \frac{c_t^2}{2\beta_t} dt\right)$$

In the Libor problem (2), (6), we must solve

$$T\mathcal{V}(L; N) = N\mathbb{E}\int_0^T (L - \gamma_t)\ell_t dt - \frac{1}{2}N^2\mathbb{E}\int_0^T \beta_t \ell_t dt \longleftarrow \max N$$

hence

$$N^* = \frac{\mathbb{E}\frac{1}{T}\int_0^T (L - \gamma_t)\ell_t dt}{\mathbb{E}\frac{1}{T}\int_0^T \beta_t \ell_t dt} \quad \text{and} \quad V(L) = \mathcal{V}(L; N^*) = \frac{\left(\mathbb{E}\frac{1}{T}\int_0^T (L - \gamma_t)\ell_t dt\right)^2}{2\mathbb{E}\frac{1}{T}\int_0^T \beta_t \ell_t dt} \tag{7}$$

We define $R = \mathbb{E}\frac{1}{T}\int_0^T r_t dt$. The sequel of the paper is devoted to the computation of a stylized LOIS defined as $(L^* - R)$, where L^* is, given the process r, the solution to (3) (assumed to exist; note that the function V is continuous and increasing in L, so that a solution L^* to (3) can only be uniquely defined).

First note that in case $\lambda = 0$, one necessarily has $U(r) \geq V(R)$, since the constant N^* solving the Libor maximization problem (2) is a particular strategy (constant process $n_t = N^*$) of the OIS maximization problem (1). As V is an increasing function, the indifference pricing equation (3) in turn yields that $L^* \geq R$.

Let $\mathcal{V}_0(\cdot;N)$ be the utility of lending Libor in case $\lambda=0$. When $\lambda>0$, for each given amount N, one has via λ which is present in γ in (6) that $\mathcal{V}(R;N)\leq\mathcal{V}_0(R;N)$ (up to the second order impact of the discount factor ℓ). Hence $V(R)\leq V_0(R)\leq U(r)$ follows, the latter inequality resulting from the inequality already proven in case $\lambda=0$ joint to the fact that τ doesn't appear in $\mathcal{U}(r;n)$. We conclude as in case $\lambda=0$ that $L^*\geq R$.

For notational convenience let us introduce the time-space probability measures $\overline{\mathbb{P}}$ product of $\mathbb{P}$ times $\frac{dt}{T}$ over $\Omega\times[0,T]$, along with $\widehat{\mathbb{P}}$ such that $\frac{d\widehat{\mathbb{P}}}{d\overline{\mathbb{P}}}\propto\ell$. For a process $f=f_t(\omega)$ we denote the corresponding time-space averages by

$$\bar{f}=\overline{\mathbb{E}}f=\mathbb{E}\frac{1}{T}\int_0^T f_t dt\,,\;\;\widehat{f}=\widehat{\mathbb{E}}f=\overline{\mathbb{E}}\left[f\frac{\ell}{\bar{\ell}}\right]$$

(so, in this notation, $R=\bar{r}$). Let similarly for every processes f,g

$$\overline{\mathbb{C}\mathrm{ov}}(f,g)=\overline{\mathbb{E}}(fg)-\overline{\mathbb{E}}f\overline{\mathbb{E}}g\,,\;\;\widehat{\sigma}_f^2=\widehat{\mathbb{E}}(f-\widehat{f})^2. \tag{8}$$

Since

$$U(r)=\overline{\mathbb{E}}\left[\frac{c^2}{2\beta}\right]\qquad\text{and}\qquad V(L)=\frac{\bar{\ell}^2\,(L-\widehat{\gamma})^2}{2\overline{\mathbb{E}}[\beta\ell]}$$

equating $V(L^*)=U(r)$ yields

$$\bar{\ell}^2\,(L^*-\widehat{\gamma})^2=\overline{\mathbb{E}}[\beta\ell]\overline{\mathbb{E}}\left[\frac{c^2}{\beta}\right] \tag{9}$$

in which

$$\overline{\mathbb{E}}[\beta\ell]\overline{\mathbb{E}}\left[\frac{c^2}{\beta}\right]=\overline{\mathbb{E}}\left[c^2\ell\right]-\overline{\mathbb{C}\mathrm{ov}}\left[\beta\ell,\frac{c^2}{\beta}\right]$$

So

$$\bar{\ell}^2(L^*-\widehat{\gamma})^2=\bar{\ell}\widehat{\mathbb{E}}\left[c^2\right]-\overline{\mathbb{C}\mathrm{ov}}\left[\beta\ell,\frac{c^2}{\beta}\right]$$

A particularly interesting point is "at-the-money" when $R=\widehat{\alpha}=\widehat{\gamma}-\widehat{\lambda}$, i.e. (recall $R\equiv\bar{r}$) $\widehat{c}=\widehat{\alpha}-\widehat{r}=\bar{r}-\widehat{r}$. Then the previous formula reads :

$$\bar{\ell}(L^*-R-\widehat{\lambda})^2=\widehat{\sigma}_c^2+\bar{\ell}^{-1}(\bar{r}-\widehat{r})^2-\overline{\mathbb{C}\mathrm{ov}}\left[\beta\frac{\ell}{\bar{\ell}},\frac{c^2}{\beta}\right] \tag{10}$$

A reasonable guess is that $\bar{\ell}^{-1}(\bar{r}-\widehat{r})^2$ and the covariance are negligible in the right-hand-side (in particular these terms vanish when the credit risk deterioration intensity λ is zero and the marginal cost-of-capital coefficient β is constant). More precisely, for the sake of the argument, let us postulate a diffusive behavior of the instantaneous funding spread process $c_t=\alpha_t-r_t$, i.e. $dc_t=\sigma^*dW_t$ for some "reference volatility" σ^* and a Brownian motion W. Let us also assume a constant $\lambda_t=\lambda^*$ ("reference credit skew" of the borrower) and a constant marginal cost of borrowing β. Neglecting the impact of the "discount factor" $\ell_t=e^{-\lambda^*t}\approx 1-\lambda^*t$ in (10) (so that $\widehat{\mathbb{P}}\approx\overline{\mathbb{P}}$), it follows that

$$\widehat{\sigma}_c^2\approx\bar{\sigma}_c^2=(\sigma^*)^2\mathbb{E}\frac{1}{T}\int_0^T W_t^2dt=(\sigma^*)^2T/2$$

and our "LOIS formula" follows from the above as[2]:

$$L^* - R \approx \lambda^* + \sigma^* \sqrt{T/2} \tag{11}$$

From a broader perspective, according to formula (10), the two key drivers of the LOIS are:

- a suitable average λ^* of the borrower's credit skew λ, which can be seen as the "intrinsic value" component of the LOIS and is a borrower's credit component;
- a suitable volatility σ^* of the instantaneous funding spread process c_t; this second component can be seen as the "time-value" of the LOIS and interpreted as a lender's liquidity component.

From a quantitative trading perspective, the formula (11) can be used for implying the value σ^* "priced" by the market from an observed LOIS $L^* - R$, and a borrower's CDS slope taken as a proxy for λ^*. The value σ^* thus implied through (11) can be compared by a bank to an internal estimate of its "realized" funding spread volatility, so that the bank can decide whether it should rather lend Libor or OIS, much like with going long or short an equity option depending on the relative position of the implied and realized volatilities of the underlying stock. Another possible application of the formula (11) is for the calibration of the volatility σ^* of the funding spread process c_t in a stochastic model for the latter, e.g. in the context of multiple-curve CVA computations.

3.1 Numerical Illustration

Fig. 2 shows the EUR market 15/08/2007 to 16/04/2012 time series of the intercept, slope and $R2$ coefficients of the linear regression of the LOIS term structure against $\sqrt{T/3\text{m}}$ or $\sqrt{T/6\text{m}}$, for T varying from 1m to 1yr (of course choosing $\sqrt{T/3\text{m}}$ or $\sqrt{T/6\text{m}}$ as a regressor only affects the slope coefficient of the regression, by a factor of $\sqrt{2}$). In our financial interpretation, the intercept represents the credit component of the LOIS (of any tenor T), while the slope coefficients represents the liquidity component of the LOIS for the Libor with tenor $T = 3\text{m}$ or 6m (we recall that 3m is the most liquid tenor on the Libor markets and 6m is the second most liquid one). So the red and blue (resp. red and purple) curves on the figure can be viewed as the credit and liquidity components of the 3m (resp. 6m) Libor. Before Aug 2007 the LOIS is negligible so that the regression (not displayed on the figure) is insignificant. Since the"advent" of the LOIS in mid-Aug 2007, we can distinguish three market regimes. In a first phase, until Q1 of 2009, the market seems to "try to understand" what happens, with an R2 becoming significant together with very large and volatile credit ("red intercept") and liquidity ("blue or purple slope") LOIS components. Note in particular the spike of both components at the turn of the credit crisis following Lehman's default in Sept 2008, during which the interbank drop of trust created both a credit and a liquidity crisis. Between Q2 of 2009 and mid-2011, the situation seems "stabilized" with an R2 close to 1, a liquidity LOIS component of the order of 30bps on the 3m or 45bps on the 6m and a much smaller credit LOIS component. The ongoing Eurozone crisis, prompted by the US downgrade mid-2011, reveals a third pattern with a much higher liquidity LOIS component, of the order of 60bps on the 3m or 90bps on the 6m, revealing increased funding liquidity concern of banks, due to harder regulatory constraints

[2] Admitting that $L^* - R - \widehat{\lambda} \geq 0$, a natural assumption as Libor lending should at least compensate for the credit risk over the tenor horizon period, cf. the comments following the equation (7).

(e.g. government bonds no longer repositable). To illustrate the three "market regimes" in this analysis, Fig. 3 shows the fit between a theoretical square root term structure and the empirical LOIS term structure corresponding to the Euribor/Eonia-swap data of 14 Aug 2008, 28 Apr 2010 and 16 Apr 2012 (data of the right panel in Fig. 1). The last two terms that we neglected in (10) to deduce (11) are a possible explanation for (minor) departures of the actual LOIS spread curve from the theoretical square root term structure implied by (11).

In relation with the discussion of the economical determinants of λ_t and α_t in Subs. 2.1, note that intercepts of e.g. 10 bp appear as reasonable for a "credit skew", that is, the differential between the one year CDS spread and the short term certificate deposit credit spread of a major bank, while the coefficient of $\sqrt{T/2}$, ranging between 100 and 200 bp/yr (corresponding to magnifying on Fig. 2 by a factor 2 the purple curve, or slope coefficients of the regression against $\sqrt{T/6\mathrm{m}}$,), is quite in line with recent orders of magnitude of the volatility of major banks' one year CDS spreads.

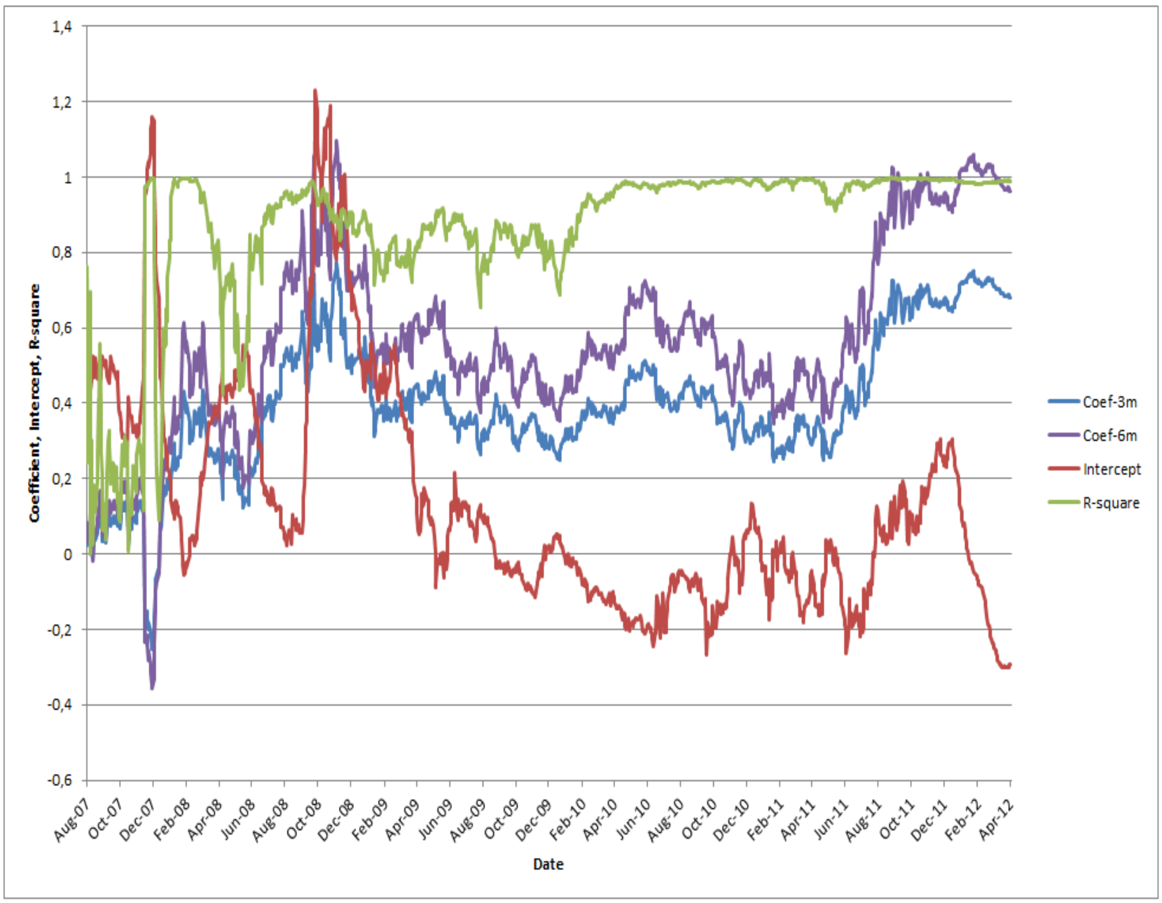

Figure 2: Time series of the "red" intercepts (in %; credit component of the LOIS), "blue" and "purple" slopes (in %; liquidity component of the 3m- and 6m- LOIS) and "green" R2 coefficients of the regressions of the 1m to 1yr LOIS against $\sqrt{T/3\mathrm{m}}$ or $\sqrt{T/6\mathrm{m}}$, over the period from 15/08/2007 to 16/04/2012.

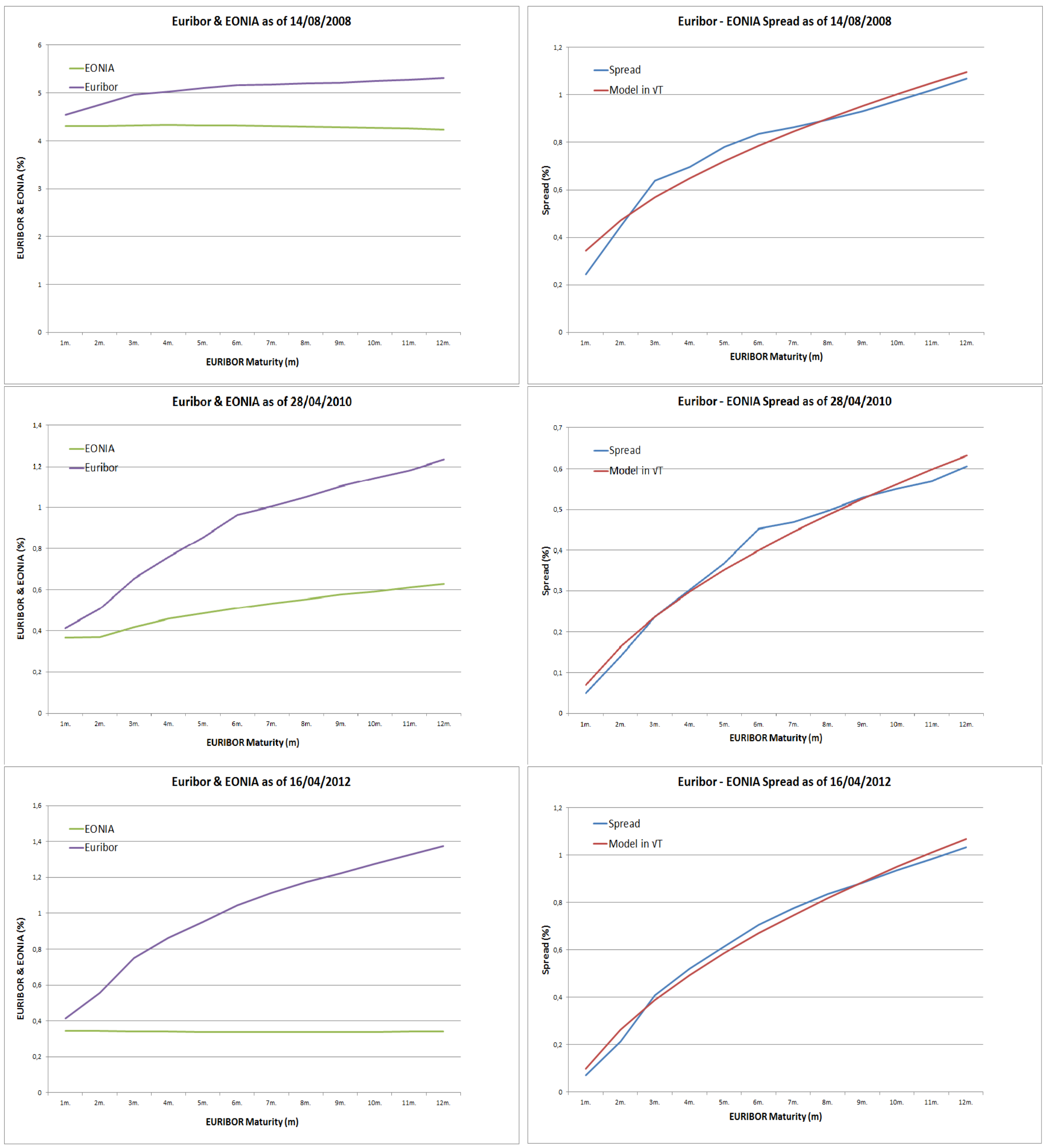

Figure 3: Euribor / EONIA-swap rates (left) and square root fit of the LOIS (right), $T = 1\text{m}$ to 12m. Top to bottom: 14 Aug 2008, 28 Apr 2010 and 16 Apr 2012 (data of the right panel in Fig. 1).

Conclusion

Since the 2007 subprime crisis, OIS and Libor markets (Eonia and Euribor in the EUR market) diverged suddenly. We show that, by optimizing their lending between Libor and OIS markets, banks are led to apply a spread (LOIS) over the OIS rate when lending at Libor. Theory implies that the LOIS has two components: one corresponding to the credit skew λ_t of a representative Libor borrower in an interbank loan, and one corresponding to the liquidity funding spread $c_t = \alpha_t - r_t$ of a representative Libor lender, where α_t and r_t respectively denote the instantaneous refinancing rate of the lender and the overnight interbank rate. Assuming a diffusive evolution of the instantaneous funding spread c_t, the above-mentioned optimization results in a square root term structure of the LOIS given by the formula (11), where the intercept λ^* can be proxied by the slope of a representative LIBOR credit curve and the coefficient σ^* is a volatility of c_t. These theoretical developments are corroborated by empirical evidence on the EUR market studied in this paper on the period half-2007 half-2012, with LOIS explained in a balanced way by credit and liquidity until the beginning of 2009 and dominantly explained by liquidity since then. The methodology of this paper is relevant for any market in which credit and funding liquidity are the main drivers of interbank risk. Residual discrepancies between the theory and the data can be explained by the existence of other features such as Libor manipulations[3]. With respect to a multiple-curve pricing approach where market OIS and Libor curves are simply fitted, the equilibrium approach of this paper allows a bank to, in principle, arbitrage the LOIS, by preferably lending Libor (resp. OIS) whenever its internally estimated funding spread is statistically found less (resp. more) volatile than σ^* implied from the market through the LOIS formula (11). Another application of this formula is for the calibration or estimation of the volatility σ^* of the funding spread process c_t in a stochastic model for the latter.

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[3]As explained at the end of Subs. 2.1, Libor manipulations can also be modeled within the framework of this paper by a spread in the borrower's credit risk skew component λ_t.

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