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The Whys of the LOIS: Credit Skew and Funding Spread Volatility

Abstract

Since the 2007 subprime crisis, OIS and Libor markets (Eonia and Euribor in the EUR market) diverged suddenly (See Fig.1 and 2). In this note we show how, by optimizing their lending between Libor and OIS markets, banks are led to apply a spread (LOIS) over the OIS rate when lending at Libor.

Introduction

The main reference rate for a variety of fixed income derivatives is Libor in the USD market and Euribor in the EUR market. Libor (resp. Euribor) is computed daily as an average of the rates at which a designated panel of banks believe they (resp. a prime bank) can obtain unsecured funding, for periods of length up to one year. In this note we shall use the term Libor and the letter *L* to denote any of these two rates. Following the recent credit crunch, which severely impacted trust between financial institutions, overnight interest rate swaps (OIS) became more and more popular. In these financial instruments, the floating rate is obtained by compounding an overnight interbank rate (O/N) r_t , i.e. a rate at which overnight unsecured loans can be obtained in the interbank market (the Federal Funds rate in the USD fixed income market and the EONIA rate in the EUR market). As a result, an OIS rate *R* can be interpreted as a suitable average of r_t .

In theory, arbitrage relations imply that $L = R$. However, the interbank loan market has been severely impacted since the 2007 subprime crisis and the ensuing liquidity squeeze.

The reference interbank rate remains Libor, though, which still underlies most vanilla interest-rate derivatives like Swaps, FRA, IRS, cap/ floor and swaptions. The resulting situation where an underlying to financial derivatives has become in a sense arbitrarily fixed by a panel of key players in the derivatives market

poses insider issues, as illustrated by the recent cases of Libor manipulation. But first of all, it poses a crucial funding issue as, on the one hand, in parallel to the drying up of the interbank loan market, Libor got disconnected from OIS rates (see Fig. 1 and Fig. 2); whilst on the other hand, as more and more trades are collateralized, their effective funding rate is the corresponding collateral rate, which is typically indexed to the O/N r_t .

This creates a situation where the price of an interest-rate product, even the simplest flow instrument like a FRA, involves (at least) two curves, a Libor fixing curve and an OIS discount curve, as well as the related convexity adjustment, which in the case of optional products can be significant (cf. (Mercurio 2010)).

Via the relations between counterparty risk and funding this also has some important CVA implications; see (Crepey 2012; Pallavicini, Perini, and Brigo 2011).

LOIS Formula

Commonly advanced explanations for the Libor-OIS spreads ($L - R$), often called LOIS in the market, are a combination of credit risk and/or liquidity risk.

Nevertheless, in these explanations the meaning of liquidity is either not precisely stated, or it is simply defined as a residual after removal of a credit component. In (Crepey and Douady 2013), we propose a stylized equilibrium model to evaluate at which rate does a bank find it worthwhile to lend at a given tenor horizon, as opposed to rolling an overnight loan which it can cancel at any moment. Let n_t represent the amount of notional that the bank is willing to lend at the O/N rate r_t between t and $t+dt$. Let N represent the (fixed) notional of a Libor loan at rate L over a period of length T . Let $U(r) = \max_n U(r; n)$ and $V(L) = \max_n V(L; N)$ represent the best

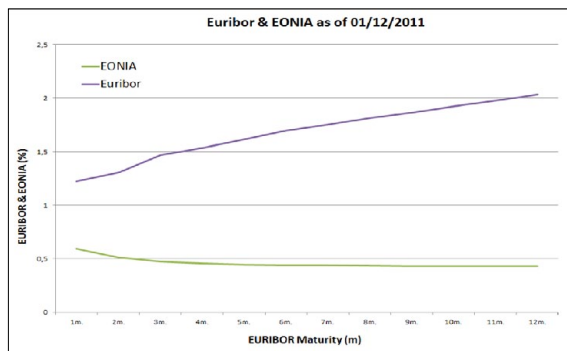


Fig. 1: Euribor / EONIA rates.

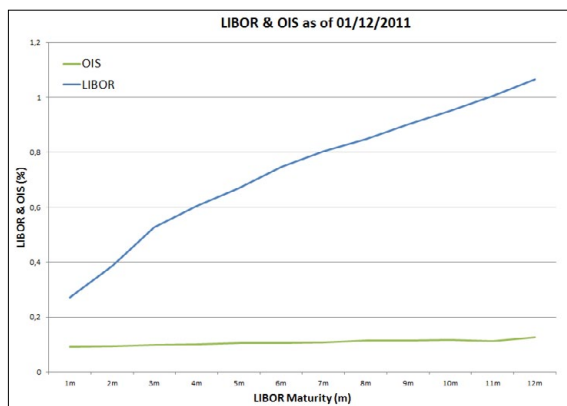


Fig. 2: Libor / OIS rates.

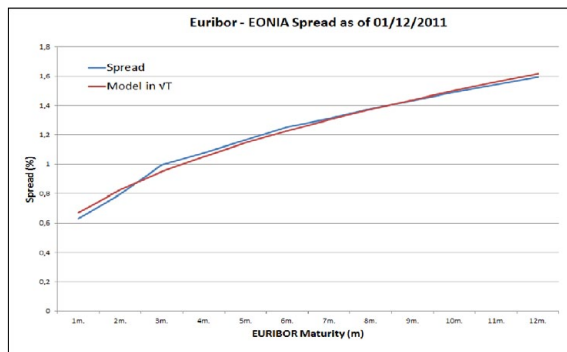


Fig. 3: Square root fit of Euribor-EONIA spread.

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tradeoffs between the gain on the remuneration of the loan and the cost of refinancing it a bank can achieve by lending OIS or Libor, respectively. Our approach for explaining the LOIS ($L - R$), where R is defined as a suitable average of the O/N process r (assumed given), consists in solving the equation

$$V(L) = U(r) \quad (1)$$

for L . This equation expresses an equilibrium relation between the utility of lending rolling overnight versus Libor for a bank involved in both markets (indifference value at the optimal amounts prescribed by the solution to the corresponding optimization problems). Then, under suitable assumptions on U and V LOIS emerges from (1) as a consequence of the slope of the credit curve of a representative Libor panelist (playing the role of the borrower in an interbank loan), and of the volatility σ^* of the spread $c_t = \alpha_t - r_t$ between the refinancing rate α_t of a representative Libor panelist (playing the role of the lender) and the O/N r_t . More specifically, we derive from (1) the following "LOIS formula":

$$L - R \approx \lambda^* + \sigma^* \sqrt{T/2} \quad (2)$$

where the first term, λ^* , is a borrower's credit component, which can be seen as the "intrinsic value" component of the LOIS; the second term, $\sigma^* \sqrt{T/2}$, is a lender's liquidity

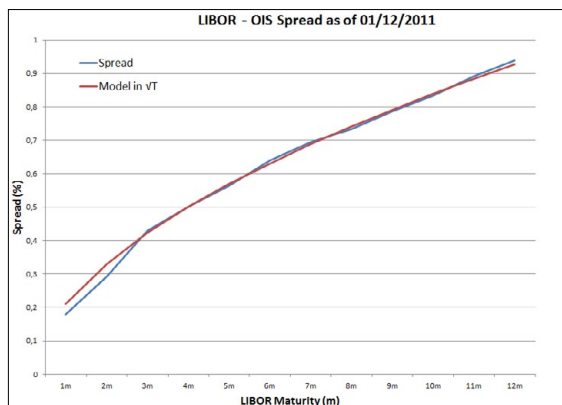


Fig. 4: Square root fit of Libor-OIS spread.

component, which can be seen as the "time-value" of the LOIS.

As illustrated by Fig. 3 and Fig. 4, empirical observations reveal a square root term structure of the LOIS consistent with this theoretical analysis, with, on the EUR market more specifically studied in (Crepey and Douady 2013) on the period half-2007 half-2012, LOIS explained in a balanced way by credit and liquidity until the beginning of 2009 and dominantly explained by liquidity since then.

Conclusion and Perspectives

Under an equilibrium approach, the Libor-OIS spread (LOIS) emerges as a consequence of a credit component determined by the skew of the CDS curve of a representative Libor panelist (playing the role of the "borrower" in an interbank loan) and a liquidity component corresponding to a volatility of the spread

between the refinancing (or funding) rate of a representative Libor panelist (playing the role of the "lender") and the overnight interbank rate.

From a quantitative trading perspective, the LOIS formula (2) can be used for implying the value σ^* "priced" by the market from an observed LOIS ($L - R$), and a borrower's CDS slope taken as a proxy for λ^* . The implied σ^* can be compared by a bank to an internal estimate of its "realized" funding spread volatility, so that the bank can decide whether it should rather lend Libor or OIS, much like with going long or short an equity option depending on the relative position of the implied and realized volatilities of the underlying stock. Another possible application of the LOIS formula is for the calibration of the volatility σ^* of the funding spread process c_t in a stochastic model for the latter, e.g. in the context of CVA computations.

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