Double Exponential Jump Diffusion Process: A Structural Model of Endogenous Default Barrier with Roll-over Debt Structure

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Abstract

In this paper, we extend the framework of Leland (1994b) who proposed a structural model of roll-over debt structure in a Black-Scholes framework to the case of a double exponential jump diffusion process. We consider a trade-off model with firm's parameters as firm risk, riskfree interest rate, payout rate as well as tax benefit of coupon payments, default costs, violation of the absolute priority rule and tax rebate. We obtain the equity, debt, firm and credit spreads values in closed form formulae. We analyze these values as functions of coupon, leverage and maturity.

Keywords: Double Exponential Jumps, Structural Approach, Credit Spreads, "Roll-over" Debt Structure.

JEL Classification: G12, G13, G33

1. Introduction

The problem of the firm's optimal capital structure and its endogenous default barrier has been considered greatly in a series of papers by Leland (1994a)[14], Leland (1994b)[15] and Leland and Toft (1996)[16]. Leland (1994b)[15] has studied a roll-over debt structure which allows simultaneous consideration of the coupon-paying bond with arbitrary maturity, while

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maintaining in a time-homogeneous environment. Leland and Toft (1996)[16] derived closed-form formulae for risky coupon paying bonds with a fixed maturity.

Due to the choice of a geometric Brownian motion process for the firm value dynamics, the credit spreads generated by their model and especially the credit spreads of short term debt were too low and did not match empirical data. Furthermore, the presence of jumps in the asset price processes was further endorsed and supported by Bates (1996)[1] and Jorion (1988)[10] publications among others. In concordance with these empirical works, a jump diffusion process is particularly helpful in correcting the two main empirical failures of the normal distribution process such as firstly, the large random fluctuations (discontinuous paths) as crashes and secondly, the non-normal features, as the negative skewness and the leptokurtic feature of the return distribution (see Papapantoleon and Senge (2002)[17] and Ramezani and Zeng (2004)[18], for example). As we shall see, the jump diffusion process also helps to increase the credit spreads.

Hilberink and Rogers (2002)[8] introduced Lévy processes[1] in the model of Leland referring to an endogenous default barrier framework. Le Courtois and Quittard-Pinon (2004)[13] have extended the Leland (1994b) framework by using a Lévy stable process having two-sided jumps without diffusion part, as the asset price process.

In this paper, we examine the debt value subjected to default risk in the structural approach. We propose a continuous time framework with a double exponential jump diffusion process which offers both negative and positive jumps. As in Leland (1994b)[15], we consider a roll-over debt structure with regular repayments and renewals of principal and of coupon. Thanks to this special debt structure, we are able to examine coupon paying bonds with arbitrary maturity, while remaining in a time homogeneous environment. This flexible roll-over debt structure proposes a wide class of possible debt structures and offers an analysis of debt value and of yield spreads with arbitrary maturity.

The double exponential jump diffusion, first proposed by Kou (2002)[11][2] for the asset process is a special case of the Lévy processes which has the two interesting properties of the exponential distribution. The first property is that the double exponential distribution, which performs two-sided jumps, has the leptokurtic feature of the jump size that provides the peak and tails of the return distribution found in reality. The second property is that the double exponential distribution has a memoryless feature which makes it easier to calculate expected means and variance terms. This memoryless property helps also to solve the problem of overshoots: this problem occurs when a jump diffusion process V crosses a barrier level L at time τ as sometimes it hits the barrier level exactly, i.e. $V_\tau = L$, and sometimes it incurs an "overshoot" over the barrier, i.e. $V_\tau \neq L$. As a consequence, we can compute the Laplace transform of the first passage times. Chen and Kou (2005)[3] also independently proposed a double exponential jump diffusion model very close to our model, emphasizing on implied volatility and different forms of credit spreads.

We further consider in our model the tax benefit of coupon payments and the reorganization costs at default while studying two additional factors usually observed in financial markets, in order to obtain a more realistic model. The first factor is the violation of the Absolute Priority Rule (APR), which means at default time, the shareholders take a part of the firm's value and do not respect the bondholders first priority. Empirical studies conducted by Franks and Torous

[1] See the book of Cont and Tankov (2003)[4] for a study of Lévy Processes

[2] The double exponential distribution was first introduced by Laplace, and the symmetric double exponential distribution was called Laplace distribution.

(1989)[7] and Eberhart, Moore, and Roenfeldt (1990)[6] showed that APR is violated in about 50% of the cases. We suppose that the shareholders will take γ of the firm's expected value, after the reorganization costs at default, and the bondholders will receive only $(1-\gamma)$ of the residual expected value at default. The second factor is the tax rebate, since it is more reasonable to account for the tax benefit or tax rebate cutoff. Leland (1994b)[15] introduced a tax cutoff level, which reduces the tax rebate to zero when the firm's assets value is lower than the tax cutoff level, while it remains the same when the firm's assets value is higher. This assumption presumes that when the coupons exceed the profits, it is not possible for the firm to claim the tax benefit on the coupon payments.

In this framework, in order to quantify the values of debt, equity, and firm, we calculate the Laplace transform of the pair first passage time of the process V at a level L and the value of V at this first passage time. when the firm's asset value process crosses a barrier and the Laplace transform of the first passage time and its value at passing time. We obtain the value of default barrier in closed form. Hence, we obtain values for the equity, the debt, the firm and the credit spreads in closed form formulae.[3]

The article is organized as follows: Section 2 describes the debt equity structure of the firm as well as the general formulae of debt, equity and firm values. Section 3 presents the model of double exponential jump diffusion processes and the computation of the Laplace transform of the first passage time. The formulae for the optimal default barriers with and without the presence of tax benefit cutoff level cases are given in Section 4. Section 5 contains the debt analysis on debt value, yield spread value, firm value and the optimal capital structure. We conclude in Section 6.

2. Debt "Roll-over" Structure

We consider a stationary, time homogeneous, debt structure as modelled in Leland (1994b)[15]. At each moment of time, the firm has the same debt structure with constant total principal $P > 0$, paying a constant total coupon rate $C > 0$ (if there is no default). The firm continuously rolls over a fraction $m > 0$ of debt. This means it continuously retires outstanding debt principal at the rate mP. The debt retirement is replaced (except in the case of bankruptcy) by newly issued debt of equal coupon, principal, and seniority. Note that the price of newly issued debt depends upon the current asset value V_t.

In the time interval $(t, t+dt)$, the firm issues a new debt principal, with a constant rate p, i.e., equal to a constant amount pdt. Since $mPdt$ is the total amount of debt principal retired in the interval, it follows that $p = mP$.

The fraction of current outstanding debt principal which is redeemed at time t is me^{-mt} and by definition the maturity profile is:

$$\varphi(t) = m\, e^{-mt} \tag{2.1}$$

[3]Other applications of the double exponential jump diffusion process in a structural approach can be found in Dao's thesis (2005)[5].

If bankruptcy never occurs, the proxy average maturity T of debt is:

$$T = \int_0^\infty t\,\varphi(t)\,dt = \int_0^\infty t\,m\,e^{-mt}\,dt = \frac{1}{m} \tag{2.2}$$

The higher the debt retirement rate (or "roll-over rate") m, the shorter the average maturity $T = 1/m$ of the debt. If $m = 0$, principal is never retired, and debt has infinite maturity as in Leland (1994)[14]. If $m \to \infty$, the average maturity of debt T approaches zero (likely short term maturity debt).

It is important to emphasize that this kind of debt structure is a very rich, flexible and time independent structure that allows an analysis of debt maturity. The real world equivalent of this debt structure is a sinking fund provision. Sinking funds are quite common in corporate debt issues. They require that a fraction of the principal value of debt be retired (or amortized) on a regular basis. Smith and Warner (1979)[19], Ho and Singer (1984)[9] stated that this kind of debt structure constitutes a large proportion of corporate debt.

This mechanism of debt, with continuous payment of coupon and principal and renewal of the same amount, is subject to default. Bankruptcy or default occurs at the first time τ when the value of the firm's assets V falls below a default barrier L. The firm's assets V is supposed to be a non negative semi-martingale càdlàg such that there exists a risk neutral probability Q. The value of the default barrier L will be determined optimally by maximizing the equity value, and the default time will be:

$$\tau = \tau_L(V) = \inf\{t \geq 0 : V_t \leq L\} \tag{2.3}$$

Bankruptcy will occur at an endogenously determined asset value L, when equity value is no longer sufficient to cover the required bond coupon and the refinancing. More precisely, bankruptcy occurs when the asset value of the firm drops to a level such that the firm can no longer raise sufficient capital (or equity can no longer be issued to meet) to retire the required amount of debt, plus pay the current total coupon (debt service requirement).

All debts are supposed to be of equal seniority, and coupon payment is at the fixed rate c (where $cP = C$) until maturity or until default (if this occurs sooner than maturity). Let α ($0 \leq \alpha \leq 1$) be the constant fraction of asset value lost in the event of default. Suppose that the APR is not respected and the shareholders will receive γ ($0 \leq \gamma \leq 1$), the constant fraction of the residual asset value at default.

The value of a bond with coupon c issued at time 0 with face value 1 and maturity t can be written under the risk neutral probability $\mathbb{Q}$ as follows:

$$\begin{aligned} d_0(V, L, t) &= \mathbb{E}_{\mathbb{Q}}\left[\int_0^{t\wedge\tau} ce^{-rs}ds\right] + \mathbb{E}_{\mathbb{Q}}\left[e^{-rt}\mathbf{1}_{t<\tau}\right] \\ &\quad + \frac{1}{P}(1-\alpha)(1-\gamma)\,\mathbb{E}_{\mathbb{Q}}\left[V_\tau e^{-r\tau}\mathbf{1}_{\tau\leq t}\right] \end{aligned} \tag{2.4}$$

The first term on the right hand-side of equation (2.4) can be interpreted as the expected present value of all coupons paid up till time t or default time τ. The second term is the expected

present value of the principal repayment if there is no default before t, and the last term is the expected present value of what the bondholders receive upon default (if this happens before maturity). Note that the value at default V_τ can be different from the default barrier L since the process of the firm's asset $(V_t, t \geq 0)$ is not assumed to be continuous. The bondholder with face value 1 gets only the fraction of $\frac{1}{P}$, since the total principal is P and the part $(1-\alpha)(1-\gamma)V_\tau$ belongs to all bondholders after taking into account the costs of default and the shareholders part (due to the violation of APR).

Let us note that $(1-\alpha)(1-\gamma) = 1-\hat{\alpha}$, where $\hat{\alpha} = (\alpha+\gamma-\alpha\gamma)$ is the fraction of asset value lost at default from the debt holders point of view. It is important for the debt holders to consider the residual fraction of asset value at default, the default costs (it is not relevant whether these costs are due to reorganization costs or due to shareholders' power).

Setting

$$\Phi(t) = \int_t^\infty \varphi(y)\,dy = e^{-mt}$$

where $\varphi(y)$ is the maturity profile form as assumed in (2.1), then the total value at time 0 of all outstanding debt is:

$$\begin{aligned} D(V,L) &= \int_0^\infty p\Phi(t)\,d_0(V,L,t)\,dt \\ &= pc\mathbb{E}_{\mathbb{Q}}\left[\int_0^\tau e^{-rs}\hat{\Phi}(s)\,ds\right] + p\mathbb{E}_{\mathbb{Q}}\left[\int_0^\tau e^{-rs}\Phi(s)\,ds\right] + \frac{p}{P}(1-\hat{\alpha})\mathbb{E}_{\mathbb{Q}}\left[V_\tau e^{-r\tau}\hat{\Phi}(\tau)\mathbf{1}_{\tau<\infty}\right] \end{aligned}$$

where we have used Fubini-Tonelli's theorem and where:

$$\hat{\Phi}(t) = \int_t^\infty \Phi(s)\,ds = \frac{1}{m}e^{-mt}$$

So the total value of the debt can be written as:

$$\begin{aligned} D(V,L) = \frac{p}{m}\frac{c}{r+m}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-(r+m)\tau}\right] + \frac{p}{r+m}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-(r+m)\tau}\right] \\ + \frac{p}{mP}(1-\hat{\alpha})\mathbb{E}_{\mathbb{Q}}\left[V_\tau e^{-(r+m)\tau}\mathbf{1}_{\tau<\infty}\right] \end{aligned}$$

Note that the expectation $\mathbb{E}_{\mathbb{Q}}\left[V_\tau e^{-(r+m)\tau}\mathbf{1}_{\tau<\infty}\right]$ is finite as V_τ is bounded above by L and below by 0. Writing V_t as $V_t = Ve^{X_t}$ and $L = Ve^l$, we obtain the following identity between first passage times of the process $(V_t, t \geq 0)$ and the process $(X_t, t \geq 0)$:

$$\tau = \tau_L(V) = \tau_l(X)$$

As $P = \frac{p}{m}$ and $C = cP$, the total value of the debt simplifies to:

$$D(V,L) = \frac{C+mP}{r+m}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-(r+m)\tau}\right] + (1-\hat{\alpha})V\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau-(r+m)\tau}\mathbf{1}_{\tau<\infty}\right] \tag{2.5}$$

In the two extreme cases of the roll-over rate value m, we have from dominated Lebesgue's theorem:

$$D(V,L) \underset{m\to 0}{\to} \frac{C}{r}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-r\tau}\right] + (1-\hat{\alpha})V\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau-r\tau}\mathbf{1}_{\tau<\infty}\right]$$

$$\text{and } D(V,L) \underset{m\to\infty}{\to} P$$

The limit as $m \to 0$ corresponds to the debt value in the perpetual coupon payment as in Leland (1994a).

The value of the firm is calculated as follows. We assume that there is a corporate tax rate θ and that the firm can benefit a tax rebate of coupon payment up to default. Indeed, the firm receives an income stream of $\theta C dt$, i.e., the expected present value of tax benefit $\frac{\theta C}{r}\left(1-e^{-r\tau}\mathbf{1}_{\tau<\infty}\right)$. This assumption is rather an idealized treatment of tax. Leland and Toft (1996)[16] introduced a tax cutoff level V_T, which reduces the tax rebates to 0 when $V < V_T$ and remains $\theta C dt$ when $V \geq V_T$. This assumption reflects the idea that when the coupons exceed the profits, it is not possible for the firm to reclaim the tax benefit on the coupon payments. Later, we shall account for this assumption in our setup.

At this stage, assuming no cutoff for tax benefits, the value of firm is:

$$\begin{aligned} v = v\left(V,L\right) &= V + \mathbb{E}_{\mathbb{Q}}\left[\frac{\theta C}{r}\left(1-e^{-r\tau}\right)\right] - \mathbb{E}_{\mathbb{Q}}\left[\alpha V_\tau e^{-r\tau}\mathbf{1}_{\tau<\infty}\right] \\ &= V + \frac{\theta C}{r}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-r\tau}\right] - \alpha V \mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - r\tau}\mathbf{1}_{\tau<\infty}\right] \end{aligned} \tag{2.6}$$

The value of the firm is composed of the three terms. The first term is the value of the firm's assets (V). The second term is the expected value of the tax benefit and the third one is the expected value of bankruptcy cost.

Using (2.5) and (2.6), the equity value of the firm, which is equal to the firm value minus the debt value, can be expressed as:

$$\begin{aligned} E = E\left(V,L\right) &= v\left(V,L\right) - D\left(V,L\right) \\ &= V + \frac{\theta C}{r}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-r\tau}\right] - \alpha V \mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - r\tau}\mathbf{1}_{\tau<\infty}\right] \\ &\quad - \frac{C+mP}{r+m}\mathbb{E}_{\mathbb{Q}}\left[1-e^{-(r+m)\tau}\right] + \left(1-\hat{\alpha}\right)V\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau-(r+m)\tau}\mathbf{1}_{\tau<\infty}\right] \end{aligned} \tag{2.7}$$

The shareholders's objective is to maximize the equity value. As in Leland (1994a)[14], Leland and Toft (1996)[16] and Hilberink and Rogers (2002)[8], the optimal default barrier L is determined endogenously by maximizing the equity value. The optimal criterion is known to be the "smooth pasting" condition (see appendix D in Hilberink and Roger (2002)[8]), that is:

$$\frac{\partial E}{\partial V}\,|_{V=L} = 0 \tag{2.8}$$

In the model of double exponential jump diffusion, we shall show later that we obtain the closed-form solutions for the equity value as well as the barrier L.

Remark: Looking at the values of debt, firm, equity and default barrier, as expressions in equations (2.5), (2.6), (2.7) and (2.8) respectively, we observe that these quantities are expressed in terms of Laplace transforms: $\mathbb{E}_{\mathbb{Q}}\left[e^{-\rho\tau}\right]$ is the Laplace transform of the time τ evaluated at $\rho \geq r$ and $\mathbb{E}_{\mathbb{Q}}\left[e^{\xi X_\tau - \rho\tau}\mathbf{1}_{\tau<\infty}\right]$ is the Laplace transform of the pair (the value of the process X at the passage time, first passage time) evaluated at $\xi = 1$ and $\rho \geq r$.

In the next section, we present the double exponential jump diffusion model and we obtain closed-form solutions of these Laplace transforms. We deduce closed-form formulae for the values of debt, firm, equity and by consequence, the default barrier. Therefore, we extend the results of Leland (1994a)[14] and Leland and Toft (1996)[16]. To the best of our knowledge, the geometric Brownian motion model and the double exponential jump diffusion model are the only one for which closed-form solutions are known.

3. Double Exponential Jump Diffusion Process

We assume now that the value of the firm's assets follows a jump diffusion process of the form:

$$\frac{dV_t}{V_{t-}} = (r-\delta)dt + \sigma dW_t + dM_t \tag{3.1}$$

where the riskfree interest rate r, the firm payout ratio δ and the volatility of the firm's asset value σ are supposed to be constant. Here, the process $(W_t, t \geq 0)$ is a one dimensional standard Brownian motion under $\mathbb{Q}$. The process $(M_t, t \geq 0)$ is the $\mathbb{Q}-$compensated martingale of a compound Poisson process, i.e.,

$$M_t = \left(\sum_{k=1}^{N_t} Z_k\right) - \lambda \mathbb{E}_{\mathbb{Q}}\left[Z_1\right] \tag{3.2}$$

where $(N_t, t \geq 0)$ is a $\mathbb{Q}-$Poisson process with a constant intensity rate $\lambda > 0$, the random variables $\{Z_k, k \geq 0\}$ are i.i.d and $\mathbb{E}_{\mathbb{Q}}\left[Z_1\right] < \infty$. All sources of randomness N, W and Z_k's are assumed to be independent under $\mathbb{Q}$.

Remark: The firm payout ratio δ is a proportional rate at which profit is distributed to investors (both shareholders and bondholders). As presented in the previous section the roll-over debt structure, the coupon is paid at rate c to bondholders, in addition, $(p-d)$ is the net cash outflow associated with redeeming a fraction m of the principal P, minus the market value d of the floating new debt of equal coupon C and principal P but whose value fluctuates with V. Thus $\delta_e = \delta V - c - p + d$ is the payout rate to stockholders. This payout rate δ_e declines as the firm's asset value V declines and may become negative (i.e., new equity must be issued to meet bond requirements). At a certain level of the firm's asset value, the contributions required are no longer met by shareholders with limited liability, and bankruptcy occurs.

The unique solution of equation (3.1) is:

$$V_t = V \exp\left(\left(r - \delta - \frac{\sigma^2}{2} - \lambda \mathbb{E}_{\mathbb{Q}}\left[Z_1\right]\right)t + \sigma W_t\right) \prod_{k=1}^{N_t} (Z_k + 1) \tag{3.3}$$

where V is the value at time 0 (the initial firm's asset value).

In the case where Z_k's are valued in $]-1,\infty[$, the process $(V_t, t \geq 0)$ takes strictly positive values and

$$V_t = Ve^{X_t} \text{ where } X_t = \left(r - \delta - \frac{\sigma^2}{2} - \lambda \mathbb{E}_{\mathbb{Q}}\left[Z_1\right]\right) t + \sigma W_t + \sum_{k=1}^{N_t} \ln\left(Z_k + 1\right) \tag{3.4}$$

In order to simplify the formula and the modeling of jump sizes, we introduce the variables $Y_k = \ln\left(Z_k + 1\right)$, $k \geq 0$. These variables are i.i.d. random variables valued in $\mathbb{R}$ and

$$X_t = \mu t + \sigma W_t + \sum_{k=1}^{N_t} Y_k \text{ where } \mu = r - \delta - \frac{\sigma^2}{2} - \lambda\left(\mathbb{E}_{\mathbb{Q}}\left[e^{Y_1}\right] - 1\right) \tag{3.5}$$

The process X is a Lévy process, i.e., a process with stationary and independent increments. The Laplace exponent of X is the function $G(\cdot)$ such that:

$$\mathbb{E}_{\mathbb{Q}}\left[e^{\beta X_t}\right] = \exp\left\{G(\beta)t\right\} \tag{3.6}$$

for any t and any β with $\mathbb{E}_{\mathbb{Q}}\left[e^{\beta X_t}\right] < \infty$.

In our model, it is well known (see Cont and Tankov (2003)[4]) that

$$\begin{aligned} G\left(\beta\right) &= \frac{1}{2}\sigma^2\beta^2 + \left(r - \delta - \frac{\sigma^2}{2} - \lambda\left(\mathbb{E}_{\mathbb{Q}}\left[e^{Y_1}\right] - 1\right)\right)\beta + \lambda\left(\mathbb{E}_{\mathbb{Q}}\left(e^{\beta Y_1}\right) - 1\right) \\ &= \frac{1}{2}\sigma^2\beta^2 + \mu\beta + \lambda\left(\mathbb{E}_{\mathbb{Q}}\left(e^{\beta Y_1}\right) - 1\right) \end{aligned} \tag{3.7}$$

We now restrict our attention to the Double Exponential Jump Diffusion Model, developed by Kou (2002)[11]. In that model, the jump sizes Y_k's has an asymmetric double exponential distribution with density:

$$f\left(y\right) = p\,\eta_1 e^{-\eta_1 y}\,1_{\{y \geq 0\}} + q\,\eta_2 e^{\eta_2 y}\,1_{\{y<0\}},\ \eta_1 > 0, \eta_2 > 0 \tag{3.8}$$

where $p, q \geq 0$, $p + q = 1$, represent the probabilities of upward and downward jumps.[4]

In particular, for $-\eta_2 < \beta < \eta_1$, we obtain:

$$\mathbb{E}_{\mathbb{Q}}\left(e^{\beta Y_1}\right) = \int_{-\infty}^{\infty} e^{\beta y} f\left(y\right) dy = \frac{p\eta_1}{\eta_1 - \beta} + \frac{q\eta_2}{\eta_2 + \beta} < \infty$$

Note that for $\beta \notin\,]-\eta_2, \eta_1[$, $\mathbb{E}_{\mathbb{Q}}\left(e^{\beta Y_1}\right) = +\infty$.

We now assume that $\eta_1 > 1$, then $\mathbb{E}_{\mathbb{Q}}(e^{Y_1}) < +\infty$ and

$$\mathbb{E}_{\mathbb{Q}}(e^{Y_1}) = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1}$$

This condition implies that the average upward jump cannot exceed 100% which is reasonable in reality.

[4] In the case of only negative jumps $p = 0$, the double exponential jump diffusion reduces to a particular case studied in Hilberink and Rogers (2002)[8]. In the case of no jumps $\lambda = 0$, the model becomes the geometric Brownian motion process as modeled in Leland (1994b)[15].

Therefore, the Laplace exponent function is defined for $-\eta_2 < \beta < \eta_1$ from (3.7) as:

$$G(\beta) = \frac{1}{2}\sigma^2\beta^2 + \mu\beta + \lambda\left(\frac{p\eta_1}{\eta_1 - \beta} + \frac{q\eta_2}{\eta_2 + \beta} - 1\right) \tag{3.9}$$

$$\text{where } \mu = r - \delta - \frac{\sigma^2}{2} - \lambda\left(\frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1\right)$$

By abuse of notation, we also denote by $G(\cdot)$ the function defined as the right hand side of (3.9) on $]-\infty, -\eta_2[\,\cup\,]-\eta_2, \eta_1[\,\cup\,]\eta_1, +\infty[$. A study of the function $G(\cdot)$ establishes that for $\rho > 0$ the equation $G(\beta) = \rho$ has exactly four real roots: $\beta_{1,\rho}$, $\beta_{2,\rho}$, $-\beta_{3,\rho}$, $-\beta_{4,\rho}$ where $-\infty < -\beta_{4,\rho} < -\eta_2 < -\beta_{3,\rho} < 0 < \beta_{1,\rho} < \eta_1 < \beta_{2,\rho} < \infty$.

Double Exponential Jump Diffusion

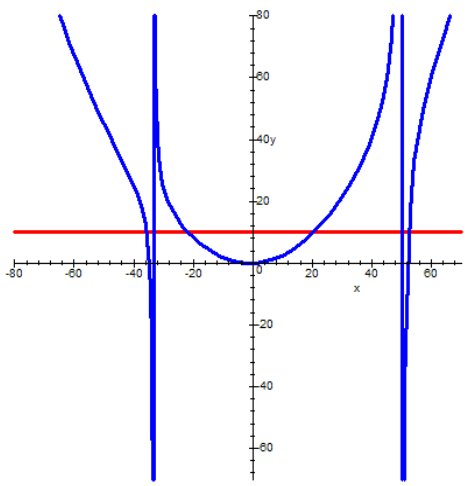


Figure 1: Graphic of $G(\beta) = \rho$ with $\eta_1 = 50$, $\eta_2 = 30$

Let $\tau_l(X)$ be the first passage time for the process X of the "downward barrier" $l < 0$

$$\tau_l(X) = \inf\{t \geq 0 : X_t \leq l\}$$

Using the results of Kou and Wang (2003)[12], Theorem 3.1 and Corollary 3.3, it is easy to

deduce the Laplace transforms of $\tau_l(X)$ as well as related results, for any $\rho > 0$:

$$\mathbb{E}_{\mathbb{Q}}\left[e^{-\rho\tau_l(X)}\right] = \frac{\eta_2 - \beta_{3,\rho}}{\eta_2}\frac{\beta_{4,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}}e^{l\beta_{3,\rho}} + \frac{\beta_{4,\rho} - \eta_2}{\eta_2}\frac{\beta_{3,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}}e^{l\beta_{4,\rho}} \tag{3.10}$$

$$\mathbb{E}_{\mathbb{Q}}\left[e^{-\rho\tau_l(X)}\mathbf{1}_{X_{\tau_l}-l<-y}\right] = e^{-\eta_2 y}\frac{\eta_2 - \beta_{3,\rho}}{\eta_2}\frac{\beta_{4,\rho} - \eta_2}{\beta_{4,\rho} - \beta_{3,\rho}}\left[e^{l\beta_{3,\rho}} - e^{l\beta_{4,\rho}}\right]; \qquad y > 0 \tag{3.11}$$

$$\mathbb{E}_{\mathbb{Q}}\left[e^{-\rho\tau_l(X)}\mathbf{1}_{X_{\tau_l}=l}\right] = \frac{\eta_2 - \beta_{3,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}}e^{l\beta_{3,\rho}} + \frac{\beta_{4,\rho} - \eta_2}{\beta_{4,\rho} - \beta_{3,\rho}}e^{l\beta_{4,\rho}} \tag{3.12}$$

$$\mathbb{E}_{\mathbb{Q}}\left[e^{\xi X_{\tau_l}-\rho\tau_l(X)}\mathbf{1}_{\tau_l(X)<\infty}\right] = e^{\xi l}\left[\frac{\eta_2 - \beta_{3,\rho}}{\beta_{4,\rho} - \beta_{3,\rho}}\frac{\beta_{4,\rho} + \xi}{\eta_2 + \xi}e^{l\beta_{3,\rho}} + \frac{\beta_{4,\rho} - \eta_2}{\beta_{4,\rho} - \beta_{3,\rho}}\frac{\beta_{3,\rho} + \xi}{\eta_2 + \xi}e^{l\beta_{4,\rho}}\right] \tag{3.13}$$

In the last equality, it is required that $\xi > 0$ for the quantity $\mathbb{E}_{\mathbb{Q}}\left[e^{\xi X_{\tau_l}}\right]$ to be finite. In what follows we shall apply the result for $\xi = 1$. Formula (3.10) is the expression of the Laplace transform of the first passage time of a downward barrier. Formula (3.11) expresses the Laplace transform of the first passage time and that the overshoot greater than $y > 0$. The next formula (3.12) is the expression of the Laplace transform of the first passage time and that no overshoot occurs at passage time. The last formula (3.13) is the expression of the Laplace transform of the pair first passage time and value at passage time and that the passage time is finite.

4. Debt, Firm, Equity Values in Double Exponential Jump Diffusion Process

In the double exponential jump diffusion, we have the following closed-form formulae of the debt value, firm value and equity value.

Proposition 4.1.

i) The debt value $D(V, L)$ is given by:

$$\begin{aligned} D(V,L) = {} & \frac{C + mP}{r + m} - \left(\frac{C + mP}{r + m}\frac{\beta_{4,r+m}}{\eta_2} - (1 - \hat{\alpha})L\frac{\beta_{4,r+m} + 1}{\eta_2 + 1}\right)\frac{\eta_2 - \beta_{3,r+m}}{\beta_{4,r+m} - \beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{3,r+m}} \\ & - \left(\frac{C + mP}{r + m}\frac{\beta_{3,r+m}}{\eta_2} - (1 - \hat{\alpha})L\frac{\beta_{3,r+m} + 1}{\eta_2 + 1}\right)\frac{\beta_{4,r+m} - \eta_2}{\beta_{4,r+m} - \beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{4,r+m}} \end{aligned} \tag{4.1}$$

ii) The firm value $v(V,L)$ is given by:

$$v(V,L) = V + \frac{\theta C}{r} - \left(\frac{\theta C}{r}\frac{\beta_{4,r}}{\eta_2} + \alpha L \frac{\beta_{4,r}+1}{\eta_2+1}\right)\frac{\eta_2-\beta_{3,r}}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{3,r}}$$
$$- \left(\frac{\theta C}{r}\frac{\beta_{3,r}}{\eta_2} + \alpha L \frac{\beta_{3,r}+1}{\eta_2+1}\right)\frac{\beta_{4,r}-\eta_2}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{4,r}} \tag{4.2}$$

ii) The equity value $E(V,L)$ is given by:

$$E(V,L) = v(V,L) - D(V,L) \tag{4.3}$$

⊡ Proof. The results of the proposition (4.1) is straightforward: Indeed, inserting the Laplace transforms (3.10) and (3.13) into the debt value defined in equation (2.5), we obtain:

$$D(V,L) = \frac{C+mP}{r+m}\left[1 - \frac{(\eta_2-\beta_{3,r+m})\beta_{4,r+m}}{\eta_2(\beta_{4,r+m}-\beta_{3,r+m})}e^{l\beta_{3,r+m}} - \frac{(\beta_{4,r+m}-\eta_2)\beta_{3,r+m}}{\eta_2(\beta_{4,r+m}-\beta_{3,r+m})}e^{l\beta_{4,r+m}}\right]$$
$$+ (1-\hat{\alpha})Ve^{l}\left[\frac{(\eta_2-\beta_{3,r+m})(\beta_{4,r+m}+1)}{(\beta_{4,r+m}-\beta_{3,r+m})(\eta_2+1)}e^{l\beta_{3,r+m}} + \frac{(\beta_{4,r+m}-\eta_2)(\beta_{3,r+m}+1)}{(\beta_{4,r+m}-\beta_{3,r+m})(\eta_2+1)}e^{l\beta_{4,r+m}}\right]$$

Replacing $l = ln\left(\frac{L}{V}\right)$ into this formula, some easy calculations lead to the formula of the debt value (4.1).

The same method applied to the firm value defined in equation (2.6) leads to (4.2). The equity value (4.3) is equal to the firm value minus the debt value.

⊡

Remark : In this model, the debt value (4.1), firm value (4.2) and equity value (4.3) are computed in closed form formulae. In the debt formula (4.1), replacing the roll-over rate m with the inverse average maturity $1/T$, we obtain the debt value as a function of the average maturity T.

The optimal default barrier L is now determined endogenously by maximizing the equity value, then we have the following proposition on the default barrier.

Proposition 4.2. *The optimal default barrier of the double exponential jump diffusion is given by:*

$$L = \frac{\frac{C+mP}{r+m}\beta_{3,r+m}\beta_{4,r+m} - \frac{\theta C}{r}\beta_{3,r}\beta_{4,r}}{1+\alpha\left[(\beta_{3,r}+1)(\beta_{4,r}+1)-1\right] + (1-\hat{\alpha})\left[(\beta_{3,r+m}+1)(\beta_{4,r+m}+1)-1\right]}\frac{\eta_2+1}{\eta_2} \tag{4.4}$$

If the APR is respected, i.e., $\gamma = 0$, then the optimal default barrier L is:

$$L = \frac{\frac{C+mP}{r+m}\beta_{3,r+m}\beta_{4,r+m} - \frac{\theta C}{r}\beta_{3,r}\beta_{4,r}}{\alpha(\beta_{3,r}+1)(\beta_{4,r}+1) + (1-\alpha)(\beta_{3,r+m}+1)(\beta_{4,r+m}+1)}\frac{\eta_2+1}{\eta_2}$$

If the debt is perpetual coupon C and the APR is respected, i.e., $m = 0$, $P = 0$ and $\gamma = 0$, then the expression of L is:

$$L = \frac{C(1-\theta)}{r}\frac{\eta_2+1}{\eta_2}\frac{\beta_{3,r}}{\beta_{3,\rho}+1}\frac{\beta_{4,r}}{\beta_{4,\rho}+1} \tag{4.5}$$

⊡ Proof. We need only to prove the proposition. The corollary is a consequence of the proposition with $\hat{\alpha} = \alpha$. The equality (4.5) is straightforward. Note that this optimal default barrier is exactly the barrier obtained in Leland (1994) for perpetual coupon-paying debt structure.

As mentioned previously, we obtain the optimal default barrier L from the condition (2.8) $\frac{\partial E}{\partial V}\mid_{V=L}= 0$. Here, the equity value E, given in the formula (4.3), is obtained by replacing v and D by their values given in (4.1) and (4.2). It follows that:

$$\begin{aligned}\frac{\partial E}{\partial V}(V,L) = 1 &+ \frac{\beta_{3,r}}{L}\left(\frac{\theta C}{r}\frac{\beta_{4,r}}{\eta_2} + \alpha L\frac{\beta_{4,r}+1}{\eta_2+1}\right)\frac{\eta_2-\beta_{3,r}}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{3,r}+1}\\ &+ \frac{\beta_{4,r}}{L}\left(\frac{\theta C}{r}\frac{\beta_{3,r}}{\eta_2} + \alpha L\frac{\beta_{3,r}+1}{\eta_2+1}\right)\frac{\beta_{4,r}-\eta_2}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{4,r}+1}\\ &- \frac{\beta_{3,r+m}}{L}\left(\frac{C+mP}{r+m}\frac{\beta_{4,r+m}}{\eta_2} - (1-\hat{\alpha})L\frac{\beta_{4,r+m}+1}{\eta_2+1}\right)\frac{\eta_2-\beta_{3,r+m}}{\beta_{4,r+m}-\beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{3,r+m}+1}\\ &- \frac{\beta_{4,r+m}}{L}\left(\frac{C+mP}{r+m}\frac{\beta_{3,r+m}}{\eta_2} - (1-\hat{\alpha})L\frac{\beta_{3,r+m}+1}{\eta_2+1}\right)\frac{\beta_{4,r+m}-\eta_2}{\beta_{4,r+m}-\beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{4,r+m}+1}\end{aligned}$$

Solving the equation

$$\frac{\partial E}{\partial V}\mid_{V=L}= \frac{\partial E}{\partial V}(V,L) = 0$$

in terms of L, we obtain the optimal default barrier L as follows:

$$L = \frac{\frac{C+mP}{r+m}\beta_{3,r+m}\beta_{4,r+m} - \frac{\theta C}{r}\beta_{3,r}\beta_{4,r}}{1+\alpha\left[(\beta_{3,r}+1)(\beta_{4,r}+1)-1\right]+(1-\hat{\alpha})\left[(\beta_{3,r+m}+1)(\beta_{4,r+m}+1)-1\right]}\frac{\eta_2+1}{\eta_2}$$

⊡

We can see immediately that the barrier L is a linear function of the coupon C. As consequence, the debt value and firm value are power functions of the coupon C.

All the results above were established under the assumption that the firm receives tax rebates until default time τ. We assume now that the firm receives tax rebates only when the value of the firm's assets exceeds some level V_T (for example $V_T = C/\delta$) and zero otherwise. Applying the procedures presented in the Annex A of Hilberink and Rogers(2002)[8], we obtain the expression for the optimal default barrier as follows:

$$L = \frac{\frac{C+mP}{r+m}\frac{\beta_{3,r+m}\beta_{4,r+m}}{\eta_2} - \frac{\theta C}{r}\frac{\beta_{3,r}\beta_{4,r}}{\eta_2}e^{-\beta_{1,r}\ln\left(\frac{V_T}{L}\right)}}{1+\alpha\left[\frac{(\beta_{3,r}+1)(\beta_{4,r}+1)}{\eta_2+1}-1\right]+(1-\hat{\alpha})\left[\frac{(\beta_{3,r+m}+1)(\beta_{4,r+m}+1)}{\eta_2+1}-1\right]} \tag{4.6}$$

where $\beta_{1,r}$ is the positive root ($\beta_{1,r} < \eta_1$) of the Laplace exponent equation $G(\beta) = r$.

5. The Debt Analysis

In the following, we present an analysis of the debt value, the yield spread value and the firm value as a function of the coupon and the leverage for different maturities.

For all the computations, we take the values of the parameters for the diffusion part as follows: the riskfree interest rate $r = 7.5\%$, the payout ratio $\delta = 7\%$, the volatility of the firm $\sigma = 20\%$. The other parameters are the tax rate $\theta = 0.35$, the reorganization costs $\alpha = 0.5$, the rate of the violation of the APR $\gamma = 0.5$ (i.e., $\hat{\alpha} = 0.75$), the firm's assets at time 0 is $V = 100$ and the coupon varies $C = (0, ..., 14)$. These values of parameters are those used by Leland (1994a)[14] and Hilberink and Rogers (2002)[8] for the graphs.

For the parameters of the jump part which is a compound Poisson process of double exponential distribution, we take the same parameters as those used in Kou and Wang (2003)[12]. The probability of upward (downward) jumps $p = 0.3$ ($q = 0.7$), the mean of upward (downward) jumps $1/\eta_1 = 0.02$ ($1/\eta_2 = 0.03$) and the intensity of the jump process $\lambda = 3$.

For the parameter of the maturity, we take m as equal to 0; 0.05; 0.1; 0.2; 1; and 4 respectively which is in turn equivalent to the "average" maturity $T = 1/m$ (if no default occurs) when maturity T equals the perpetual (consol); 20 years; 10 years; 5 years; 1 year and 0.25 years (3 months) respectively.

From equations (4.4) and (4.6), the optimal barrier L can be substituted into equation (4.1) as the debt value and we obtain the closed form solution for the total debt value D, given an arbitrage coupon C and principal P.

However, when the debt is first issued, there is a further constraint in relating the market value D, coupon C, and principal P. That is, coupon C is set so market value D equals principal value P (the debt is sold at par value). If V is the asset value when the debt is first issued at time 0, this constraint requires that C be the smallest solution to the equation $D(V; L, C, P) = P$. Using the formulae of (4.4), (4.6) and (4.1), we can find the solution of this equation numerically.

The "roll-over" debt structure in this analysis means that at every moment only a small amount of debt ($mPdt$) is issued. However, since all outstanding debt from previous dates are identical in value, it can be assumed that the total debt is issued at the current moment, and thereafter rolled over at rate m.

Figure 1 plots the value of newly-issued debt D (as in formula 4.1) as a function of the coupon C for different maturities $T = 1/m$, given that principal P and debt value D coincide at current value $V = 100$. Figure 2 plots the same value of the newly-issued debt D as functions of the leverage (D/v) and coupon C.

The leverage is the proportion of borrowed money (the debt) in the total assets of the firm (the firm value). The ratio of leverage $Lev = \frac{D(V,C)}{v(V,C)}$ is obtained from the formulae of the debt value (4.1) and firm value (4.2), i.e.:

$$\begin{aligned} D(V,C) \quad = \quad & \frac{C+mP}{r+m} - \left(\frac{C+mP}{r+m}\frac{\beta_{4,r+m}}{\eta_2} - (1-\hat{\alpha})\, L\frac{\beta_{4,r+m}+1}{\eta_2+1}\right)\frac{\eta_2-\beta_{3,r+m}}{\beta_{4,r+m}-\beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{3,r+m}} \\ & - \left(\frac{C+mP}{r+m}\frac{\beta_{3,r+m}}{\eta_2} - (1-\hat{\alpha})\, L\frac{\beta_{3,r+m}+1}{\eta_2+1}\right)\frac{\beta_{4,r+m}-\eta_2}{\beta_{4,r+m}-\beta_{3,r+m}}\left(\frac{L}{V}\right)^{\beta_{4,r+m}} \end{aligned}$$

and

$$
\begin{aligned}
v(V,C) &= V + \frac{\theta C}{r} - \left(\frac{\theta C}{r}\frac{\beta_{4,r}}{\eta_2} + \alpha L \frac{\beta_{4,r}+1}{\eta_2+1}\right)\frac{\eta_2-\beta_{3,r}}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{3,r}} \\
&\quad - \left(\frac{\theta C}{r}\frac{\beta_{3,r}}{\eta_2} + \alpha L \frac{\beta_{3,r}+1}{\eta_2+1}\right)\frac{\beta_{4,r}-\eta_2}{\beta_{4,r}-\beta_{3,r}}\left(\frac{L}{V}\right)^{\beta_{4,r}}
\end{aligned}
$$

The leverage by definition, measures the partition of the funds used by the firm between debt and equity. The leverage represents in fact, the percentage of firm's assets financed by the debt. The leverage value is hereby defined by the proportion of debt value over firm value. A firm that is highly leveraged may increase its bankruptcy probability and may also decrease its opportunities to find new lenders in the future. The leverage can increase the shareholders' return on their investment and often there are tax benefits from the coupon payment associated with debt. Here, in our model, the leverage ($Lev = D/v$) is a function of the coupon C. The higher the coupon C, the higher the leverage Lev for a certain level.

Figure 1: Debt Value as a Function of the Coupon

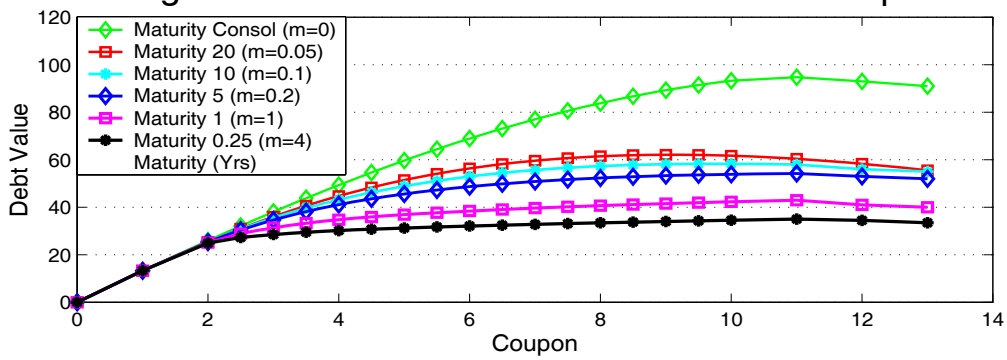
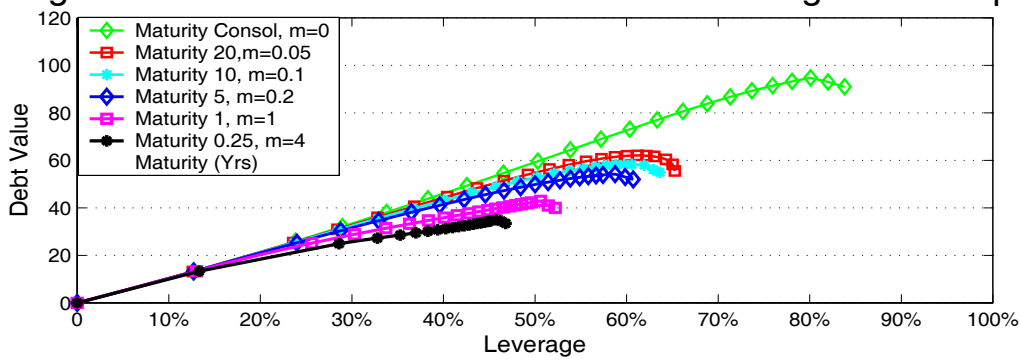


Figure 2: Debt Value as Functions of the Leverage and Coupon



The riskfree interest rate is $r = 7.5\%$, the payout ratio $\delta = 7\%$, the volatility of the firm $\sigma = 20\%$, the tax rate $\theta = 0.35$, the reorganization costs $\alpha = 0.5$, the rate of the violation of

the APR $\gamma = 0.5$, the probability of upward (downward) jumps $p = 0.3$ ($q = 0.7$), the mean of upward (downward) jumps $1/\eta_1 = 0.02$ ($1/\eta_2 = 0.03$) and the intensity of the jump process $\lambda = 3$, the firm's assets at time 0 is $V = 100$.

From the Figure 1 and 2, it is observed that the debt capacity ($Dmax$) is the maximal value of the debt value curve as a function of the leverage or coupon. For example, the maximal debt capacity is around 61 for a maturity of 20 years when the coupon $C = 8$ and the leverage $Lev = 63\%$. The debt capacity is smaller for shorter maturities. The maximal debt is at a very different level of leverage, from 45% to 80% for debt of different maturities, from $T = 0.25$ of a year (3 months) to infinity. It can be shown that debt capacity falls as volatility σ, bankruptcy costs α, and/or the intensity λ rise. The Figure 2 has almost the same form as the Figure 1 in Leland (1994b)[15] and the Figure 2 in Hilberink and Rogers (2002)[8]. However, the behavior of the debt capacity is very different. In their models, the maximal debt is at approximately the same level of leverage (about 75-80%) for debt of different maturities.

The yield spread of newly issued debt is defined as $C/P - r$. The coupon C is set such that the market value of debt D equals principal value P, so the coupon C has to satisfy the condition of the right-hand side of the Debt Value formula (2.5) which equals P, i.e.:

$$\frac{C + mP}{r + m}\mathbb{E}_{\mathbb{Q}}\left[1 - e^{-(r+m)\tau}\right] + (1 - \hat{\alpha})\, V\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - (r+m)\tau}\mathbf{1}_{\tau<\infty}\right] = P$$

When we arrange the terms as follows, we obtain the value of C:

$$C = \frac{P\left(r + m - m\mathbb{E}_{\mathbb{Q}}\left[1 - e^{-(r+m)\tau}\right] - (1 - \hat{\alpha})\,\frac{V}{P}\,(r + m)\,\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - (r+m)\tau}\mathbf{1}_{\tau<\infty}\right]\right)}{\mathbb{E}_{\mathbb{Q}}\left[1 - e^{-(r+m)\tau}\right]}$$

Hence, the yield spread is:

$$YS = \frac{C}{P} - r = \frac{r + m}{P}\,\frac{P\mathbb{E}_{\mathbb{Q}}\left[e^{-(r+m)\tau}\right] - (1 - \hat{\alpha})\, V\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - (r+m)\tau}\mathbf{1}_{\tau<\infty}\right]}{\mathbb{E}_{\mathbb{Q}}\left[1 - e^{-(r+m)\tau}\right]} \tag{5.1}$$

This yield spread is a closed form formula as all the terms in expectations are known ($\mathbb{E}_{\mathbb{Q}}\left[e^{-(r+m)\tau}\right]$ and $\mathbb{E}_{\mathbb{Q}}\left[e^{X_\tau - (r+m)\tau}\mathbf{1}_{\tau<\infty}\right]$ are as in (3.10) and 3.13), respectively).

Figure 3: Yield Spreads as a Function of the Coupon

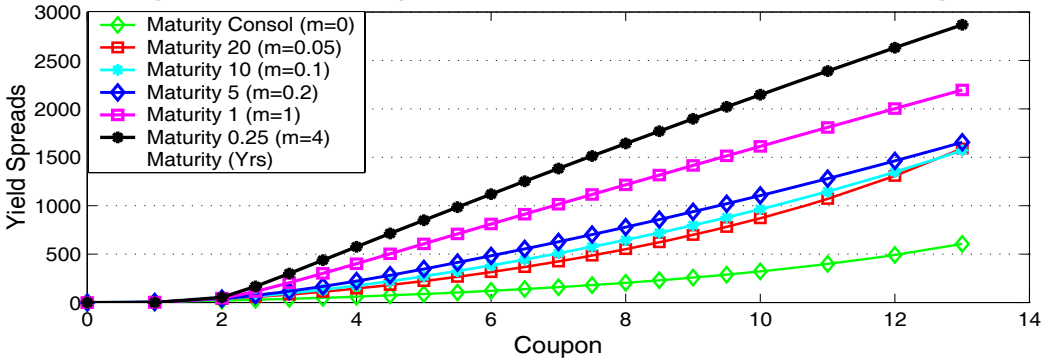
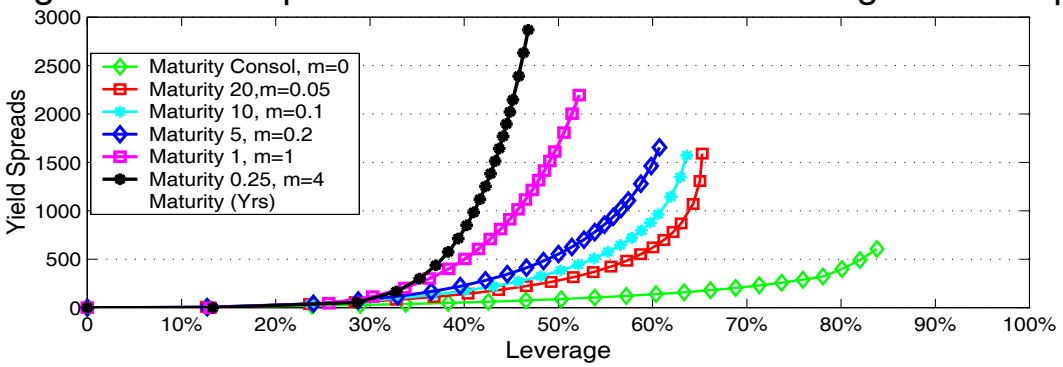
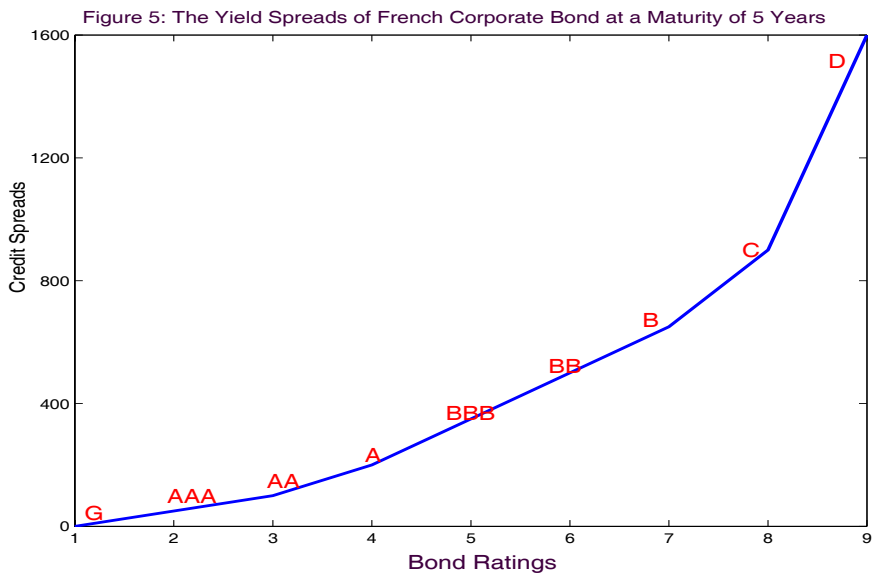


Figure 4: Yield Spreads as Functions of the Leverage and Coupon



The riskfree interest rate $r = 7.5\%$, the payout ratio $\delta = 7\%$, the volatility of the firm $\sigma = 20\%$, the tax rate $\theta = 0.35$, the reorganization costs $\alpha = 0.5$, the rate of the violation of the APR $\gamma = 0.5$, the probability of upward (downward) jumps $p = 0.3$ ($q = 0.7$), the mean of upward (downward) jumps $1/\eta_1 = 0.02$ ($1/\eta_2 = 0.03$) and the intensity of the jump process $\lambda = 3$, the firm's assets at time 0 is $V = 100$.

The Figure 3 and 4 plot the yield spreads (as in 5.1) of newly issued debt as a function of the coupon and the leverage (respectively), for an alternative maturity $T = 1/m$. For higher leverage levels yield spreads are higher, but the propensity for yield spread to increase is smaller for higher maturities. The yield spreads for a short term debt of one year are higher than the yield spread for a longer maturity of five years. This phenomena is also observed in Leland (199b) and Hilberink and Rogers (2002). For higher leverage levels yield spreads are higher, but they decrease as maturity increases beyond 0.5 years. The yield spreads generated in the double exponential jump diffusion are higher in comparison with the geometric Brownian motion of Leland (1994b)[15] for shorter maturity debt. The yield spreads generated are almost the same at the following yield spread curve of the French corporate bonds at maturity of 5 years (i.e., $m = 2$):



The Figure 5 plot the yield spreads of the French Corporate Bonds varies from the ratings AAA, AA, A, BBB, BB, B, C and D (Standard and Poor's class order) in 2002, by the Datastream Database.

We can also observe in formula (5.1) that the yield spreads are functions of the bankruptcy costs, violation of APR, firm's asset risk, and riskfree rate. The yield spread is an increasing function of the riskfree rate r, the inverse maturity m, the bankruptcy costs α, and the violation of APR γ.

Figure 6 and Figure 7 plot the firm value (as in formula 4.2) as a function of the coupon and the leverage for alternative maturities from 3 months, to 20 years, and to infinity. Due to scale, the graphs for short maturities (3 months and 1 year) seem not to be regular. However, they have the same forms as longer maturities, for example 10 year maturity.

Figure 6: Firm Value as a Function of the Coupon

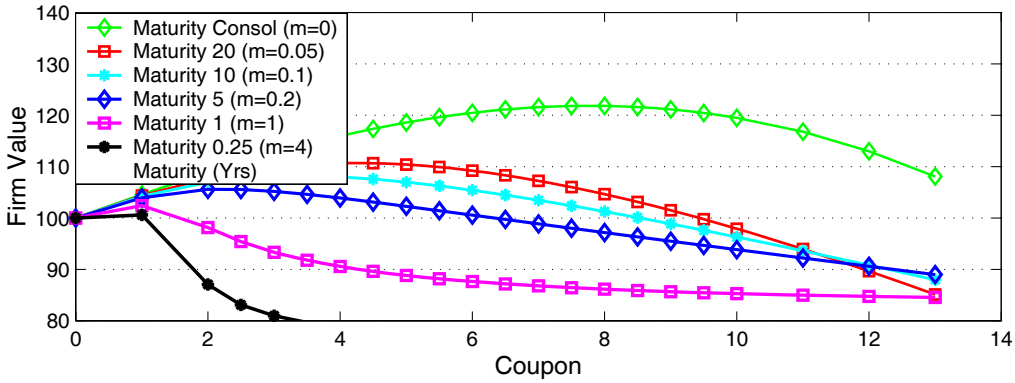
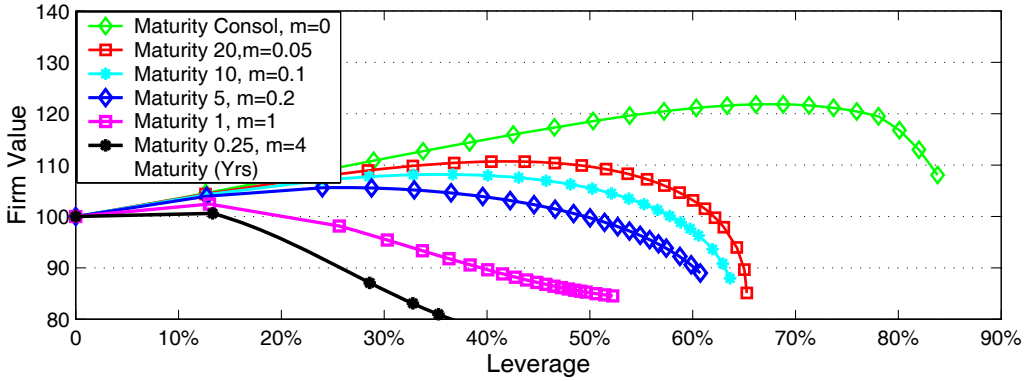


Figure 7: Firm Value as Functions of the Leverage and Coupon



The riskfree interest rate $r = 7.5\%$, the payout ratio $\delta = 7\%$, the volatility of the firm $\sigma = 20\%$, the tax rate $\theta = 0.35$, the reorganization costs $\alpha = 0.5$, the rate of the violation of the APR $\gamma = 0.5$, the probability of upward (downward) jumps $p = 0.3$ ($q = 0.7$), the mean of upward (downward) jumps $1/\eta_1 = 0.02$ ($1/\eta_2 = 0.03$) and the intensity of the jump process $\lambda = 3$, the firm's assets at time 0 is $V = 100$.

From the Figure 6 and 7, the optimal firm value is the maximal value of the firm's value curve. The optimal firm value is higher for higher maturity and also for higher leverage. The optimal firm value in these figures Figure 6 and 7 capacity ($Dmax$) is the maximal value of the debt value curve as a function of the leverage or coupon. For example, the optimal firm value for a maturity of 20 years is around 110 when coupon $C = 5$ and leverage $Lev = 47\%$. The optimal firm value is also smaller for shorter maturities. Therefore the leverage ratio which maximizes firm value for alternative choices of debt maturity is lower for shorter maturities.

6. Conclusion

We propose a structural model of a perpetual roll-over debt structure with regular repayments and renewals of principal and coupon in a framework where the firm's asset value follows a double exponential jump diffusion process. As a result of this special debt structure, we are able to examine coupon paying bonds with arbitrary maturity, while remaining in a time homogeneous environment. As consequence, we can take an analysis of debt value and yield spreads as functions of leverage and arbitrary average maturity.

We consider a framework with firm risk, riskfree interest rates, payout rates, tax benefit of coupon payments (at a certain value of the firm), reorganization costs, and violation of the APR. We observe that the values of debt, equity and firm and the optimal default barrier depend on the Laplace transforms of the first passage time and its firm value. We obtain the Laplace transforms of a downward barrier from the results derived by Kou and Wang (2003)[12] for the passage time of a upward barrier. Hence, we obtain values for the equity, the debt and the firm in closed form formulae. To the best of our knowledge, only the models of Leland (1994a, b)[14] and [15], Leland and Toft (1996)[16] and others who used the geometric Brownian motion process and our model with the double exponential jump diffusion process have obtained closed form formulae for the debt value, equity value, firm value as well as for the optimal default barrier in the structural approach of credit risk with endogenous default barrier.

Analysis of comparative results on the debt value, the yield spreads and the firm value as function of coupon and leverage were done. The double exponential jump diffusion generates yield spreads much higher than the model of Leland (1994b)[15] and Leland and Toft (1996)[16] for the same parameters. It highlights also that the yield spreads in the double exponential model are different from zero when the maturity is zero (or court term) by the influence of jumps.

This approach presented here has, at the same time, the advantages of tractability, analytic and closed form formulae (as in the Black Scholes approach) and the reality demanded of a model. This model can be an alternative solution to the reference model of geometric Brownian motion for risk management and default claims evaluations.

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